Design and Analysis of a Classical Angular Acceleration Apparatus

Azaka Onyemazuwa Andrew[1], Nwadike Emmanuel Chinagorom[2], Umeobi Ijeoma Happiness[3], Ezenwa Obiora Nnaemeka[4]

[1,2,3,4]Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka

[1]oa.azaka@unizik.edu.ng

***Abstract*– In the design of machine component parts, it is a good practice to determine the cause of failure of all designed mechanical components and resolve the cause(s) of the failure in the final design. This is to prevent frequent failure of the components parts and increase the performance of the structure and machine life. It also enables, quality assurance engineers, reliability engineers, design engineers, etc. in the development of quality and efficient products. In this study, the static and dynamic analyses of mechanical and structural parts of a classical Angular Acceleration Apparatus which will be used in performing classical experiments in mechanics (kinematics and dynamics) of machines was designed and constructed. These types of analyses are performed with the purpose of estimating the behavior of the mechanical parts under specific operational conditions. The optimum safe and economical design of the machine shaft (s) were ascertained. The 3D model of the shafts was produced with inventor using absolute coordinates. The results of the commercial finite element analysis (FEA) were reported on the shafts stress, displacement, and safety factor. The bearing strength check was seen to be positive when static equivalent load P_O and dynamic equivalent load P were 1N each. From the simulation result, since the von mises stress value is not up to half of the yield strength of the material with a factor of safety of 15, the design of the shafts is considered safe.**

***Keywords*– Design, Static, Dynamic and Angular Acceleration**

I. INTRODUCTION

Modern design of classical experimental apparatus is an essential aid in the teaching and demonstration to undergraduate students of engineering the principles and applications of established scientific theories. In this work, an Angular Acceleration Apparatus which will be used in performing classical experiments in mechanics (kinematics and dynamics) of machines was designed and constructed. In modern design, static and dynamic analyses of mechanical and structural parts play a very important role in determining the efficiency and precision of design work performed by any engineer to aid in the accuracy of construction and fabrication of mechanical parts and equipment. This analysis enables, quality assurance engineers, reliability engineers, design engineers, etc. in the development of quality and efficient products. These types of analyses are performed with the purpose of estimating the behavior of the mechanical parts under specific operational conditions. In the past, these analyses were performed through tests that were carried out on prototypes of the product, which meant an increase on the time needed to develop the product, as well as an increase on the related costs. However, technological advancements have allowed developers and engineers to depend upon computational tools to carry out these tests virtually via the Finite Elements Method (FEM).

Knowing fully well that the study of the angular acceleration of a cylindrical solid body caused by the action of a falling mass allows a number of simple experiments to be performed relating to the kinematics and dynamics of rotating systems, the need for this apparatus in the mechanics laboratory in institutions of higher learning cannot be over emphasized. Angular acceleration apparatus is used in measuring the centripetal force of an object of a mass which moves in a uniform circular motion. Understanding the concept and applications of Centripetal force, would enable young scientist and engineers to understand the applications of circular motion in planetary bodies around the sun, motion of satellites, the motion in a centrifuge, the motion of a vehicle at a bend, etc. Centripetal force is usually measured by inputting the various variable quantities needed and using the appropriate formula needed for determining centripetal force. The aim of this project is to design and construct a laboratory scale angular acceleration apparatus for use in mechanics laboratory to aid students in understanding the principles of uniform angular acceleration and its applications. This project would be very useful to institutions of higher learning as it could be used for illustrations to students to aid their learning experience and differentiate between these forces seeing that centrifugal force is an apparent force while centripetal force is an actual force. This laboratory apparatus no doubt, will expose students to the applications and understanding of the fundamental concepts and principles of mechanics, principle of transmissibility, parallelogram law of forces, mechanics of rigid bodies, planar kinetics of rigid bodies, translational motion, impulse and momentum, principle of work and energy, rotation about a fixed axis and general plane motion, eccentric impact, three-dimensional kinetics of a rigid body, three dimensional kinematics of a rigid body, moments and products of inertia, gyroscopic motion, planar kinematics of rigid bodies, relationship between linear and angular variables, absolute motion analysis, relative motion using translating axes, instantaneous center of zero velocity, uniform circular motion, applications of centripetal force. The experimental capability of the apparatus includes but not limited to; Newton's second law applied to a rotational system, Angular acceleration proportional to torque, Angular acceleration inversely proportional to moment of inertia, Rate of change of moment of momentum proportional to applied torque, change in momentum proportional to impulse, interchange between potential energy and kinetic energy, moment of inertia, moment of inertia proportional to mass, confirmation of parallel axis theorem applied to moment of inertia etc. Hence, there is a compelling need for this apparatus in the mechanics laboratory of the Department of Mechanical Engineering in institutions of higher learning.

