A hybrid Power Series – Cuckoo Search Optimization Algorithm to Electrostatic Deflection of Micro Fixed-fixed Actuators

Aminreza Noghrehabadi, Mohammad Ghalambaz*, Mehdi Ghalambaz and Amir Vosough

Abstract— **A hybrid Power series and Cuckoo Search via L´evy Flights optimization algorithm (PS-CS) method is applied to solve a system of nonlinear differential equations arising from the distributed parameter model of a micro fixed-fixed switch subject to electrostatic force and fringing filed effect. To aim this purpose, a trial solution of the differential equation is defined as sum of two polynomial parts. The first part satisfies the boundary conditions and does not contain any adjustable parameter and the second part which is constructed so as not to affect the boundary conditions and involves adjustable parameters. The cuckoo search via l´evy flights optimization algorithm is applied to find adjustable parameters of trial solution. The obtained results are compared with numerical results and found in good agreement. Further more the present method can be easily extended to solve a wide range of boundary value problems.**

Keywords— Micro, Pull-in, Fix-fix, Electromechanical Switches, Power Series, Cuckoo Search via L´evy Flights and Optimization

I. INTRODUCTION

Conductive actuators have been widely used in micro electro mechanical systems (MEMS) [1]. On the micro scale, suspended beams or plates serve as the active component of accelerometers, electrical switches, pressure sensors, optical switches, resonators, electrostatic actuators, pumps and valves [1]. A typical MEMS actuator is a micro-beam which is suspended above a conductive ground plate (electrode). Applying voltage difference between the suspended beam and the ground plane causes the micro-beam to deflect and be attracted toward to the ground electrode. At a certain voltage, which is known as pull-in voltage, the micro-beam becomes unstable and pulls-in onto the ground plane and the instability occurs [1, 2]. Peterson for the first time in 1978 was studied the nonlinear pull-in phenomenon of an electrostatic micro-actuator [3]. After that, different models such as lumped model, distributed model, and three-dimensional model are widely used to investigate the pull-in behavior of different beam structures [3-8]. Chan *et al.* considered the effects of fringing fields and finite beam thickness on the pull-in voltage and capacitance-voltage of a micro beam [9].

Recently, power series methods are used to obtain solution for pull-in instability of micro and nano actuators. Ghalambaz *et al.* applied a power series method basis on Taylor polynomials to achieve electrostatic pull-in instability of beam actuators [10]. Ghalambaz *et al.* in a different work obtained buckling of beam actuators using a monotone solution [11]. Number of researchers applied the Adomian decomposition method to evaluate deflection and pull-in parameters of beam actuators [4 -7].

Lagaris *et al.* represented a new method to solve ordinary and partitial differential equations using artificial neural networks [12]. Malek and Shekari used a hybrid artificial neural network and Nelder-Mead optimization algorithm to solve high order differential equations [13]. A hybrid artificial neural network- swarm intelligence method was used by Ghalambaz *et al.* to solve Wessinger's differential equation [14].

In the present study integrated of power series and Cuckoo Search via L´evy Flights optimization algorithm is applied to obtain a solution for buckling and deflection of fixed-fixed micro beam type actuators.

II. MATHEMATICAL MODEL

Fig. 1 shows a micro fixed-fixed beam, of length L with uniform rectangular cross section of thickness h and width w. the initial gap between the movable beam and the ground plane is g. The constitutive material of the micro-fixed-fixed is assumed linear elastic and only the static deflection of the micro-beam is considered. The electrostatic force per unit length of micro-fixed-fixed, (electrical force and fringing field effect) can be defined as [4, 5]:

$$f_{elec} = \frac{\varepsilon_0 w V^2}{2(g-y)^2}\left(1 + 0.65\frac{(g-y)}{w}\right) \qquad (1)$$

Where $\varepsilon_0 = 8.854\times10^{-12} c^2 N^{-1} m^{-2}$ the permittivity of vacuum, V is is the applied external voltage and y is the deflection of the beam. E_{eff} is the effective Young's modulus

Manuscript received July 3, 2011.

