Modification of SST Turbulence Model on the Basis of Delay Time Concept for Unsteady Flows

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Abstract— **In this paper, the modification of SST turbulence model for an accelerated flow in a pipe is studied. In order to determine the characteristics of the fluid under the non-periodic accelerating conditions and using water as the working fluid, SST model has been investigated. Ramp-up experiments were performed in which the ramp rate of the bulk velocity was varied by imposing excursions of flow rate during which the Reynolds number increased linearly from an initial value of 7000 to a final value of 45200 in periods of time which ranged from 5 sec to 45 sec. It has been presented that the numerical values of turbulent kinetic energy, before the end of the delay time are less than the corresponding experimental data. In this research, it has been shown that by adding suggested factors to the third term of non-dimensional specific dissipation equation of SST model, which introduces the turbulence diffusion, the numerical results will be very close to the present experimental data. The modifying factor has been extracted on the basis of the delay time concept. This process is performed in two stages; before the end of the delay period and after that. This method of modification has been also applied for three different fluids and radius, and the results have been compared with the original turbulence model.**

Keywords– Unsteady Turbulent Flow, SST Turbulence Model, Delay Time and Pipe Flow

I. INTRODUCTION

Almost all fluid flows which are encountered in daily life are turbulent. There is no definition on turbulent flow, but it has a number of characteristic features such as; Irregularity, Diffusivity, Large Reynolds numbers, Being Three-Dimensional, Dissipation, and Continuity. Apart from being of practical importance in connection with various engineering applications, the study of unsteady turbulent pipe flow is of value in providing information which may led to an improved understanding of the phenomenon of turbulence. Unsteady turbulent pipe flows could be classified conveniently into two groups, namely periodic pulsating flows and non-periodic transient flows. In contrast to pulsating pipe flow, non-periodic transient pipe flow has received relatively little attention. The study of Maruyama, Kuribayashi & Mizushina (1976) was concerned with transient turbulent pipe flow. Delays were observed in the response of turbulence, which were found to be greater at the center of the pipe than close to the wall. It should be noticed that in this research He and Jackson's experimental results are used [1], [2]. Some striking features are evident in the response of the turbulence field to the imposed excursions of flow rate. The period of time from the start of an excursion to the point at which the faster response starts will be described here as the delay period τ .

This is clearly a function of radial position. Very near the wall the delay is less than 1s. The further the position is away from the wall, the longer is the delay. At the centre it approaches 4s [1]. But using the movable averaging method with 5 data (involving 2 points adjacent from the sides and the main considered point) and then time differentiation to calculate the slope of the alterations of the velocity fluctuations, the value of delay time is accurately obtained; τ =2.91s [3]. Three different delays have been identified: a delay in the response of turbulence production; a delay in turbulence energy redistribution among its three components; and a delay associated with the radially propagation of turbulence. The latter is the most pronounced under the conditions of the present study [1].

In this paper three turbulence models will be considered; k-ε Lam-Bremhorst [4], k-ω Wilcox [5] and Shear Stress Transport (SST) k-ω model [6]. Since SST model predicts the period of delay time as well as the existed experimental data, so this model has been modified on the basis of delay time. This is performed by adding a factor to the third term of non-dimensional specific dissipation equation. The special manner of modifying SST model will be represented later.

II. SHEAR STRESS TRANSPORT k-ω MODEL (SST)

The shear stress transport k-ω model is a two equation k-ε/k-ω hybrid designed in the form of k-ω scheme which was created by Menter in 1994 [6]. The formulation of k-ω is superior to other formulations with regard to numerical stability. Baseline Model (BSL) is the basis of SST. The idea behind the BSL was to retain the robust and accurate formulation of the Wilcox k-ω model in the near wall region, and to take advantage of the free stream independence of the k-ε model in the outer part of the boundary layer. To achieve this, the k-ε model is transformed into a k-ω formulation. The difference between BSL and the original k-ω model is that an additional cross-diffusion term appears in the ω equation.

A modification to the eddy viscosity on the basis of the philosophy underlying the Johnson-King model [7] brought into Shear Stress Transport (SST). Johnson-King model holds that the transport of the principal turbulent shear stress is of vital importance in the prediction of the severe adverse pressure gradient flows. SST k-ω model has been carefully fine tuned and tested for a large number of challenging research flows.