II. MATERIALS AND METHODS

A) Description of the Apparatus

The apparatus consists of a base (500×200×30mm) manufactured from plywood. The base is fitted with stoppers that offer support to the entire apparatus to prevent unnecessary vibration and movement. The crossbar with the counter weight at its end can be moved to change the position of the rotating mass. The steel ruler attached to the wooden board base serves as a means of measuring the radius of the circular path of the rotating mass. In this angular acceleration apparatus, the centripetal force is provided by the spring. When the spring is attached to the rotating mass M_R, it gets pulled in towards the fixed steel rod; to bring the rotating mass back above the ruler so that it is perpendicular to it, the string is passed over the pulley and enough mass is added at the end of the string to bring the rotating mass back over the desired radius.

The weight $m_h g$ of the hanging mass m_h just balances the elastic force of the spring when the rotating mass rests above the desired radius. The spring is stretched by the same amount as that of the rotating mass in a path with the same radius. The magnitude of the two forces F_c and $m_h g$ should be in close agreement since both forces produce the same elongation of the spring.

B) Design and construction method

Solid works software, Logan's spring stiffness matrix model, Autodesk Inventor, and Arc/Gas welding process were employed in the design/construction and analyses of the apparatus. Solid works software was used for the engineering design, spring stiffness matrix model was used for calculating and determining the stiffness of the spring used and also the load bearing capacity of the spring before deformation takes place, Autodesk Inventor was used in the static and dynamic stress analysis on the steel components and the roller bearing. Arc and Gas welding process was used for the construction of the apparatus.

C) Experimental validation of the apparatus

Text running the efficiency of the apparatus involves experimentally verifying the linear speed of the object(s) and differentiating it from the angular speed as it undergoes uniform circular motion. The determination of the following parameters are necessary:

1. The radius of the circular motion formed by the object in rotation.
2. The mass of the object.
3. The total time of rotation.
4. The number of rotations.
5. The velocity of each mass during rotation. $v = {}^{2\pi r}\!/_{T}$
6. The centripetal force. $F_c = m\, {}^{v^2}\!/_{r}$

D) Material selection and specification

Good material selection was undertaken to reduce vibration and wear, ease of operation during experimentation and quick replacement of parts for efficient demonstration.

1. Ply wooden board (500×200×30mm)
2. Assorted steel weights
3. Steel Cross bar (400mm)
4. Steel Pulley (35^0)
5. Steel Meter rule (300mm)
6. Steel spring
7. Steel fixed rod (400mm)
8. Rolling bearing (20mm)

E) Construction process of the apparatus

After purchasing the materials needed for the construction of the apparatus, the wooden base is cut into a 500×200 (mm) dimension and marked out for the insertion of the bearing. The metal rods are cut into their respective lengths. The 400mm rod is welded to the bearing and inserted into the wooden base. A hole is drilled at the top of this rod to enable a smaller threaded rod of 300mm which is to carry the rotating mass and the counter mass, to be inserted into it. A rivet is threaded and screwed in place to prevent the rod from falling out of place. Another smaller rod of 250 mm is placed and fixed to the wooden board in front of this 400mm rod; it is bent at an angle of 35° for the placement of the pulley. The various masses were turned on a lathe machine for smoothness and aesthetics. The counter mass is threaded and screwed in place. A thin rope is attached to the top of the hung rotating mass and a spring is attached at its side. The rods are spray painted an aluminum color to give it a nice look and the meter rule is fixed to the wooden base in between the two rods.