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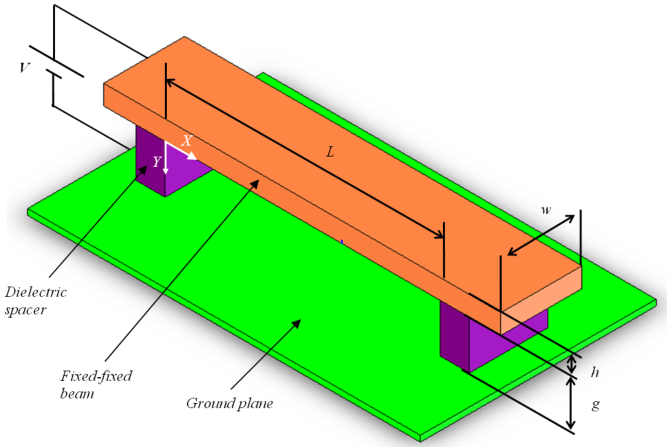

Fig. 1. Schematic of a micro fixed-fixed beam

which is equal to $wh^3/12$, and I is the moment of inertia of the beam cross section [4, 5]. In the absence of non-conservative forces and neglecting axial forces and by considering only the static elastic small deflection of the micro fixed-fixed beam the appropriate approximation of the beam deflection can be evaluated by applying the virtual work principle as follow [4].

$$\delta W = \delta E_{elast} - \delta W_{elec} - \delta W_{mole} = \int_0^L \left(E_{eff} I \frac{d^2 y}{dX^2} \delta \frac{d^2 y}{dX^2} - f_{elec} \delta Y \right) dX \qquad (2)$$

By integrating (2) one may found

$$\delta W = E_{eff} I \frac{d^2 Y}{dX^2} \delta \frac{dY}{dX}\Big|_0^L - E_{eff} I \frac{d^3 Y}{dX^3} \delta y \Big|_0^L + \int_0^L \left(E_{eff} I \frac{d^4 Y}{dX^4} - f_{elec} \right) \delta Y dX \qquad (3)$$

As there are no deflection and rotation at the fixed ends deflection of micro beam can be defined as following boundary value differential equation

$$E_{eff} I \frac{d^4 Y}{dX^4} = f_{elec} \qquad (4\text{-a})$$

Subject to

$$\begin{aligned} Y(0) &= Y'(0) = 0, & at \ \ x &= 0, \\ Y(1) &= Y'(1) = 0, & at \ \ x &= 1 \end{aligned} \qquad (4\text{-b})$$

Where X is the position along the beam and prime denotes differentiation with respect to X. $\varepsilon_0 = 8.854 \times 10^{-12} c^2 N^{-1} m^{-2}$ is the permittivity of vacuum, V is the applied external voltage and y is the deflection of the beam. E_{eff} is the effective Young's modulus which is equal to $wh^3/12$, and I is the moment of inertia of the beam cross section [4, 5].

The equation (4) can be rewrite in the nondimensional form for convenience. By substituting (1) in (4) and introducing the following nondimensional variables

$$\beta = \frac{\varepsilon_0 w V^2 L^4}{2 g^3 E I}, \ \gamma = 0.65 \frac{g}{w}, \ x = \frac{X}{L}, \ u = \frac{y}{g} \qquad (5)$$

we obtain

$$\frac{d^4 u}{dx^4} = \frac{\beta}{(1-u(x))^2} + \frac{\gamma \beta}{(1-u(x))} \qquad (6\text{-a})$$

subject to the following boundary conditions

$$\begin{aligned} u(0) &= u'(0) = 0, & at \ \ x &= 0, \\ u(1) &= u'(1) = 0, & at \ \ x &= 1 \end{aligned} \qquad (6\text{-b})$$

where x and y are nondimensional length and nondimensional deflection respectively. β and γ, are the nondimensional applied voltage and the nondimensional fringing field respectively.