This model leads to a significant improvement for all flows involving adverse pressure gradients and should be the model of choice for aerodynamic applications. It is the only available two-equation model that has demonstrated the ability to accurately predict pressure-induced separation and the resulting viscous-inviscid interaction [8], [9]. Non-dimensional form of SST model is:

$$k^+ = \frac{k}{u_\tau^2} \quad , \quad \omega^+ = \frac{\nu}{u_\tau^2}\omega \qquad (1)$$

$$\nu_t^+ = \frac{a_1 k^+}{\max\left(a_1\omega^+, \left|\omega^+ F_2\right|\right)} \qquad a_1 = 0.31 \qquad (2)$$

$$\frac{\partial k^+}{\partial t^+} = \nu_t^+\left(\frac{\partial u^+}{\partial y^+}\right) - \beta^* k^+\omega^+ + \frac{\partial}{\partial y^+}\left[\left(1+\sigma^*\nu_t^+\right)\frac{\partial k^+}{\partial y^+}\right] \qquad (3)$$

$$\frac{\partial \omega^+}{\partial t^+} = \overbrace{\alpha \nu_t^+ \frac{\omega^+}{k^+}\left(\frac{\partial u^+}{\partial y^+}\right)^2}^{term\ 1} - \overbrace{\beta \omega^{+2}}^{term\ 2} + \overbrace{\frac{\partial}{\partial y^+}\left[\left(1+\sigma\nu_t^+\right)\frac{\partial \omega^+}{\partial y^+}\right]}^{term\ 3} + \overbrace{2(1-F_1)\sigma_{w2}\frac{1}{\omega^+}\frac{\partial k^+}{\partial y^+}\frac{\partial \omega^+}{\partial y^+}}^{term\ 4} \qquad (4)$$

In the above equations, F_1, F_2 are known as the blending functions and the below variables are applied in the 1D non-dimensional specific dissipation equation:

k	Kinetic Turbulent Energy	u_τ	Frictional Velocity
ω	Specific Dissipation	ν_t	Kinematic Turbulent Viscosity
T	Time	u	Axial Velocity

III. THE MANNER OF MODIFICATION OF SST MODEL

Specifying the features of SST model, three types of turbulence models, involved; k-ε Lam-Bremhorst, k-ω Wilcox and SST k-ω were compared with the present experimental data, in a 5s time period ramp-up excursion. Figs. 1 and 2 show the corresponding modeled turbulent kinetic energy for different models and compared to the experimental data.

As a result , we found that SST turbulence model represents a good coincidence with the experimental data near the wall region, specifically for high Reynolds number, and also predicts delay time (as shown in Fig. 1) accurately rather than other considered models. But, in the core region the difference between the numerical results of SST model and the experiments is clearly obvious.

Just as observed from Fig. 1 and Fig. 2, the values of turbulent kinetic energy, before the end of the delay time, are less than the corresponding values for the experiments. Each term in the equation (4) plays a rule in the concept of turbulent flow.

Term 3 represents the diffusion specification of turbulence and Term 4, which has been obtained from transforming standard k-ε model into a k-ω formulation, is termed cross-diffusion. So adding a factor to these terms established the criterion of the present modification. Figs. 3 and 4 show the variations of the third and fourth terms of non-dimensional specific dissipation equation, for a 5s ramp-up excursion, with time.

As can be recognized from Figs. 3 and 4 the magnitude order of term 3 is larger than term 4, thus term 3 was considered to this manner of process. Applying a weighting factor to the third term will be performed in two stages; before the end of the delay period and also after that. The relation between Reynolds number related to the end of the delay time ($\mathrm{Re}_{d.t.}$) and the radial position is obtained by considering the experimental results for 5s ramp-up excursion as the following:

$$\mathrm{Re}_{d.t.}(y^+) = -0.1768 \times y^{+2} + 116.59 \times y^+ + 15161 \qquad (5)$$

In which:

$$y^+ = \left(\frac{y u_{\tau 0}}{\nu}\right) \qquad (6)$$

Using the experimental data for different imposed accelerations to the mean flow, the equation (5) can be developed to the other ramp-up excursions larger than T=5 sec. The below equation shows the above mentioned relation:

$$\mathrm{Re}_{d.t.} = (\mathrm{Re}_{d.t.})_{for\ T=5} - (0.000571\times T^3 - 0.0619\times T^2 + 2.2762\times T - 7.2)\times 1000 \qquad (7)$$