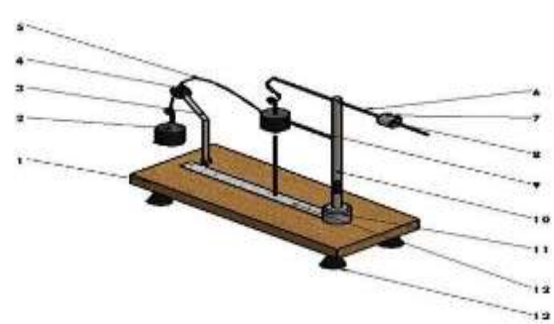


Fig. 1: Schematics of the Angular Acceleration Apparatus

Table I: Components of the apparatus

1	Wooden board	8	Counter mass
2	Hanging mass	9	Spring
3	Pulley rod	10	Steel rod
4	Pulley	11	Roller bearing
5	Rope	12	Steel rule
6	Cross bar	13	Stand
7	Screw		

F) Description of the apparatus

The bottom of the base is fitted with stoppers that offer support to the entire apparatus to prevent unnecessary vibration and movement. The crossbar with the counter weight at its end can be moved to change the position of the rotating mass. The steel ruler attached to the wooden board base serves as a means of measuring the radius of the circular path of the rotating mass. In this angular acceleration apparatus, the centripetal force is provided by the spring. When the spring is attached to the rotating mass M_R, it gets pulled in towards the fixed steel rod; to bring the rotating mass back above the ruler so that it is perpendicular to it, the string is passed over the pulley and enough mass is added at the end of the string to bring the rotating mass back over the desired radius.

The weight $m_h g$ of the hanging mass m_h just balances the elastic force of the spring when the rotating mass rests above the desired radius. The spring is stretched by the same amount as that of the rotating mass in a path with the same radius. The magnitude of the two forces F_c and $m_h g$ should be in close agreement since both forces produce the same elongation of the spring.

G) Derivation of the stiffness matrix for a spring element

A stiffness matrix can be defined as a representation of a system of linear equations that must be solved in order to ascertain an approximate solution to the differential equation. As one of the methods of structural analysis, which is the determination of the effects of loads on physical structures and their components, the direct stiffness method which is also referred to as the stiffness matrix method or displacement method, is a matrix method that makes use of the stiffness relations of the member for computing member forces and displacements in structures.

The direct stiffness method is the most common implementation of the finite element method (FEM). In applying the method, the system must be modeled as a set of simpler, idealized elements interconnected at the nodes. The material stiffness properties of these elements are then compiled into a single matrix equation through matrix mathematics which governs the behavior of the entire idealized structure. The structure's unknown displacements and forces can then be determined by solving the equation.

The direct stiffness method using logan's eight finite element method:

When trying to derive the matrix of an element, the displacements of key points (node points) are to be treated as state variables. A state variable is a set of values that define the state of the entire element. As stated earlier, the stiffness matrix relates forces acting at the nodes to the displacements of the nodes. A generalized force is either of a force or moment while a generalized displacement is either a translation or a rotation.
The state of the structural system is defined by a matrix of displacements (generalized displacements),

$$D = \left\{d_{1x} d_{1y} \theta_1 d_{2x} \dots d_{Ny} \theta_N\right\}^T \quad (1)$$

(Global or assembled displacement matrix)

The external factors acting on the system are given by a force (generalized force) matrix, $\underline{F}$.

$$F = \left\{f_{1x} f_{1y} m_{1z} f_{2x} \dots f_{Ny} m_{Nz}\right\}^T \quad (2)$$

(Global or assembled force matrix)

The quantities are related by a stiffness matrix, $\underline{K}$ by the global or assembled system of equations:

