Exergy Analysis of Forced Convection through an Elliptical Duct with Internal Longitudinal Fins

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Abstract— **In the present study, the exergy transfer characteristics of forced convection through an elliptical duct with internal longitudinal fins under constant wall heat flux for hydrodynamic and thermally fully developed laminar flow have been considered. Affecting parameters such as heat flux rate, Reynolds number, aspect ratio (α*) and relative length of fin (L*) are studied for exergy destruction. It is concluded that with the increasing relative length of fin (L*) values, non-dimensional exergy flux and non-dimensional heat transfer rate at fixed Reynolds number increases, with increasing wall heat flux values, non-dimensional exergy flux increases, however, non-dimensional heat transfer rate decreases and with increasing aspect ratio (α*) values, total non-dimensional exergy flux and non-dimensional heat transfer rate at fixed Reynolds number decreases.**

Keywords— Exergy, Laminar Flow, Elliptical Duct with Longitudinal Fin and Constant Wall Heat Flux

I. INTRODUCTION

The second law analysis is the gateway for optimization in thermal equipments and systems. Entropy generation or exergy destruction due to heat transfer and fluid flow through a duct has been investigated by many researchers and non-dimensional entropy generation number is always employed in the irreversibility examination of convective heat transfer. A study of entropy generation in fundamental convective heat transfer process was carried out by Bejan [1]. He demonstrated spatial distribution of irreversibility and entropy generation maps in the flow field and indicated that the flow geometric parameters could be selected in order to minimize irreversibility associated with the specific problem. Sahin [2] presented the second law analysis for different shaped duct such as triangular, sinusoidal etc, in laminar flow and constant wall temperature boundary conditions. He made a comparison between these ducts to find an optimum shape.

He found that the circular duct geometry is the favorable one among them. He made another study in order to investigate the constant heat flux effects on these cross-sectional ducts without taking into account the viscosity variation in the analysis [3]. Viscosity variation was considered by Sahin [4] for turbulent flow condition for circular ducts [5]. Since Soma [6] and Dunbar et al. [7] put forward the concept of exergy transfer and its equation, the research on exergy transfer has led to some researchers' attention.

Exergy is an important parameter for increase the system efficiency. To the best of the author's knowledge the exergy destruction of elliptical duct with internal longitudinal fins under constant wall heat flux for hydrodynamic and thermally fully developed laminar flow has not yet been investigated.

The present paper reports an analytical study of exergy transfer characteristics in laminar flow. The effects of Reynolds number, wall heat flux and geometrical dimensions on exergy transfer characteristics are analyzed.

II. PHYSICAL MODE OF PROBLEM

The physical model of elliptical duct with internal longitudinal fins is depicted in Fig. 1. The hydraulic diameter of any duct given by:

$$D_h = \frac{4A}{p} \tag{1}$$

where A is cross-sectional area and P is perimeter. The hydraulic diameter for annular sector cross-sectional area can be written as

$$D_h = \frac{\sqrt{\pi}\sqrt{\alpha^*}}{\frac{\pi}{2}\sqrt{\frac{1+\alpha^{*2}}{2}} + L^*\left(1+\alpha^*\right)}\sqrt{A} \tag{2}$$

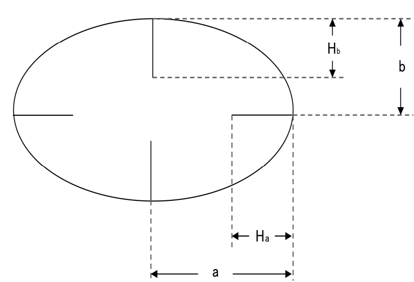

Fig. 1. Cross section of duct

III. EXERGY TRANSFER ANALYSIS

Consider the constant cross-sectional area duct shown schematically in Fig. 2. The wall heat flux q_w keeps constant and the hydraulic diameter of duct is D_h. An incompressible viscous fluid with mass flow rate, $\dot{m}$, and the inlet temperature, T_i, enters the duct of length L, and the exit temperature is T_e. The convective heat transfer and flow processes are in a steady state and thermally and hydrodynamic fully developed. The average convective heat transfer coefficient, h, is constant. In addition, the physical properties of the fluid are assumed to be constant within the range of temperature considered in this study, the axial heat conduction, viscous dissipation and heat losses of duct are neglected.