III. CUCKOO SEARCH L´EVY FLIGHT

Cuckoo Search (CS) is a new meta-heuristic algorithm, to solve optimization problems which first proposed by Yang and Deb [15]. This algorithm is inspired by the obligate brood parasitic behavior of some cuckoo species in combination with the L´evy flight behavior of some birds and fruit flies in nature [15]. This section introduces main steps of the cuckoo search via l´evy flights optimization algorithm.

For simplicity in describing the Cuckoo Search, consider the following three idealized rules: 1) Each cuckoo lays one egg at a time, and dump its egg in randomly chosen nest; 2) The best nests with high quality of eggs will carry over to the next generations; 3) The number of available host nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability $p_a \in [0 \ \ 1]$.

In the case of minimization problems, the quality or fitness of a solution can simply be proportional to the minus value of the objective function. For convenience, one may use the following simple representations in which each egg in a nest represents a solution, and a cuckoo egg represent a new solution, the aim is to use the new and potentially better solutions (cuckoos) to replace a not so good solution in the nests. Based on these three rules, the basic steps of the Cuckoo Search (CS) can be summarized as the pseudo code shown in figure 2. When generating new solutions $x^{(t+1)}$ for, say a cuckoo i, a L´evy flight is performed.

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \otimes t^{-\lambda} \qquad 1 \le \lambda \le 3 \qquad (7)$$

Where α > 0 is the step size which should be related to the scale of variables of the problem of interests. In most cases, we can use α = 1. The product $\otimes$ means entry-wise

begin
Objective function f(x), $x = (x_1, \ldots, x_d)^T$
Generate initial population of n host nests x_i *(i = 1 2, ..., n)*

While *(t <MaxGeneration) or (stop criterion)*
Get a cuckoo randomly by L´evy flights
evaluate its quality/fitness F_i
Choose a nest among n(say, j) randomly
if *(*$F_i > F_j$*),*
replace j by the new solution;
End
A fraction (p_a*) of worse nests*
Are abandoned and new ones are built;
Keep the best solutions
(or nests with quality solutions);
Rank the solutions and find the current best
end while
Post process results and visualization
end

Fig. 2. Pseudo code of the Cuckoo Search (CS)

multiplications. This entrywise product is similar to those used in particle swarm optimization algorithm [16, 17]. For more details about CS method readers are referred to [15].

IV. PROBLEM FORMULATION

Consider governing equation of micro beam is expressed by (6). In order to solve (6), assume a discretization of the solution domain D with m arbitrary points. Here, the problem can be written as the following set of equations [12-14].

$$\frac{d^4u(x_i)}{dx^4} - \left(\frac{\beta}{(1-u(x_i))^2} + \frac{\gamma\beta}{(1-u(x_i))} \right) = 0, \qquad (8)$$
$$x_i \in D, \quad i = 1,2,\ldots,m$$

subject to given boundary conditions (i.e. 6-b). Let's assume $y_T(x,\vec{a})$ as an approximate solution to (6-a) where, $\vec{a}$ is a vector which contains adjustable parameters. These parameters (i.e. adjustable parameters) should be determined by minimizing the following sum of squared errors, subject to given conditions in (6-b)

$$E(\vec{a}) = \sum_{i=1}^{m} \left(\frac{d^4 y_T(x_i,\vec{a})}{dx^4} - \left(\frac{\beta}{(1-y_T(x_i,\vec{a}))^2} + \frac{\gamma\beta}{(1-y_T(x_i,\vec{a}))} \right) \right)^2, \qquad (9)$$

where $x_i \in [0 \ 1]$. In order to transform (9) to an unconstrained problem $y_T(x,\vec{a})$ can be written in the following form

$$Y_T(x,\vec{a}) = x^2(x-1)^2 + x^2(x-1)^2 N(x,\vec{a}) \qquad (10)$$

where $N(x,\vec{a})$ is an infinite power series ($N(x,\vec{a}) = \sum_{i=0}^{n} a_{i+1}x^i$) which involves adjustable parameters ($a_1 \ldots a_n$). Now equation (10) is in the form of an infinite power series with adjustable coefficients which exactly satisfy given boundary conditions of (6-b).

