Incidence Matrix-Based LMP Calculation: Algorithm and Applications

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Abstract— **This paper presents a new approach for calculating Locational Marginal Price (LMP) based on incidence matrix. This approach is an effective tool which can be implemented for short-term and long-term power system analysis, especially for economic analysis of restructured power systems. A DC-Optimal Power Flow (DCOPF) methodology has been considered for LMP calculation. This approach can be applied in market simulation and planning owing to its robustness and speed. Unlike, previous admittance based matrix methodologies, which solidly depended on the network topology, independency of network in the presented approach; it would be an effective tool for long-term expansion planning criteria. The simulation results show that the presented method is both satisfactory and consistent with expectation.**

Keywords— Incidence Matrix, Locational Marginal Price, Pricing Mechanism and Power System Economic

List of symbols:

i	Index for bus
j	Index for line
ug	Index for generation unit
ud	Index for load demand
NB	Total number of buses
NL	Total number of lines
NU	Total number of units
ND	Total number of loads
$P(i,ug)$	Power produced by unit ug at bus i
$D(i,ud)$	Power demanded by consumer ud at bus i
$C(i,ug)$	Offered price of unit ug at bus i
$PG(i)$	Total generation at bus i
$PD(i)$	Total demand at bus i
$A(i,j)$	Incidence matrix (node and branch)
$X(j,j)$	Diagonal reactance matrix
$\delta(i)$	Voltage angle of bus i
$\lambda(i)$	Dual variable of the balance constraint at bus i
$PL(j)$	Transmission line j capacity

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I. INTRODUCTION

The competitive environment of electricity markets necessitates wide access to transmission and distribution networks that connect dispersed customers and suppliers. Moreover, as power flows influence transmission charges, transmission pricing may not only determine the right of entry but also encourage efficiencies in power markets. For example, transmission constraints could prevent an efficient generating unit from being utilized. A proper transmission pricing scheme that considers transmission constraints or congestion could motivate investors to build new transmission and/or generating capacity for improving the efficiency. In a competitive environment, proper transmission pricing could meet revenue expectations, promote an efficient operation of electricity markets, encourage investment in optimal locations of generation and transmission lines, and adequately reimburse owners of transmission assets. Most important, the pricing scheme should implement fairness and be practical.

However, it is difficult to achieve an efficient transmission pricing scheme that could fit all market structures in different locations. The ongoing research on transmission pricing indicates that there is no generalized agreement on pricing methodology. In practice, each country or each restructuring model has chosen a method that is based on the particular characteristics of its network. Measuring whether or not a certain transmission pricing scheme is technically and economically adequate would require additional standards.

When the transmission becomes congested, meaning that no additional power can be transferred from a point of injection to a point of extraction, more expensive generating units may have to be brought on-line on one side of the transmission system. In a competitive market, such an occurrence would cause different locational marginal prices (LMPs) between the two locations. If transmission losses are ignored, a difference in LMPs would appear when lines are congested. Conversely, if flows are within limits (no congestion), LMPs will be the same at all buses and no congestion charges would apply. The difference in LMPs between the two ends of a congested line is related to the extent of congestion and MW losses on this line. Since LMP acts as a price indicator for both losses and congestion, it should be an elementary part of transmission pricing [1].

The locational marginal pricing is a dominant approach in energy market operation and planning to identify the nodal price and to manage the transmission congestion LMP has been implemented under consideration at the number of ISO's such as PJM, New York ISO , ISO-New England, California ISO, and Midwest ISO [2-4].

Locational marginal prices may be decomposed into three components: marginal energy price, marginal congestion price, and marginal loss price [1, 5-6]. The LMP can be calculated by the Optimal Power Flow (OPF) and DCOPF-based simulations. The DCOPF has been used by many utilities for price forecasting and system planning [5], [7].

In many paper LMP calculated as a deterministic variable [5]. Considering the uncertainties associated with the input data of load flow, the LMP can be considered as a stochastic variable. Therefore calculation of LMP as a random variable can be very useful in power market forecasting studies [7].

Other method is Point Estimation Method (PEM) [7-8]. This method used two or more point to calculate mean and variance of desired variable and estimate PDF and CDF of this variable.

Point Estimation Method (PEM) has lack of accuracy although has a good speed. It can be seen that the results of point estimation method in [7] have a few differences from deterministic calculation .