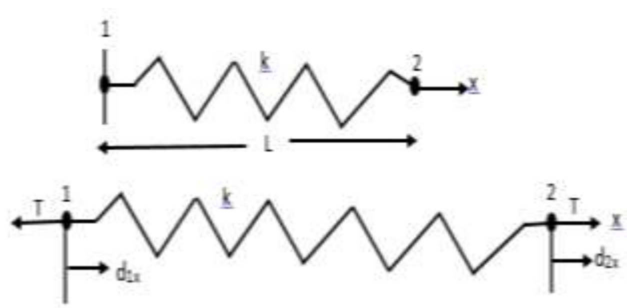
$$F = KD \quad (3)$$

$$Where\ K = \begin{matrix} k_{11} & k_{12} \dots & k_{1N} \\ k_{21} & k_{22} \dots & k_{2N} \\ k_{N1} & k_{N2} \dots & k_{NN} \end{matrix} \quad (4)$$

When deriving the stiffness matrix for a spring element, the first line element put into consideration is an ideal linear spring. The spring obeys Hooke's Law and resists forces only in the direction of the spring

The eight steps of Logan's finite element method are as follows:

Set the element type:



$|f1x| = T \qquad |f1x| = -T$

$|f2x| = T \qquad |f1x| = +T$

Fig. 2: Linear spring subjected to tensile forces

L is the length, k is the spring stiffness and T is the tension in the spring (compression)

It is worth noting that the spring can only deform in the x direction.

Then tension in the spring can be seen as a type of force which prevents the spring from undergoing deformation.

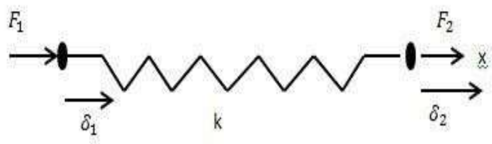


Fig. 3: Linear spring subjected to tensile forces

The spring stiffness matrix can be derived as:

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{Bmatrix} k & -k \\ -k & k \end{Bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (5)$$

The equation can be generalized for a linear spring as:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} k_{11} k_{12} \\ k_{21} k_{22} \end{Bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \quad (6)$$

Where k_{ij} is the force in the i^{th} node induced by a unit displacement in the j^{th} node.

H) Component Stress Analysis

Static stress analysis was performed on the threaded shaft, pulley shaft and main shaft assembly of the equipment. Maximum mass of 100g (equivalent of 0.981N) is used to analyze the various stresses in the components. The material is assumed to be steel.

Fig. 4: Threaded shaft

Fig. 5: Pulley shaft

Fig. 6: Main shaft Subassembly

I) Stress Analysis Report for the Shaft Housing the Pulley

The basic properties of the shaft housing the pulley are shown in Table II. However, the physical values could be different from physical values used by FEA as reported below. In carrying out the finite element analysis, single point static analysis was done on the pulley shaft on a fixed constraint condition with the elimination of rigid body modes. The mesh settings, materials and operating conditions are shown in Table III to Table V. The analysis was done assuming a fixed constraint condition. The finite element meshes of the shaft in the structural model were 992 elements with 1832 nodes.

Table II: Physical properties of the pulley shaft

Material	Steel
Density	1 g/cm^3
Mass	0.0331831 kg
Area	10475.6 mm^2
Volume	33183.1 mm^3
Center of Gravity	x=16.2028 mm y=-5.37849 mm z=0 mm

Table III: Mesh settings for the pulley shaft

Avg. Element Size (fraction of model diameter)	0.1
Min. Element Size (fraction of avg. size)	0.2
Grading Factor	1.5
Max. Turn Angle	60 deg
Create Curved Mesh Elements	Yes

Table IV: Pulley shaft material properties

Name	Steel	
General	Mass Density	7.85 g/cm^3
	Yield Strength	207 MPa
	Ultimate Tensile Strength	345 MPa
Stress	Young's Modulus	210 GPa
	Poisson's Ratio	0.3 ul
	Shear Modulus	80.7692 GPa
Stress Thermal	Expansion Coefficient	0.000012 ul/c
	Thermal Conductivity	56 W/(m K)
	Specific Heat	460 J/(kg c)
Part Name(s)	Pulley Shaft	