Finally, The CS optimization technique will be applied in order to determine optimal adjustable parameters of $y_T(x,\vec{a})$ (i.e. $\vec{a}$) to minimize $E(\vec{a})$ in (10). The present algorithm was coded with MATLAB 2007.

V. RESULTS

By taking step size of 0.05 in the domain of solution, 21 collocation points are selected for all cases (x_i ϵ {0, 0.05, 0.01 ... 1}) in the following text. For best given results, the following combination of user-specified parameters of CS method are used for this problem

The number of nests (nests): 15
A fraction of worse nests (P_a): 0.25
Number of Iteration (Generation): 300

In order to verify the convergence of obtained series, the deflection of a typical micro-actuator with β=20 and g/w=1 is computed and the solutions are compared with the numerical results. Numerical results are obtained using a combination of trapezoid as base scheme and Richardson extrapolation as enhancement scheme [18, 19]. Table 1 presents the variation of the mid point deflection (u_{tip}) of beam, using different selected terms in the series. This table ensures the convergence of the results. As seen, higher accuracy can be obtained by evaluating more terms of the solution $u(x)$.

The relative error in Table 1 is computed from the following equation:

$$Error = \left| \frac{u_{PS-CS} - u_{Numerical}}{u_{Numerical}} \right| \qquad (20)$$

Where $u_{PS\text{-}CS}$ *and* $u_{Numerical}$ are the micro fixed-fixed mid point (u_{tip}) deflection computed from present method (*i.e.* PS-CS) and the tip deflection computed using numerical method respectively. The *Error* represents relative error and $f_{opt} = \sqrt{E(\vec{a})}/m$.

By using eight terms of power series, the global error between the present solution and numerical results is less than 0.03%. The result of eight terms of power series with 0.03% error is within the acceptable range for most engineering applications.

TABLE 1.THE VARIATION OF THE MID-POINT (U_{TIP}) DEFLECTION OF A TYPICAL BEAM USING DIFFERENT SELECTED TERMS OF POWER SERIES B=20 , AND G/W =1

Method	Utip	%Errors	fopt
Numerical	0.097731		
5 Terms series	0.092772	5.044063	4.42E-01
6 Terms series	0.09727	0.439846	1.30E-01
7 Terms series	0.09727	0.439846	1.30E-01
8 Terms series	0.097727	0.027357	9.68E-03
9 Terms series	0.097727	0.027357	9.68E-03
10 Terms series	0.097730	0.031271	1.50E-03

Therefore, eight terms are selected in the following section for the convenience. The obtained power series with eight term for β=20 and g/w=1 is as follow:

$$u(x) = 1.54069107x^2 - 3.00559753x^3 + 1.37718398x^4 - 0.02833965x^5 + 0.30786213x^6 - 0.25560000x^7 + 0.06380000x^8 + \ldots \quad (11)$$

A. *Instability Study*

In order to study the instability of the micro fixed-fixed actuator, equation (1) is solved numerically simulated and the results are compared with presented method using eight terms of series. For any given β and g/w, the mid point deflection (u_{tip}) at the onset of pull-in instability can be obtained by setting $du(1)/d\beta \to \infty$. No physical solution exists for u by increasing β beyond β pull-in (β_{PI}).

B. *Electrostatic Force at Micro-Scale Separation*

Fig. (3) shows the centerline deflection a narrow micro-beam which considered in table 1 under electrostatic loading for different size of power series. This figure reveal the present method with seven terms of series or more is in very good agreement with numerical results to predict the deflection and instability of micro-fixed-fixed beams.

