Second Law Analysis of Parallel Plate Ducts with Span Wise Periodic Triangular Corrugations at one Wall

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Abstract— **In this study, entropy generation for hydro dynamically and thermally developing laminar flow in a parallel plate ducts with span wise periodic triangular corrugations at one wall at constant heat flux is investigated. Entropy generation is obtained for various aspect ratio (α), various corrugation angle (2Φ), various number of corrugation (n) and various Reynolds number. It is found that with the increasing aspect ratio (α) and various number of corrugation (n) values and decreasing corrugation angle (2Φ), total entropy generation decreases , with increasing aspect ratio (α) and corrugation angle (2Φ)values and decreasing number of corrugation (n), pumping power decreases.**

Keywords— Second Law, Corrugation Duct, Laminar Flow and Aspect Ratio

I. INTRODUCTION

Many studies in engineering area involve fluid flow and heat transfer inside a duct. To analyze the energy transfer in a thermal system, the first law of thermodynamics is always used. However, the investigation of exergy or irreversibility of a thermal system requires the second law analysis, relating to an entropy generation. In fluid flow and heat transfer processes, the total entropy generation consists of the entropy generation due to viscous friction between the wall and the fluid, and that due to heat transfer across finite temperature differences between the wall and the fluid. For internal flow, there are two conditions, constant wall temperature condition and constant wall heat flux condition. These are often investigated in the entropy generation studies. Parallel plate ducts with span wise periodic triangular corrugations are used in liquid–gas heat exchangers.

Entropy generation due to heat transfer and fluid flow has been investigated by many researchers. Bejan [1] – [3] analyzed heat transfers from ducts with constant heat flux for flat plates, cylinders in cross flow and rectangular ducts. Perez-Blanco [4] integrated the entropy generation rate along the entire surface of the heat exchanger, and then evaluated the effect of the heat transfer augmentation technique on the total entropy-generation rate.

Sahin [5] presented the second law analysis for different shaped duct such as triangular, sinusoidal etc, in laminar flow and constant wall temperature boundary conditions. He made a comparison between these ducts to find an optimum shape. He found that the circular duct geometry is the favorable one among them. He made another study in order to investigate the constant heat flux effects on these cross-sectional ducts without taking into account the viscosity variation in the analysis [6]. Viscosity variation was considered by Sahin [7] for turbulent flow condition for circular ducts [8].

Recently, Oztop [9] made a study on entropy generation in semicircular ducts. Dagtekin et al. [10] investigated the problem for circular duct with different shaped longitudinal fins for laminar flow using the similar methods of Sahin [6]. Oztop et al. [11] made a study on entropy generation in rectangular ducts with semicircular ends ducts with two boundary conditions: constant wall temperature and constant wall heat flux and Jarungthammachote [12] investigated entropy generation for hexagonal duct subjected to constant heat flux.

The main aim of this study is to investigate entropy generation through parallel plate ducts with span wise periodic triangular corrugations at one wall at constant heat flux. To the best of the author's knowledge the entropy generation in parallel plate ducts with span wise periodic triangular corrugations with constant heat flux has not yet been investigated. The duct is assumed to be infinite in the span wise direction. Therefore, the end effects due to the short bounding walls are neglected. The corrugated wall is subjected to the thermal boundary condition, while the flat wall is considered to be adiabatic. The present paper reports an analytical study of entropy generation in laminar flow. The effect of aspect ratio, corrugation angle, number of corrugation and Reynolds number on entropy generation and pumping power are analyzed.

II. DEFINITION OF PHYSICAL MODEL

The physical model of parallel plate ducts with span wise periodic triangular corrugations at one wall elliptical duct is depicted in Fig. 1. The hydraulic diameter of any duct given by

$$D_h = \frac{4A_c}{p} \tag{1}$$

where A_c is cross-sectional area and P is perimeter. The hydraulic diameter for annular sector cross-sectional area can be written as:

$$D_h = \frac{2\sqrt{n(1-\alpha^2)\tan\phi}}{n(1-\alpha)\dfrac{1+\sin\phi}{1+\cos\phi}}\sqrt{A_c} \quad (2)$$

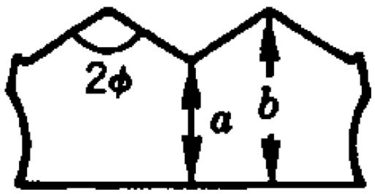

Fig. 1. Cross section of parallel plate ducts with span wise periodic triangular corrugations at one wall