Table V: Pulley shaft Operating conditions

Load Type	Remote Force
Magnitude	0.981 N
Vector X	-0.694 N
Vector Y	-0.694 N
Remote Point X	150.000 mm
Remote Point Y	95.000 mm
Remote Point Z	150.000 mm

Fig. 7: Pulley shaft showing the selected face of the applied force

J) *Stress Analysis Report for the threaded shaft and main shaft sub assembly*

The main shaft assembly is the principal component that transmits circular motion from the bearing derived from the motor. The physical properties are shown in Table VI. In carrying out the static analysis, rigid body modes detected were considered, and stresses across contact surfaces were not separated. Motion load analysis were also neglected. The finite element meshes of the threaded shaft and main shaft sub assembly were of 2073 elements with 4016 nodes. The mesh settings, material properties and operating conditions (remote & gravity forces) showing the selected faces are shown in Table VII to Table IX.

Table VI: Physical properties of the threaded shaft and main shaft assembly.

Mass	0.262693 kg
Area	44144.9 mm^2
Volume	155399 mm^3
Center of Gravity	x=27.3169 mm y=9.02729 mm z=206.665 mm

Table VII: Mesh settings for static simulation of the threaded shaft and main shaft sub assembly

Avg. Element Size (fraction of model diameter)	0.1
Min. Element Size (fraction of avg. size)	0.2
Grading Factor	1.5
Max. Turn Angle	60 deg
Create Curved Mesh Elements	No
Use part based measure for Assembly mesh	Yes

Table VIII: Mechanical properties of the shaft material

Name	Steel	
General	Mass Density	7.85 g/cm^3
	Yield Strength	207 MPa
	Ultimate Tensile Strength	345 MPa
Stress	Young's Modulus	210 GPa
	Poisson's Ratio	0.3 ul
	Shear Modulus	80.7692 GPa
Stress Thermal	Expansion Coefficient	0.000012 ul/c
	Thermal Conductivity	56 W/(m K)
	Specific Heat	460 J/(kg c)
Part Name(s)	bearing shaft screw shaft	

Table IX: Operating conditions showing gravity & remote forces 1 and 2 for static simulation of the threaded shaft and main shaft sub assembly

Load Type	Remote Force 1	Remote Force 2	Gravity
Magnitude	0.981 N	0.981 N	9810.000 mm/s^2
Vector X	0.000 N	0.000 N	0.000 mm/s^2
Vector Y	0.000 N	0.000 N	0.000 mm/s^2
Vector Z	0.981 N	0.981 N	9810.000 mm/s^2
Remote Point X	165.000 mm	-80.000 mm	--
Remote Point Y	0.000 mm	0.000 mm	--
Remote Point Z	-60.000 mm	-60.000 mm	--

Fig. 8: Threaded shaft and main shaft sub assembly showing the selected face of the applied remote force 1

Fig. 9: Threaded shaft and main shaft sub assembly showing the selected face of the applied remote force 2

Fig. 10: Threaded shaft and main shaft sub assembly showing the selected face of the applied gravity force during simulation

K) Selection of ball bearing

The rolling ball bearing of type 70000AC (46304) which is fitted on the base and connected to the threaded shaft and main shaft sub assembly was technically selected to avoid failure of the apparatus. Bearing radial and axial loads, including speed of rotation and the required static safety factor were considered. ANSI/AFBMA 9-1990 (ISO 281-1990) calculation method and lubricant (grease) frictional factor μ of 0.0015ul was used in the bearing life calculation. The Loads acting on the bearing, bearing parameters and bearing life calculation are reported in Table X to Table XII.