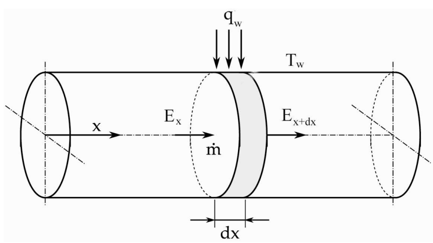

Fig. 2. Control volume for exergy destruction

The exergy transfer equations of convective heat transfer may be written as follows on the basis of the linear non-equilibrium thermodynamics theory

$$E = h\,A\,\Delta T \tag{3}$$

$$e = \frac{E}{A} = h_e\,\Delta T \tag{4}$$

where

$$\Delta T = T_w - T_b = \frac{q_w}{h} \tag{5}$$

The bulk temperature variation of the fluid along the duct can be obtained as:

$$T_b = T_i + \frac{4\,q_w}{\rho\,C_P\,u_m\,D_h}x = T_i + \frac{4\,q_w\,x}{h\,D_h}St \tag{6}$$

The expression of outlet temperature is:

$$T_e = T_i + \frac{4\,q_w}{h}St\,\lambda \tag{7}$$

where

$$St = \frac{h}{\rho\,C_P} \tag{8}$$

and

$$\lambda = \frac{L}{D_h} \tag{9}$$

Using the equation (4), T_w is

$$T_w = T_b + \frac{q_w}{h} \tag{10}$$

The exergy transfer rate over a differential element of length dx is given in the following equation [8]:

$$dE = h_{ex}\,dA\,\Delta T_x \tag{11}$$

where

$$dA = \pi\,D_h\,dx \tag{12}$$

Using specific exergy transfer rate definition

$$de' = dh - T_o\,ds \tag{13}$$

and from thermodynamic equation

$$T\,ds = dh - v\,dp \tag{14}$$

rearranging equation (13) by (14) one obtains

$$de' = C_P(1-\frac{T_o}{T_b})dT + \frac{v}{T_b}T_o\,dp \tag{15}$$

Thus, the exergy change rate of working fluids over a differential element of length is given by

$$dE = \dot{m}\left[C_P\left(1-\frac{T_o}{T_b}\right)dT_b + \frac{v}{T_b}T_o\,dp\right] \tag{16}$$

From equations (11) and (16)

$$h_{ex} = \frac{\rho\,h\,d_i\,u_m}{4q_w}\left[C_P\left(1-\frac{T_o}{T_i+\frac{4St\,q_w\,x}{h\,d_i}}\right)\frac{dT_b}{dx} + \frac{v\,T_o}{T_i+\frac{4\,St\,q_w\,x}{h\,d_i}}\frac{dp}{dx}\right] \tag{17}$$

where

$$\frac{dp}{dx} = -\frac{f\,u_m^2}{2d_i}\rho \tag{18}$$

Putting equations (6) and (18) into equation (17) gives:

$$h_{ex} = h\left[1-\frac{T_o}{T_i+\frac{4q_w\,St\,x}{h\,d_i}}\left(1+\frac{f\,\mathrm{Re}^3\,\mu^3}{8q_w\,d_i^3\,\rho^2}\right)\right] \tag{19}$$

Combining equations (4) and (19), the local exergy flux becomes

$$e_x = h_{ex}\,\Delta T_x = q_w \left[1 - \frac{1}{\theta}\,\frac{1}{1 + \dfrac{4 N_q X}{\mathrm{Re\,Pr}}} \left(1 + \frac{f\,\mathrm{Re}^3}{8 N_{qw}} \right) \right] \quad (20)$$

The non-dimensional exergy flux is defined as:

$$e^* = \frac{e}{T_o \dfrac{\lambda}{L}} = N_q \theta\,\lambda - \frac{\mathrm{Re\,Pr}}{4} \left(1 + \frac{f\,\mathrm{Re}^3}{8 N_{qw}} \right) \times \ln\left(1 + \frac{4 N_q \lambda}{\mathrm{Re\,Pr}} \right) \quad (21)$$