Fig. (4) show the variation of β as a function of mid point deflection (u_{tip}) for narrow and wide micro beam types. As seen in narrow beams instability occurs with smaller values of applied voltage (β) than wide beams. Therefore wide beams are more stable than narrow beams because of fringing filed effects. As we seen in figure (4), present method is in very good agreement with numerical results.

In the case of narrow beams (*i.e.* g/w=1) instability occurs when β exceeded 48. In this case the obtained power series is

$$u(x) = 5.65292623x^2 - 10.15322902x^3 + 3.53047449x^4 - 1.88380395x^5 + 6.85988795x^6 - 5.34167425x^7 + 1.33541856x^8 \quad (12)$$

In the case of wide beams (i.e. g/w=1) by increasing the value of β parameter beyond the 69 in-stability occurs. In this case the obtained power series is as follow:

$$u(x) = 4.91599388x^2 - 8.81385372x^3 + 3.00302510x^4 - 1.51133122x^5 + 5.83046595x^6 - 4.56573332x^7 + 1.14143332x^8 \quad (13)$$

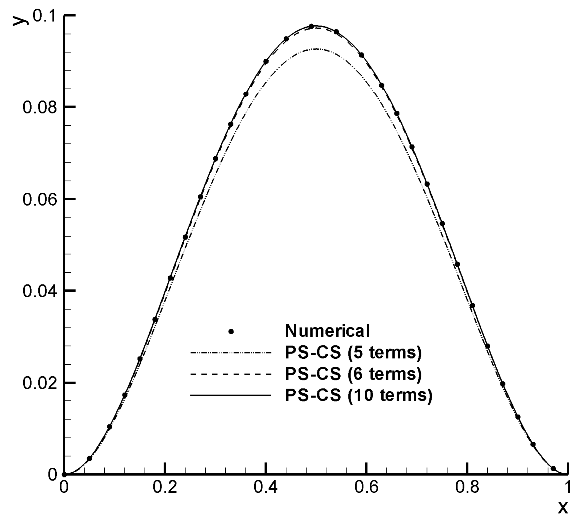

Fig. 3. Centerline deflection of a narrow micro-beam under the electrostatic load when β=20 for different size of power series.

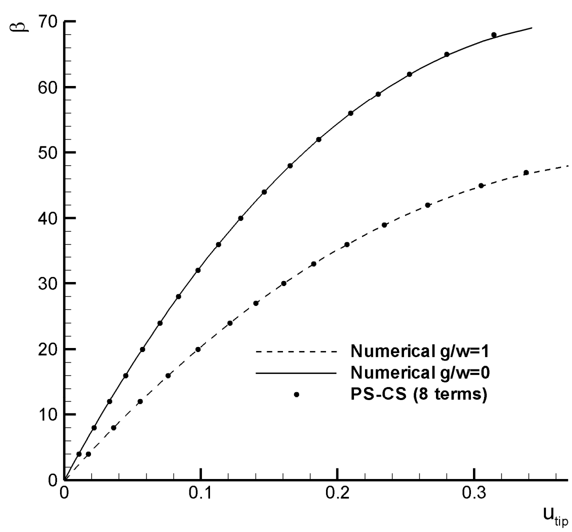

Fig. 4. Relationship between non dimensional electrostatic parameter (β) and the mid point tip deflection (u_{tip}) of fixed-fixed micro beams.

VI. CONCLUSIONS

Buckling of distributed model of micro fixed- fixed beams were computed using a combination of power series and heuristic Cuckoo Search optimization algorithm. The obtained power series solution satisfied all boundary conditions. The PS-CS results are compared with the numerical results which show the PS-CS method using eight terms is in very good agreement with numerical results. The solution of distributed model using PS-CS method provides more details about deflection shape of micro beams than lumped models. Furthermore, the present method is capable to obtain magnitude of bending moment and shear forces of micro fixed-fixed beam which will be discussed in future works.

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