Table X: Loads acting on the bearing

Bearing radial load	F_r	1 N
Bearing axial load	F_a	1 N
Speed	n	15 rpm
Required static safety factor	s_0	2.0 ul

Table XI: Bearing parameters

Designation		Rolling bearing GB/T 292-2007 Type 70000AC (46304)
Bearing inside diameter	d	20.000 mm
Bearing outside diameter	D	52.000 mm
Bearing width	B	15.000 mm
Nominal contact angle of the bearing	α	25 deg
Basic dynamic load rating	C	15000 N
Basic static load rating	C_0	13000 N
Dynamic radial load Factor	X	0.60 ul / 0.60 ul
Dynamic axial load Factor	Y	0.50 ul / 0.50 ul
Limit value of F_a/F_r	e	0.40 ul
Static radial load Factor	X_0	0.60 ul
Static axial load Factor	Y_0	0.50 ul
Limiting speed lubrication grease	nLim1	0 rpm
Limiting speed lubrication oil	nLim2	0 rpm

Table XII: Bearing Life Calculation

Calculation Method		ANSI/AFBMA 9-1990 (ISO 281-1990)
Required rating life	Lreq	10000 hr
Required reliability	Rreq	90 ul
Life adjustment factor for special bearing properties	a_2	1.00 ul
Life adjustment factor for operating conditions	a_3	1.00 ul
Working temperature	T	100 c
Factor of Additional Forces	f_d	1.00 ul

III. RESULTS

A) Results of the Reaction Force and Moment on Constraints Stress Analysis for the Shaft Housing the Pulley

A force of 0.981N was used to run a simulation study on the shaft above, one face of the shaft is fixed while the above-mentioned force is applied at the other end. The magnitude result of reaction force and reaction moment are 0.980237 N and 0.175476 N m respectively as shown in Table XIII. The von mises failure criteria were used to examine the strength of the shaft, from the study, the maximum von mises stress value is 0.878994 MPa. Since the von mises stress value is not up to half of the yield strength of the material and the factor of safety is 15, the design of the component is considered safe.

Table XIII: Fixed constraint result table showing the magnitude of the reaction force & moment

Constraint Name	Reaction Force		Reaction Moment	
	Magnitude	Component (X,Y,Z)	Magnitude	Component (X,Y,Z)
Fixed Constraint:1	0.980237 N	0.693133 N	0.175476 N m	-0.103878 N m
		0.693132 N		0.103888 N m
		0 N		0.0959613 N m

Table XIV: Result Summary

Name	Minimum	Maximum
Volume	33183.1 mm^3	
Mass	0.260487 kg	
Von Mises Stress	0.00225322 MPa	0.878994 MPa
Displacement	0 mm	0.0157134 mm
Safety Factor	15 ul	15 ul

Fig. 11: FEA result of the Von Mises Stress

Fig. 12: FEA result of the shaft Displacement

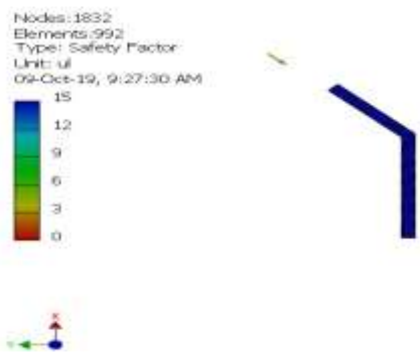


Fig. 13: FEA result of the shaft's Safety Factor

B) Results of the Stress Analysis Report for the threaded shaft and main shaft sub assembly

For the threaded shaft and main shaft sub assembly, a force of 0.981N was used to run a simulation study on the shaft above under a fixed and pin constraints position. The magnitude result of reaction force and reaction moment are 4020066 N and 0.00528455 N m respectively for the fixed constraint condition, and 7.33761N and 0.0150746 N m for the pin constraint condition as shown in Table XV. The von mises failure criteria was used to examine the strength of the shaft, from the study, the maximum von mises stress value is 0.128307 MPa which is not up to half of the yield strength of the material with a factor of safety of 15. Therefore, the design of the threaded shaft and main shaft sub assembly is considered safe.