In these equations some parameters can be made dimensionless as follows:

$$\theta = \frac{T_i}{T_o} \quad (22)$$

$$X = \frac{x}{D_h} \quad (23)$$

$$N_q = \frac{q_w\, d_i}{k\, T_i} \quad (24)$$

$$N_{qw} = \frac{q_w\, \rho^2\, d_i^3}{\mu^3} \quad (25)$$

IV. HEAT TRANSFER ANALYSIS

If the duct is in constant wall heat flux, the local and mean heat fluxes are:

$$q_x = q = q_w \quad (26)$$

The total heat transfer rate is

$$Q = \pi\, D_h\, L\, q_w \quad (27)$$

The non-dimensional heat flux and heat transfer rate are defined as follows:

$$q^* = \frac{q_w}{T_o k / L} = N_q\, \theta\, \lambda \quad (28)$$

$$Q^* = \frac{Q}{\dot{m} C_P T_o} = \frac{4 N_q\, \theta\, \lambda}{\mathrm{Re\,Pr}} \quad (29)$$

V. RESULT AND DISCUSSION

In this section, exergy and energy transfer characteristics are conducted for elliptical duct with internal longitudinal fins in laminar flow regime. Water has been used as working fluid. The thermophysical properties used are shown in Table 1. Respectively, the friction factors for fully developed laminar flow are given in Table 2.

TABLE 1
THERMOPHYSICAL PROPERTIES OF WATER

SYMBOL	QUANTITY
$C_P\,(J/kg\,K)$	4182
Pr	7
$T_w\,(K)$	293
$\mu(Ns/m^2)$	9.93×10^{-4}
$\rho(kg/m^3)$	998.2

TABLE 2
FRICTION FACTORS FOR FULLY DEVELOPED FLOW IN AN ELLIPTICAL DUCT WITH INTERNAL FINS [9]

α^*	L^*	$(f\,\mathrm{Re})$
	0.0	72.20
0.25	0.5	45.36
	0.8	69.34
	0.9	71.53
	1.0	67.26
0.5	0.0	67.26
	0.5	50.02
	0.9	68.78
	1.0	61.71
0.8	0.0	64.33
	0.5	50.72
	0.9	67.34
	1.0	58.91

Fig. 3 shows non-dimensional exergy flux for different ratio of the relative length of fins (L^*) at different Reynolds number values. As the relative length of fins (L^*) is increased non-dimensional exergy flux increases for fixed Reynolds number. With increasing of Reynolds number non-dimensional exergy flux values also decrease. For a fixed Reynolds number as relative length of fins (L^*) values are increased. Fig. 4 shows variation of non-dimensional heat transfer rate for different relative length of fins (L^*) and Reynolds numbers. As the relative length of fins (L^*) is increased non-dimensional heat transfer rate increases. With increasing of Reynolds number non-dimensional heat transfer rate values also decrease.

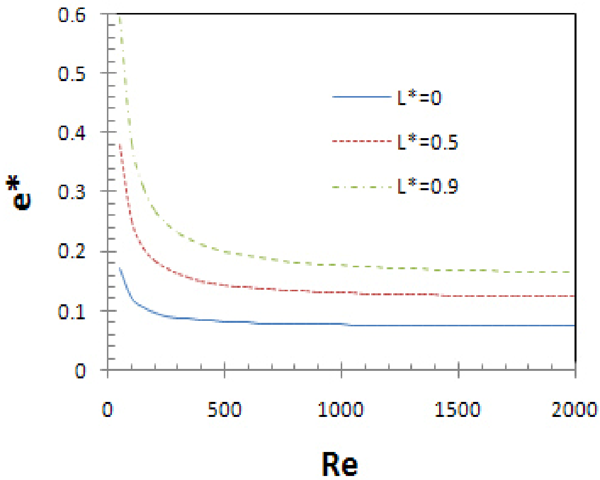

Fig. 3. Variation of non-dimensional exergy flux for different L* values and Reynolds number