III. ENTROPY GENERATION

The total entropy generation within a control volume of thickness dx, shown in Fig. 2, along the duct can be written as follows:

$$d\dot{S}_{gen} = \dot{m}\,ds - \frac{\delta\dot{Q}}{T_w} \quad (3)$$

and

$$\delta\dot{Q} = \dot{m}C_P\,dT = qp\,dx \quad (4)$$

For an incompressible fluid we have

$$T\,ds = C_P\,dT - v\,dP \quad (5)$$

Pressure drop in Eq. 8 is given in the following equation.

$$dP = -\frac{f\rho U^2}{2D_h}dx \quad (6)$$

The bulk temperature variation of fluid along a duct can be written as follows

$$T = T_0 + \left(\frac{4q}{\rho U D_h C_P}\right)x \quad (7)$$

The non-dimensional entropy generation number Ns can be defined as:

$$Ns = \frac{\dot{S}_{gen}}{\dot{Q}/\Delta T} = \frac{\dot{S}_{gen}}{\dot{m}C_P} \quad (8)$$

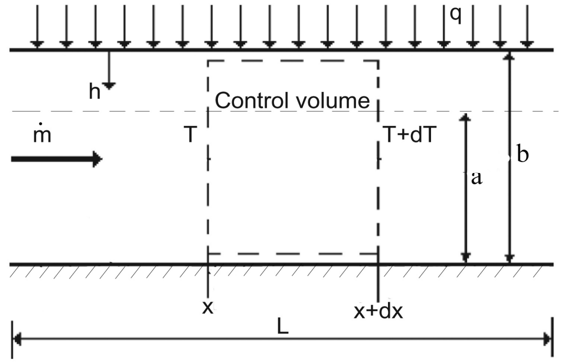

Fig. 2. control volume for entropy generation

Integrating Eq. 3 along the duct length L and by using Eq. 4-7, the dimensionless total entropy generation becomes

$$Ns = \ln\left[(1+4St\,\tau\,\lambda)(1+\tau)/(1+\tau+4St\,\tau\,\lambda)\right] + \frac{f\,Ec}{8St}\ln(1+4St\,\tau\,\lambda) \quad (9)$$

In these equations some parameters can be made dimensionless as follows:

$$St = \frac{h}{\rho U C_P} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} \quad (10)$$

$$Ec = \frac{U^2}{C_P(T_w - T_i)} \quad (11)$$

$$\tau = \frac{T_w - T_i}{T_w} = \frac{(q/h)}{T_i} \quad (12)$$

$$\lambda = \frac{L}{D_h} \quad (13)$$

The fully developed $(f\,\mathrm{Re})$ and Nu values obtained by Sparrow and Charmchi [13] for variety of duct geometries.

IV. REQUIRED PUMPING POWER

The power required to overcome the fluid friction in the duct in dimensionless form is

$$PPR = \frac{A_c\,\Delta P\,U}{\dot{Q}} \quad (14)$$

For constant wall heat flux boundary conditions, the pumping power to heat transfer ratio for fully developed laminar flow becomes

$$PPR = \frac{\mu(f\,\mathrm{Re})}{8\rho^2 D_h q}\mathrm{Re}^2 \quad (15)$$

V. RESULT AND DISCUSSION

In this section, the entropy generation due to heat transfer and viscous friction is investigated for fully developed laminar flow in the parallel plate ducts with span wise periodic triangular corrugations at one wall at constant heat flux for water. The thermo-physical properties used are shown in Table 1:

Table 1: Thermo-physical properties of water

SYMBOL	QUANTITY
$C_P\,(J/kg\,K)$	4182
Pr	7
$T_w(K)$	293
$\mu(Ns/m^2)$	9.93×10^{-4}
$\rho(kg/m^3)$	998.2

A. Effect of Aspect Ratio

Fig. 3 shows dimensionless entropy generation for different aspect ratio (α) of parallel plate ducts with span wise periodic triangular corrugations at one wall at different Reynolds number values. In this figure, total entropy generation decreases considerably while the Reynolds number is increased. As the value of aspect ratio (α) is increased total entropy generation decreased for fixed Reynolds number.

Fig. 4 shows variation of pumping power for different aspect ratio (α) and Reynolds numbers. As aspect ratio (α) is increased pumping power to heat transfer ratio decreases. With increasing of Reynolds number pumping power to heat transfer ratio values also increase. For a fixed Reynolds number as aspect ratio (α) values are increased pumping power to heat transfer ratio values are decreased, especially for higher values of Reynolds number. These results indicate that for larger aspect ratio friction losses are smaller than that of lower aspect ratio as expected.