Table XV: Reaction Force and Moment on Constraints

Constraint Name	Reaction Force		Reaction Moment	
	Magnitude	Component (X,Y,Z)	Magnitude	Component (X,Y,Z)
Fixed Constraint:1	4.20066 N	0.00274471 N	0.00528455 N m	-0.00526872 N m
		-0.0408435 N		-0.000408759 N m
		-4.20046 N		0 N m
Pin Constraint:1	7.33761 N	0 N	0.0150746 N m	0 N m
		0 N		0.0150746 N m
		-7.33761 N		0 N m

Table XVI: Result summary

Name	Minimum	Maximum
Volume	155399 mm^3	
Mass	1.0971 kg	
Von Mises Stress	0.000135512 MPa	0.128307 MPa
Displacement	0 mm	0.0000464479 mm
Safety Factor	15 ul	15 ul

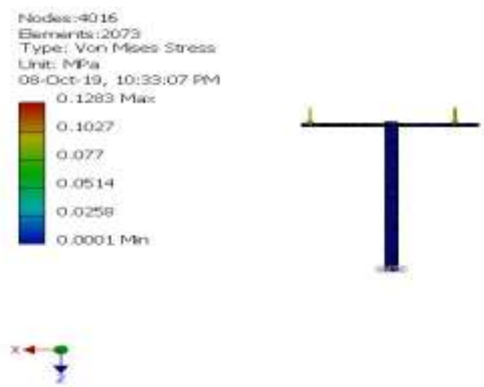


Fig. 14: FEA result of the shaft's Von Mises Stress result

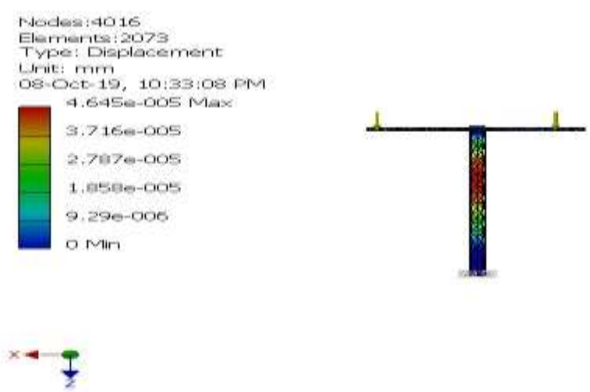


Fig. 15: FEA result of the shaft's Displacement

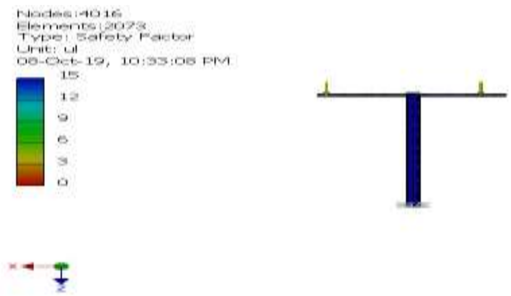


Fig. 16: FEA result of the shaft's Safety Factor

C) Results of bearing basic life rating

The Bearing load resulting from the threaded shaft and main shaft sub assembly and the bearing parameters used for bearing life calculations were tabulated in Table X and Table XI respectively. With the required life rating (L_{req}) at 100000hrs, required reliability (R_{req}) of 90, Life adjustment factor for special bearing properties of 1, life adjustment factor for operating conditions of 1, working temperature T at 100^0C, and factor of additional forces f_d of 1, the strength check was seen to be positive when static equivalent load P_O and dynamic equivalent load P were 1N each. The results are tabulated in Table XVII.