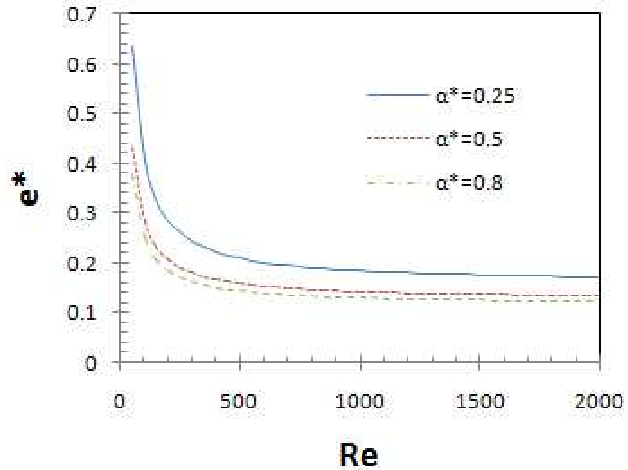

Fig. 5. Variation of non-dimensional exergy flux for different α* values and Reynolds number

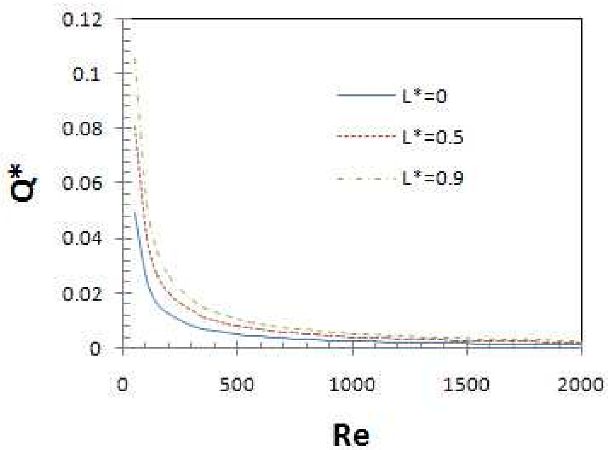

Fig. 4. Variation of non-dimensional heat transfer rate for different L* values and Reynolds number

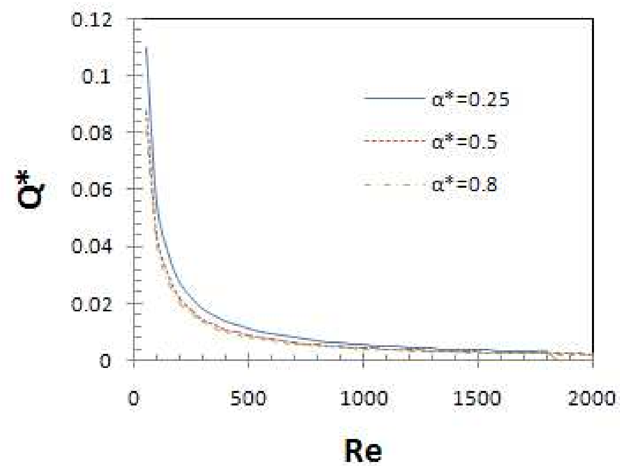

Fig. 6. Variation of non-dimensional heat transfer rate for different α* values and Reynolds number

Fig. 5 shows the effects of aspect ratio (α^*) on non-dimensional exergy flux at different Reynolds numbers for a fixed relative length of fins. As Reynolds number is increased, the non-dimensional exergy flux decreases. The decrease of non-dimensional exergy flux depends on the increase of aspect ratio (α^*) as expected. Fig. 6 shows variation of non-dimensional heat transfer rate for various aspect ratio (α^*) and Reynolds numbers. Increasing Reynolds number yields lower non-dimensional heat transfer rate values for fixed aspect ratio (α^*) values. As the aspect ratio (α^*) is increased non-dimensional heat transfer rate values decreases for fixed Reynolds number.

VI. CONCLUSION

In this present study, exergy and energy transfer characteristics are conducted for elliptical duct with internal longitudinal fins in laminar flow regime. Some conclusions can be given as follows:

As Reynolds number is increased non-dimensional exergy flux and non-dimensional heat transfer rate are increasing. With increasing value of relative length of fins (L^*) is increased non-dimensional exergy flux for fixed Reynolds number. The non-dimensional heat transfer rate increases as the relative length of fins (L^*) is increased.

When aspect ratio (α^*) is increased non-dimensional exergy flux and non-dimensional exergy flux are decreasing for fixed Reynolds number.

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