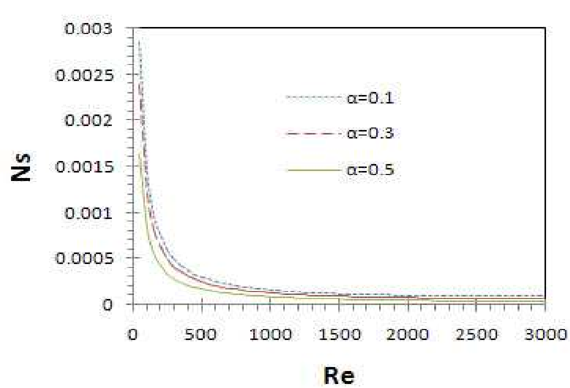

Fig. 3. Variation of dimensionless entropy generation for different α values and Reynolds number (2Φ=600, τ=0. 1, n=3)

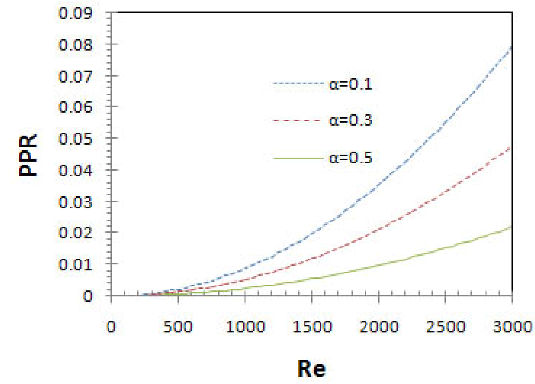

Fig. 4. Variation of pumping power for different α values and Reynolds number (2Φ=600, τ=0. 1, n=3)

B. Effect of Corrugation Angle

Fig. 5 shows dimensionless entropy generation for different corrugation angle (2Φ) of parallel plate ducts with span wise periodic triangular corrugations at one wall at different Reynolds number values. As the corrugation angle (2Φ)is increased total entropy generation increases for fixed Reynolds number, especially for lower values of Reynolds number. With increasing of Reynolds number total entropy generation values also decrease. Fig. 6 shows variation of pumping power for different corrugation angle (2Φ) and Reynolds numbers. As the corrugation angle (2Φ) is increased pumping power to heat transfer ratio decreases. With increasing of Reynolds number pumping power to heat transfer ratio values also increase. For a fixed Reynolds number as corrugation angle (2Φ) are increased pumping power to heat transfer ratio values are decreased, especially for higher values of Reynolds number. These results indicate that for larger corrugation angles (2Φ) friction losses are smaller than that of lower sector angels (2Φ) as expected.

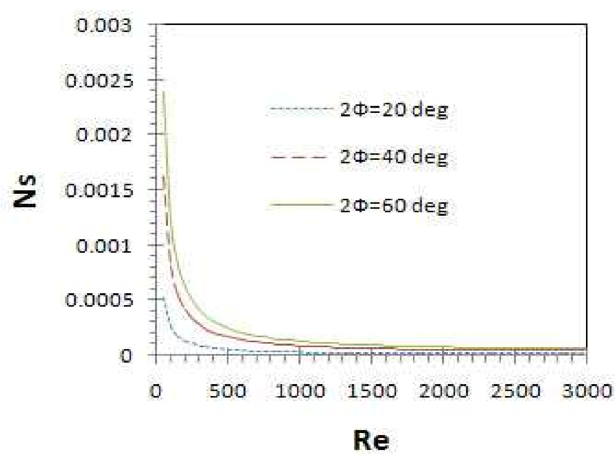

Fig. 5. Variation of dimensionless entropy generation for different corrugation angles values and Reynolds number (α =0.3, τ=0. 1, n=3)

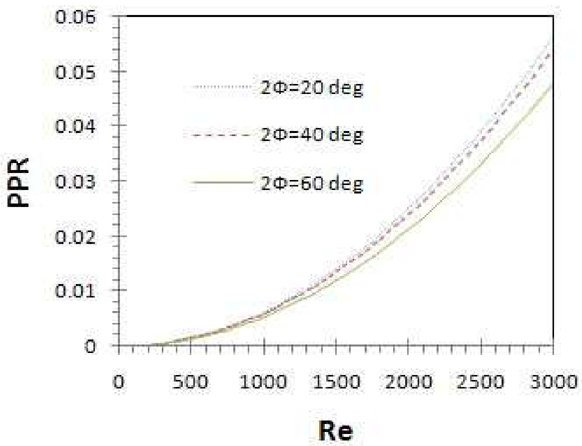

Fig. 6. Variation of pumping power for different corrugation angles values and Reynolds number (α =0.3, τ=0. 1, n=3)