Before the mean flow responds to the imposed fluctuations, the modifying factor that should be added to the diffusivity term of specific dissipation equation is obtained as the below equation:

$$\alpha_3 = \mathrm{Re}_{d.t.} \times \mathrm{Re} \times 10^{-9} + 11.175 \times 10^{-6} \times y^{+2} - 4.809 \times 10^{-6} \times y^+ + 0.65 \qquad (8)$$

After the time-period of delay, α_3 is changed weakly with Reynolds number and y^+ as following:

$$\alpha_3 = (2.5 \times y^+ - 3.5833) \times \mathrm{Re} \times 10^{-7} + 0.002165 \times y^+ + 0.8 \qquad (9)$$

The modification process will be completed by applying the equations (8) and (9) into the code. In Fig. 5 to Fig. 10, the modified SST model will be compared with original SST and the existed experimental data for different turbulent parameters and also for different time period of accelerating pipe flows.

In this research, the modification method is also applied on three types of fluids with different viscosities and three different radius of pipe for turbulent kinetic energy. The results of comparison between modified and original SST models are represented in Fig. 11 and Fig. 12. Fig. 11 shows that trend the turbulence kinetic is similar to each other and is not dependant to the kind of fluid, but it depends to the Reynolds number. Fig. 12 illustrates that the turbulence characteristics are significantly dependant to size of pipe diameter. It is clear that for the first pipe (a) the delay time of center line passed at Reynolds number about 20000 and for the second and third ones this is about 25000 and 35000.

Therefore, the fluid flow responses are very different. Further, the numerical predictions using original and modified SST turbulence models are considerably different too. The trend of this difference is similar to the one found for pipe radius equal to 0.0253 (Fig. 5).

IV. NUMERICAL ASPECTS

The relevant model equations are solved using a finite–difference time-marching code. The code employs the second-order Crank-Nicolson discretization [10] on a non-uniform grid (stretched in the r direction) including the range of turbulence models. Because we invoke symmetry, therefore we solve the momentum and model equations in the pipe and impose zero-slope boundary conditions at the centerline on u and the model transport variables. Different mesh grids and time steps have been evaluated. The independency processes have been obtained for the characteristics such as axial mean velocity and turbulent shear stress. As can be seen from Fig .13 the characteristics with the least grids in radial direction do not have appropriate coincidence, so it means that some changes in the mesh grids can alter the results. It is shown that the results of increasing the grids from 400 to 600 have enough consistency to each other. Thus, we utilized 400 grid points between the wall and pipe centerline for each model.

A geometric grid stretching monotonically clusters the points near the wall, with the first grid point 0.1 initial wall units above the wall for all the models. It can be observed from Fig .14 that non-dimensional time step of 0.05 is the best selection. Therefore, the results are grid independent sufficiently with these values of grid points and time step.

V. CONCLUSIONS

In this paper SST model was tested for non-periodic transient turbulent flows in a pipe with working fluid of water and then the results have been compared with the present experimental data. As a result of this comparison for turbulent kinetic energy, we found that SST turbulence model represents a good coincidence with the experimental results near the wall region, specifically for high Reynolds number flows. This model also predicts delay effects accurately rather than other basic models. But, at the centre line, the difference between the numerical results of SST model and the experiments is considerable. It has been shown that the values of turbulent kinetic energy, before the end of the delay time are less than the corresponding values for the experiments. So adding a weighting factor to the third term of non-dimensional specific dissipation equation that represents the diffusivity specification of turbulent flows established the criterion of the modification of SST on the basis of the delay time concept. The presented factor is applied in two stages; before passing the delay time and after that. Using experimental data, a relation between Reynolds number of the end of the delay period and the radial position extracted. The modifying factor is determined on the basis of the mentioned Reynolds number in two noticed stages. So, we could represent 2 equation relation for the factor that should be added to the turbulence diffusion term of specific dissipation equation. It seems that these modified equations will satisfy the different time period ramp-up excursions, and also different working fluids and geometric conditions.