Several earlier works [9-13] have reported the modeling of LMPs, especially in marginal loss model and related issues. Reference [9] points out the significance of marginal loss price, which may differ up to 20% among different zones in New York Control Area based on actual data. Reference [10] presents a slack-bus-independent approach to calculate LMPs and congestion components.

Reference [11] presents a real-time solution without repeating a traditional power flow analysis to calculate loss sensitivity for any market-based slack bus from traditional Energy Management System (EMS) products based on multiple generator slack buses. Reference [12] demonstrates the usefulness of dc power flow in calculating loss penalty factors, which has a significant impact on generation scheduling. The authors of [12] also point out that it is not advisable to apply predetermined loss penalty factors from a typical scenario to all cases. Reference [13] presents LMP simulation algorithms to address marginal loss pricing based on the dc model.

From the viewpoint of generation and transmission planning, it is always crucial to simulate or forecast LMPs, which may be obtained using the traditional production (generation) cost optimization model, given the data on generation, transmission, and load [14], [6]. Typically, dc optimal power flow (DCOPF) is utilized for LMP simulation or forecasting based on production cost model via linear programming (LP) owing to LP's robustness and speed. The popularity of DCOPF lies in its natural fit into the LP model. Moreover, various third-party LP solvers are readily available to plug into DCOPF model to reduce the development effort for the vendors of LMP simulators [5].

This paper as follows. Theoretical consideration of modeling the DCOPF based on incidence matrix is presented in next section. Simulation results are presented in section III and conclusion of this paper is conducted in last section.

II. THEORETICAL CONSIDERATIONS

Earlier studies of LMP calculation with DCOPF ignore the line losses. Thus, the energy price and the congestion price follow a perfect linear model with a zero loss price. However, challenges arise if nonlinear losses need to be considered in LMP calculations.

The lossless DCOPF can be modeled as the minimization of the total production cost subject to energy balance and transmission constraints. The voltage magnitudes are assumed to be unity and reactive power is ignored. Also, it is assumed that there is no demand elasticity. This model may be written as LP:

$$Min \sum_{i=1}^{NB}\sum_{ug=1}^{NU}[P(i,ug)*C(i,ug)] \tag{1}$$

Subject to:

$$PG(i)=\sum_{ug=1}^{NU}P(i,ug) \tag{2}$$

$$PD(i)=\sum_{ud=1}^{ND}D(i,ud) \tag{3}$$

$$PG(i)-PD(i)=\sum_{j=1}^{NL}A(i,j)*PL(j)\ \perp\ \lambda(i) \tag{4}$$

$$\sum_{i=1}^{NB}A(i,j)*\delta(i)=\sum_{j=1}^{NL}X(j,j)*PL(j) \tag{5}$$

$$PL^{\min}(j)\le PL(j)\le PL^{\max}(j) \tag{6}$$

$$P^{\min}(i,ug)\le P(i,ug)\le P^{\max}(i,ug) \tag{7}$$

Aggregated generation and demand at each bus are represented in (2) and (3), respectively. Generation and demand balance addressed in (4) by implementing the incidence matrix, this equation corresponds with injection power through power transmission lines connected to bus *i*. Locational marginal price is the dual variable of the balance constraint at bus *i* and indicated as λ(i). Power transmitted through transmission lines is indicated as (5) using correspondence diagonal reactance matrix, X. Transmission line limits and power generation boundary

Constraints (6) and (7) enforce the transmission capacity limits of each line and each generation unit, respectively.

The first step is extracting corresponding incidence matrix of the network. Fig. 1 shows a simple network which consists of three buses and three lines. Each network can be represented as a graph and such a directional graph. Each bus indicated as a node and each transmission line addressed as a directed branch. In the corresponding incidence matrix, nodes and branches indicated as rows and columns, respectively. In the incidence matrix, "1" indicates if branch leaves node, "-1" if branch arrives at node and "0" if no connection.