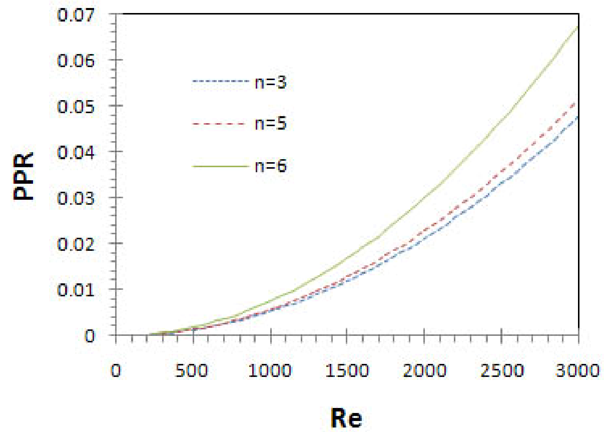

Fig. 8. Variation of pumping power for different number of corrugation values and Reynolds number (α =0.3, 2Φ=600 , τ=0. 1)

C. Effect of Number of Corrugation

Fig. 7 shows the effects of number of corrugation on entropy generation at different Reynolds numbers for a fixed aspect ratio (α =0.3) and corrugation angle (2Φ=600), namely, for a fixed cross sectional area. Entropy generation is influenced by number of corrugation. As Reynolds number is increased, the dimensionless entropy generation decreases. Thus, lower entropy generation is obtained for smaller value of number of corrugation. The decrease of total entropy generation depends on the decrease of number of corrugation as expected. Fig. 8 shows variation of pumping power to heat transfer ratio for various number of corrugation and Reynolds numbers. Increasing Reynolds number yields higher pumping power to heat transfer ratio values for fixed number of corrugation. As the number of corrugation is increased pumping power to heat transfer ratio values increases for fixed Reynolds number, especially for higher values of Reynolds number.

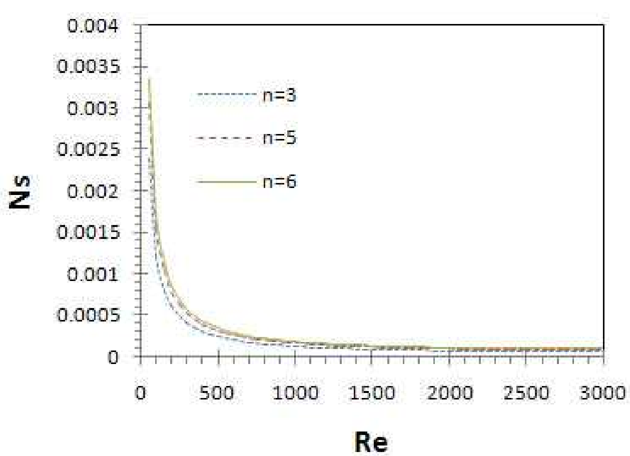

Fig. 7. Variation of dimensionless entropy generation for different number of corrugation values and Reynolds number (α =0.3, 2Φ=600 , τ=0. 1)

VI. CONCLUSION

In this study second law analysis of laminar flow subjected to constant wall heat flux has been obtained for parallel plate ducts with span wise periodic triangular corrugations at one wall. Some conclusions can be given as follows:

As Reynolds number is increased total entropy generation is decreasing. With increasing value of aspect ratio (α) and decreasing corrugation angle (2Φ) are decreased total entropy generation for fixed Reynolds number. The pumping power ratio decreases as the aspect ratio (α), and decreases as corrugation angle (2Φ) are increased.

When number of corrugation is increased total entropy generation and pumping power ratio are increasing for fixed Reynolds number.

NOMENCLATURE

A_c	cross-sectional area of duct, m^2
a	hight of duct, m
b	hight of duct, m
C_P	specific heat, $J/kg\,K$
D_h	hydraulic diameter, m
Ec	Eckert number
f	Darcy friction factor
h	heat transfer coefficient, W/m^2K
k	thermal conductivity of fluid, W/m^2K
L	Length of duct, m
$\dot{m}$	mass flow rate, kg/s
n	number of corrugation
Ns	dimensionless entropy generation
Nu	Nusselt number
P	Pressure, Pa
p	Perimeter of duct, m
Pr	Prandtl number
PPR	pumping power to heat transfer rate ratio
q	heat flux, W/m^2

$\dot{Q}$	Total heat flux, W
Re	Reynolds number
St	Stanton number
s	entropy, W/K
$\dot{S}_{gen}$	entropy generation rate, W/K
T	Temperature, K
T_i	Inlet fluid temperature, K
T_w	Wall temperature of the duct, K
U	velocity in coil tube, m/s
x	axial distance, m

Greek alphabets

ΔT	increase of fluid temperature, K
ΔP	Total pressure drop, N/m^2
α	Aspect ratio, a/b
ρ	density, kg/m^3
Φ	half of corrugation angle
τ	dimensionless temperature difference
μ	viscosity, $Pa\,s$
λ	dimensionless axial distance

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