VI. FIGURES

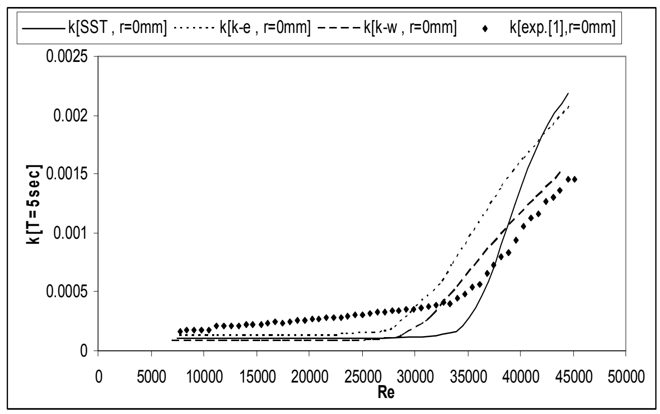


Fig . 1. Variation of turbulent kinetic energy in a 5s time period ramp-up excursion at the centre line

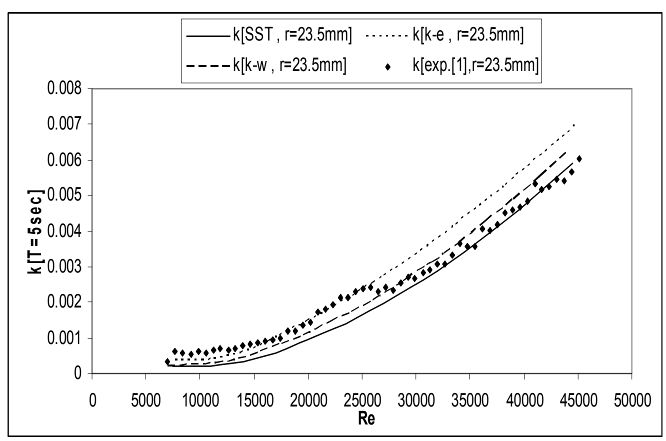


Fig. 2. Variation of turbulent kinetic energy in a 5s time period ramp-up excursion near the wall region

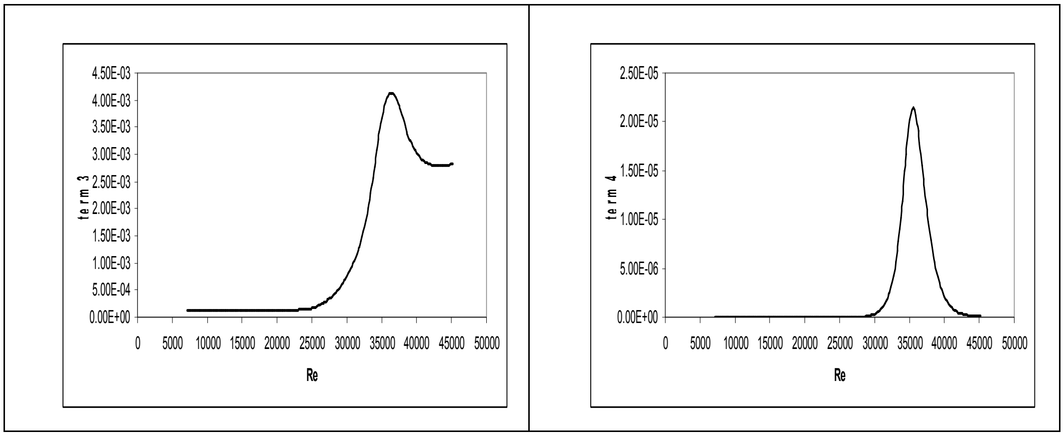


Fig. 3. Comparing the order magnitude of the terms in non-dimensional specific dissipation equation at the centre line in a 5s time-period ramp-up excursion