It should be noted that the mathematical formulation in this paper extends the general formulation of single generator and single load for each bus. Aggregated production and load demand are modeled in this paper. Despite of recent papers which claim that actual implementation can be more

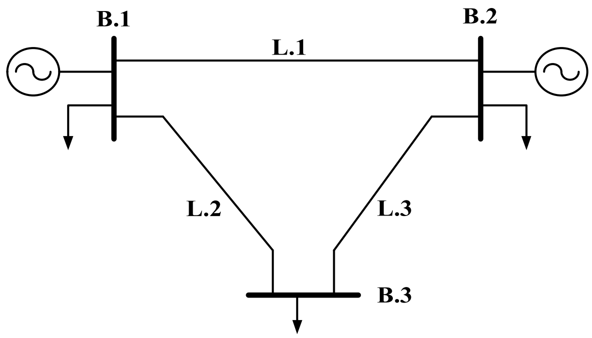

Fig. 1. Simple power system

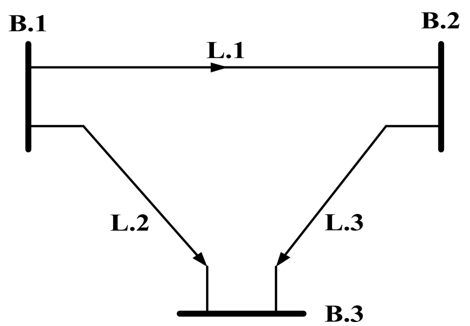

Fig. 2. Directional graph of simple power system

Table I: Incidence matrix of simple power network

A(i,j)		Lines		
		1	2	3
Buses	1	1	1	0
	2	-1	0	1
	3	0	-1	-1

complicated considering multiple generators and loads [5], the incidence matrix based formulation ignores both multiple generation units and multiple transmission lines between buses. It also should be noticed that implementing the incidence matrix methodology eliminates the network interdependencies because of admittance matrix structure in conventional power flow. This approach would be useful in contingency analysis of power network. In contingency analysis it is very important to utilize a fix algorithm and eliminating the topological changes. For multiple generation units which installed in each bus, contingency analysis would be easily carried out, but for transmission line contingencies because of changing the admittance elements but in the incidence matrix formulation this objection has been removed. The incidence graph is illustrated as Fig. 2, and Table I represents the corresponding incidence matrix.

The diagonal reactance matrix is easily extracted from grid. For example X(1,1) indicates the first line, L.1 in the grid. Similarly, X(2,2) and X(3,3) imply L.2 and L.3, respectively. One of the advantages of this network representation by using incidence matrix is appeared in contingency analysis which outages of both generation units and transmission lines would be modeled easily. For example, when a transmission line outage is occurred, by assigning "0" in line capacity, the entire impacts of corresponding transmission line is eliminated easily.

Another application of this methodology is in the transmission and generation expansion planning, which examined in the simulation section.

III. SIMULATION STUDIES

In order to validate the proposed incidence matrix based LMP calculation, a PJM five bus, six lines test system, which is a standard test case, is considered here. The benchmark parameters are listed in tables II and III. Demanded load at buses B, C and D, are similarly 300MW. The system is slightly modified from the PJM 5-bus system [2] and will be used for the rest of this paper. The generation cost at Sundance (unit 4.1) is modified from the original \$30/MWh to \$35/MWh to differentiate its cost from the Solitude (unit 3.1) for better illustration.

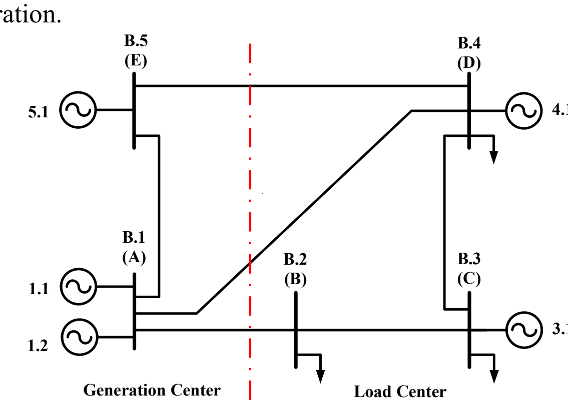

Fig. 3. Base case of the PJM 5-Bus example

The system can be roughly divided into two areas, a generation center consisting of Buses A and E with three low-cost generation units and a load center consisting of Buses B, C, and D with 900 MWh load and two high-cost generation units. The transmission line impedances are given in Table I, where the reactance is obtained from [2] and the resistance is assumed to be 10% of the reactance. Here only the thermal flow limit of Line DE (Line 6) is considered for illustrative purpose.