Table XVII: Result summary of bearing basic life rating

Basic rating life	L10	7.591465888734543E+015 hr
Adjusted rating life	Lna	7.591465888734543E+015 hr
Calculated static safety factor	s0c	16445.28779 ul
Power lost by friction	P_z	0.00002 W
Necessary minimum load	Fmin	0 N
Static equivalent load	P_0	1 N
Dynamic equivalent load	P	1 N
Over-revolving factor	k_n	0.000 ul
Life adjustment factor for reliability	a_1	1.00 ul
Temperature factor	f_t	1.00 ul
Equivalent speed	n_e	15 rpm
Minimum speed	nmin	15 rpm
Maximum speed	nmax	15 rpm
Strength Check	Positive	

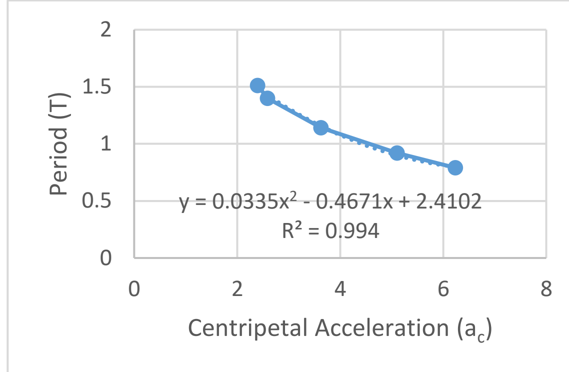


Fig. 17: Graph of period of rotation against centripetal acceleration

The relationship between period of rotation and centripetal acceleration could perfectly be described using a second order polynomial equation $T = 0.0335{a_c}^2 - 0.4671a_c + 2.4102$ with R^2 value of 0.994.

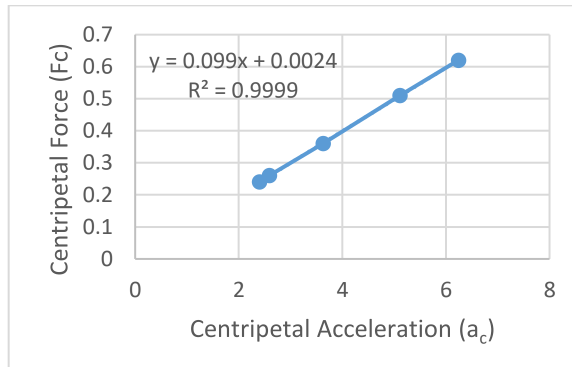


Fig. 18: Graph of centripetal force against centripetal acceleration

This graph shows that a linear relationship exist between centripetal force and centripetal acceleration and can be described using a linear equation of the form $F_C = 0.099a_c + 0.0024$ with R^2 value of 0.9999.

D) Experimental Results

Trial	Radius (m)	Mass (kg)	Total rotation time (s)	Number of rotations (N)	Velocity V $V=2\frac{\pi RN}{t}$ $(\frac{m}{s})$	$F_c = mv^2/r$	Period of rotation T	Centripetal acceleration a_c
1	0.1	0.05	5.5	7	0.79	0.62	0.79	6.24
2	0.11	0.07	6.4	7	0.75	0.51	0.92	5.11
3	0.12	0.09	6.8	6	0.66	0.36	1.14	3.63
4	0.13	0.11	7.1	5	0.58	0.26	1.40	2.59
5	0.14	0.13	7.5	5	0.58	0.24	1.51	2.40

IV. CONCLUSION

The angular acceleration apparatus was successfully designed, constructed and used for experimentation. This apparatus will serve as a visual learning aid for students in the understanding of centripetal force and acceleration as it is the basis of rotational motion. This apparatus is very diverse as it can be used in the fields of physics and in the engineering through application of already established theories. The graph of period of rotation and centripetal acceleration described using a second order polynomial equation $\mathrm{T} = 0.0335\mathrm{ac}^2 - 0.4671\mathrm{a}_\mathrm{c} + 2.4102$ with R^2 value of 0.994, validates an established fact that a rigid body undergoing uniform circular motion constantly accelerates (centripetal acceleration), and the direction of the acceleration of the rigid body will always be towards the center of the circular path (Colwell 2011). From the simulation result, since the von mises stress value is not up to half of the yield strength of the material with a factor of safety of 15, the design of the component is considered safe.

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