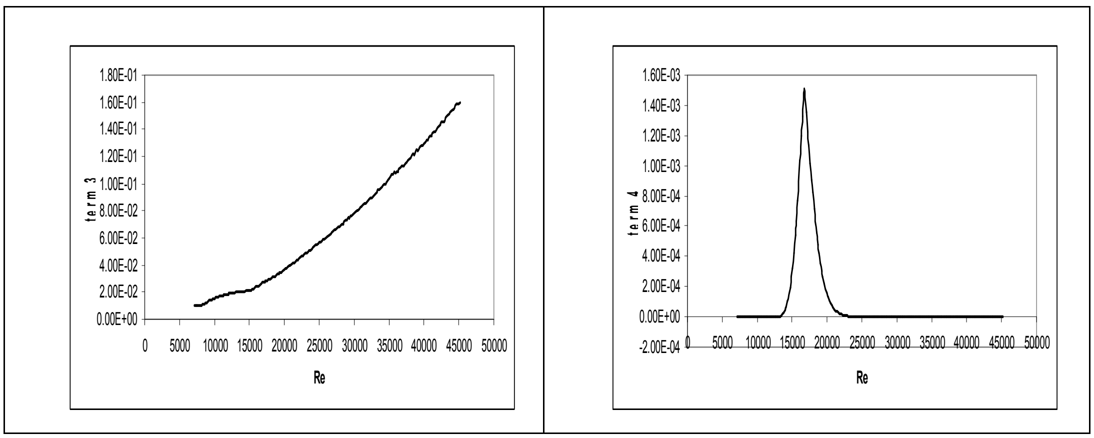


Fig. 4. Comparing the order magnitude of the terms in non-dimensional specific dissipation equation near the wall region in a 5s time-period ramp-up excursion

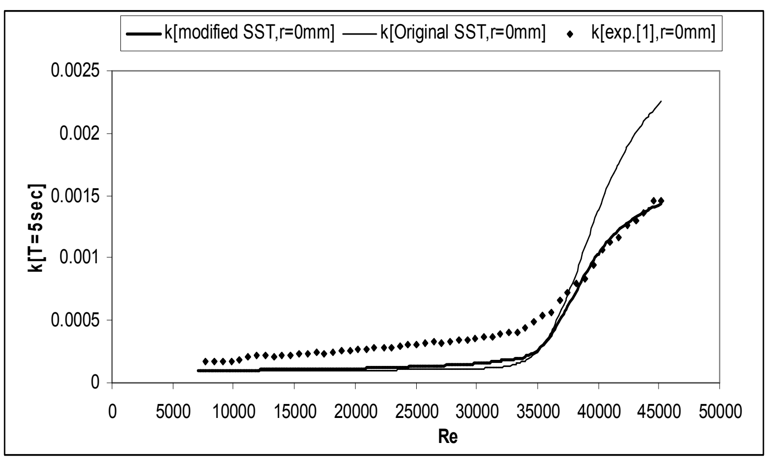


Fig. 5. Comparing modified and original SST models with the experimental data at the centre line in a 5s ramp-up excursion

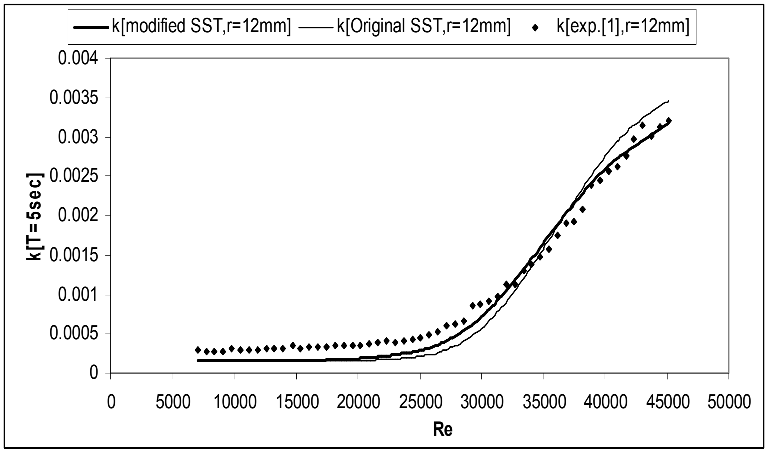


Fig. 6. Comparing modified and original SST models with the experimental data at 12 mm from the centre line in a 5s ramp-up excursion