Table II. Line impedance and flow limits

Line Number	1	2	3	4	5	6
Connection	AB	AD	AE	BC	CD	DE
R%	0.281	0.304	0.064	0.108	0.297	0.297
X%	2.81	3.04	0.64	1.08	2.97	2.97
Limit(MW)	999	999	999	999	999	240

Table III. Generation unit's data

Unit	Location	Indication	Pmax	Pmin	Offer
Alta	A	1.1	110	0	14
Park City	A	1.2	100	0	15
Solitude	C	3.1	520	0	30
Sundance	D	4.1	200	0	35
Brighton	E	5.1	600	0	10

A. LMP Calculation

Based on previous consideration, LMP of each bus is the dual variable of load balance equation. In this case, total demand is 900MW and installed capacity is 1530MW. Each generation company, GENCO, offer its price to maintaining consumer load. Incidence and reactance matrixes are addressed in appendix tables A1 and A2 respectively. Summary of load dispatch is presented in table IV.

Table IV. Generation dispatch results and LMP

Bus	Indication	Generation	LMP
1	1.1	110.00	15.826
	1.2	100.00	
2	-	-	23.680
3	3.1	0	26.699
4	4.1	116.079	35.000
5	5.1	573.921	10.000

B. Contingency Analysis

In contingency analysis it is possible to considering all generation units and transmission lines. In this paper two possible contingencies are considered, at first, outage of the unit 1.2, which called Case I, is considered. The simulation results are presented in table V. Operation cost is 13427.755 $/h in this case. Second case, Case II, considers the outage of line AE. Operation cost in this case is 22317.987 $/h and generator's dispatch are addressed in table VI.

These high LMP which has been occurred in Case II imply that transmission lines have important role in power system. It also should be noted that congestion of the transmission lines has an economic signal for network expenditure.

C. Expansion Planning

In the recent case, it has been noticed that congestion of available transmission cost enforced the additional cost of operation. In this case, the expenditure of transmission lines is considered.

Table V. Generation dispatch results and LMP, for Case I

Bus	Indication	Generation	LMP
1	1.1	110.00	23.451
	1.2	**Out**	
2	-	-	28.182
3	3.1	152.449	30.000
4	4.1	37.551	35.000
5	5.1	600.00	19.942

Table VI. Generation dispatch results and LMP, for Case II

Bus	Indication	Generation	LMP
1	1.1	110.00	52.732
	1.2	100.00	
2	-	-	45.468
3	3.1	520.00	42.677
4	4.1	79.119	35.000
5	5.1	90.881	10.000

Suppose that the planner decides to expand the DE line. For this sake, he decides to add a parallel similar transmission line in this corridor. It is very simple to takes into accounts in the incidence matrix DCOPF model. The reactance of corresponding line (Line 6) is modified from the original 0.0297 to 0.01485 and also, transmission line capacity is modified from the original 240MW to 480MW. The simulation results show that this expenditure plan eliminates network constraints and this case is equally with a traditional economic dispatch (EDC). The marginal operation cost is 30$/MWh for entire network.

IV. CONCLUSION

The proposed Incidence Matrix-Based LMP calculation is simple approach to implementing for large scale power system analysis regardless of time horizon analysis. In short-term analysis, such as Day A-head market clearing, mid-term analysis such as maintenance scheduling or fuel allocation and in the long-term analysis such as expansion planning studies; this approach would be applicable.

This can reduce the computational effort since it does not require the algorithm to run till convergence. Therefore, it fits a simulation or planning purpose well if the accuracy is reasonably acceptable. The simulation results on the benchmark PJM 5-bus system show the feasibility and applicability of the proposed method in short-term, mid-term and long-term analysis. Simulation results also show that the presented method is both satisfactory and consistent with expectation.

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Appendix (A)

Table A1: Incidence matrix of PJM 5-bus test system

A(i,j)	**1**	**2**	**3**	**4**	**5**	**6**
1	**1**	**1**	**1**	0	0	0
2	**-1**	0	0	**1**	0	0
3	0	0	0	**-1**	**1**	0
4	0	**-1**	0	0	**-1**	**1**
5	0	0	-1	0	0	**-1**

Table A2: Reactance matrix of PJM 5-bus test system

X(j,j)	**1**	**2**	**3**	**4**	**5**	**6**
1	**0.0281**	0	0	0	0	0
2	0	**0.0304**	0	0	0	0
3	0	0	**0.0064**	0	0	0
4	0	0	0	**0.0108**	0	0
5	0	0	0	0	**0.0297**	0
6	0	0	0	0	0	**0.0297**