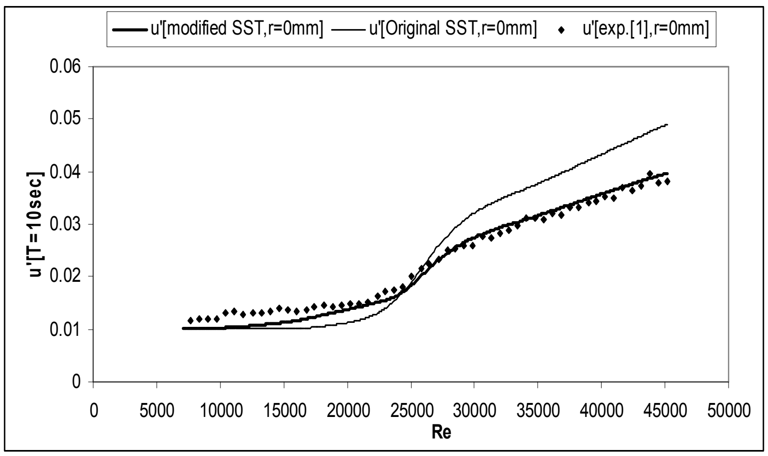


Fig. 7. Comparing modified and original SST models with the experimental data at the centre line in a 10s ramp-up excursion

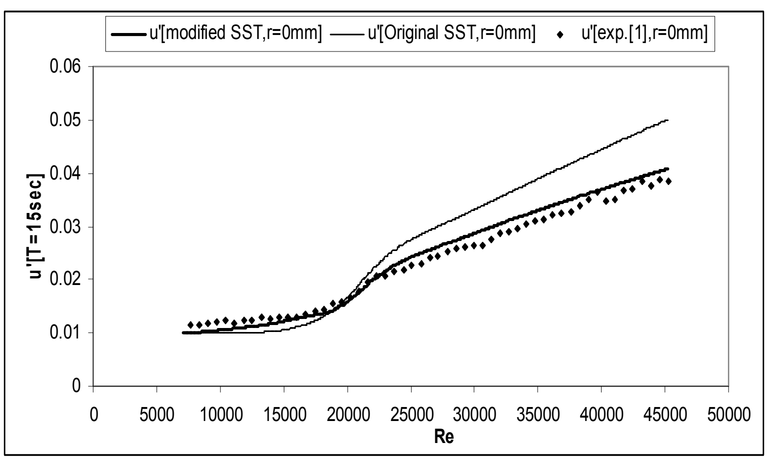


Fig. 8. Comparing modified and original SST models with the experimental data at the centre line in a 15s ramp-up excursion

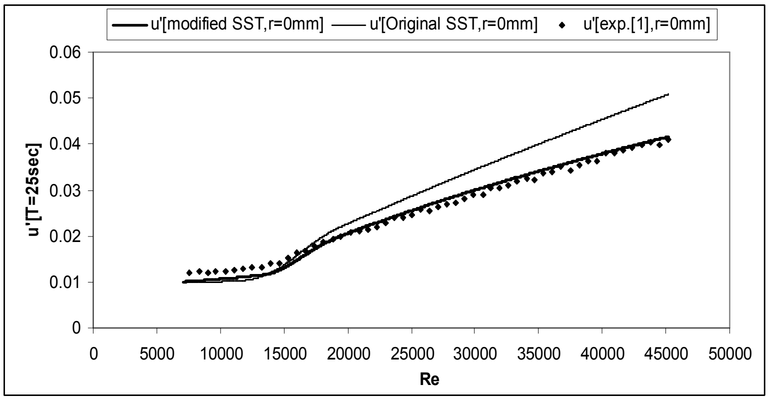


Fig. 9. Comparing modified and original SST models with the experimental data at the centre line in a 25s ramp-up excursion

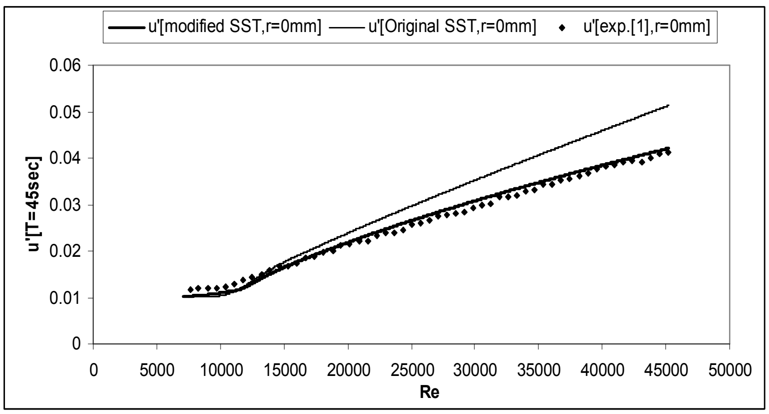
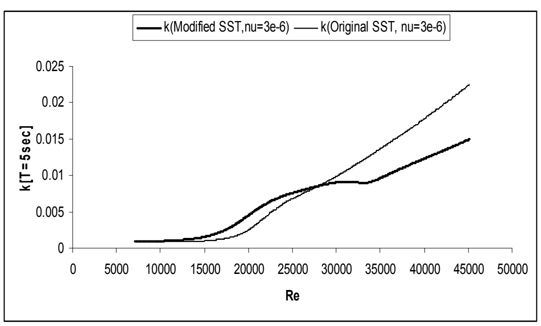
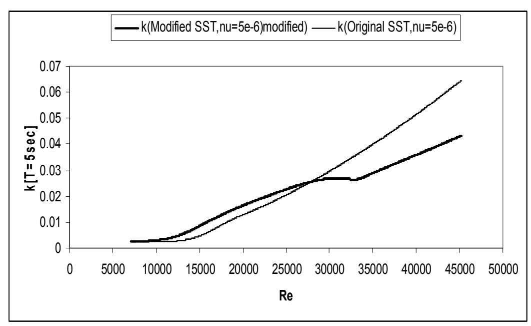


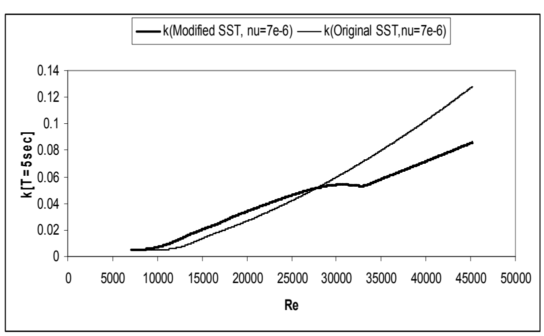
Fig. 10. Comparing modified and original SST models with the experimental data at the centre line in a 45s ramp-up excursion



(a)

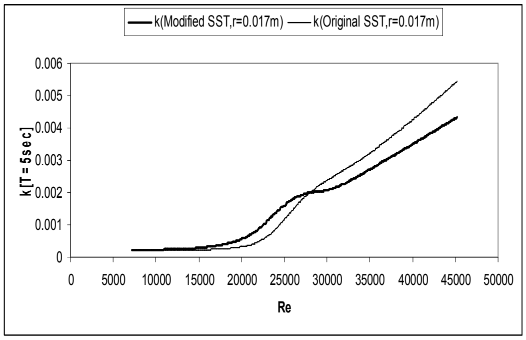


(b)

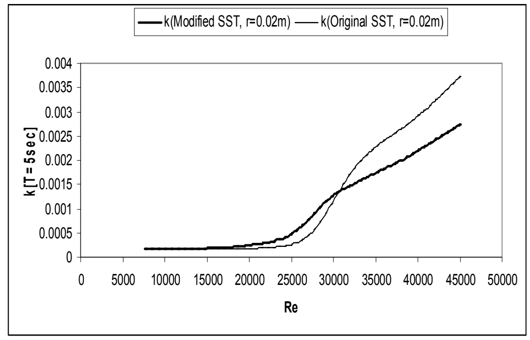


(c)

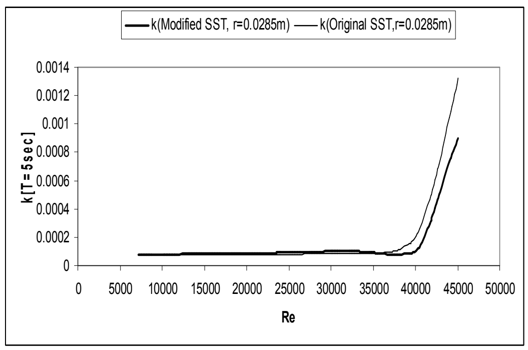
Fig. 11. Comparing modified and original SST models for different fluids at the centre line in a 5s ramp-up excursion



(a)



(b)



(c)

Fig. 12 . Comparing modified and original SST models for different radius of the pipe and water as the working fluid at the centre line in a 5s ramp-up excursion (a) r=0.017m , (b) r=0.02m (c) r=0.0285m

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