Analysis of a Laboratory Bicycle Ergometer System Using Grand Canonical Ensemble

Chwukwuneke J.L., Achebe C.H., Okolie P.C., and *Chukwumuanya E.O.

Department of Mechanical Engineering, NnamdiAzikiwe University, PMB-5025, Awka, Nigeria

*Department of Industrial/Production Engineering, NnamdiAzikiwe University, PMB-5025, Awka, Nigeria

(Corresponding Author's email: jl.chukwuneke@unizik.edu.ng)

Abstract– **This research work, "analysis of a laboratory bicycle ergometer system using grand canonical ensemble", was employed to study the muscular work, the heartbeat and the rate of energy loss when a body undergoes a particular work. The essence was to understand and to interpret measurable macroscopic properties of a system and thus establish valid relationships among measureable variables in analytical manner over time. The system was integrated with the use of free surface energy while applying appropriate thermodynamic ensemble to get absolute values for the measured properties. The system was analysed and regression line equation of R=2.4W-2.24 was obtained. The results obtained from both composite trapezoidal rule and regression approach to the system analysis was compared. Absolute values of energy and heartbeat were obtained as E_{abs} = 27711.936 Calorie and $H_{b(abs)}$ = 3955.5 per min respectively.**

Keywords– Ergometer, Ensemble, Grand Canonical, Macroscopic Properties, Regression and Statistical Thermodynamics

I. INTRODUCTION

Bicycle ergometer is equipment in human factor engineering for measuring muscle work. In friction-type bicycle ergometer, the subject is made to work against friction created on ergometer flywheel by a brake belt mechanism. As the subject pedals the bicycle, the flywheel rotates against the brake belt around its rim. The brake belt is tensioned by the addition of weights on one end while the other end is attached to a spring balance.

The difference between the weight added and the spring balance reading when cycling gives the frictional force, F, on the flywheel [1].

$$W = Fs = Fr\theta \quad (1)$$

$$\text{Where; } S = r\theta \quad (2)$$

R = Radius of flywheel, θ = Angular displacement in radians.

With the heart beat type, the bike is pedalled at a constant speed of 18-20 rpm for about 3minutes. A stop watch is used to measure the time and a stethoscope for the measurement of the heartbeat. The stethoscope is connected to the ear of the subject (human) to measure the time for rotation of the pedal after each set-point. The readings for the 10 heart beats and energy lost are measured.

The most important feature of the bicycle ergometer is the ability to set a constant external work rate that is maintained independent of pedalling rhythm; many of the cycling related investigations in the laboratory have involved a manipulation of pedalling rhythms and the result of such research would be much more difficult to interpret if the subjects are working at different power output for each rhythm. The advantage of the bicycle ergometer is the ability to have subjects use their own bicycle frame for testing and also enables the subject to work at a constant work load or power output which is independent of rhythm. This implies that the rhythm and the rear wheel resistance are inversely proportional and this provides a valuable tool for conducting experiments at constant power output laboratories.

The underlying theory is based on the principle that for a rotating flywheel bicycle the difference between the weights added and the spring balance reading when cycling gives the frictional force "F" on the flywheel [1]:

$$W = Fs = Fr\theta \quad (3)$$

$$\text{Where; } S = r\theta \quad (4)$$

r = radius of flywheel, θ = Angular displacement in radians

But when a number of revolutions are made

$$W = 2F\pi rn = F\pi Dn \quad (5)$$

Where; D = Diameter of flywheel, n = number of revolution of flywheel.

To calibrate a subject on a bicycle ergometer, the subject's cycle at a given temperature and relative humidity for 3-5mins at different load levels (5 to 40N) from which the work done is calculated is required.

The heart rate of the subject is measured after each load level cycling. The heart rate is calculated from the first 10 heartbeats counted with the stethoscope and timed with a stopwatch immediately after 3-5mins cycling. There may be 2 or 3 replications at each load level.

The regression is computed to find the intercept and slope. The energy expenditure of any work done in the calibration environment can be obtained from the calibration graph if the heart rate is measured during the exercise. In every heart rate measurement the subject is allowed to work for 3-5mins before measurement is taken. This is to enable him attain the steady state of metabolism before such measurement is obtained.

Therefore, this principle provides the regression equation which shows that [2], [3];

$$R = mW + R_o \quad (6)$$

Where; R = heart rate, m = slope, W = workdone, R_o = intercept

$$R_o = \frac{[N(\Sigma R)(\Sigma W^2)] - [(\Sigma R)(\Sigma WR]}{[(N(\Sigma W^2) - (\Sigma W)^2]} \quad (7)$$

$$m = \frac{[N(\Sigma WR) - (\Sigma W)(\Sigma R)]}{[(N\Sigma W^2) - (\Sigma W)^2]} \quad (8)$$

Where N = number of observations.

II. MATERIALS AND METHODS

A. The Apparatus

The apparatus include the following: bicycle ergometer, stethoscope, stop watch/set watch, and spring balance.

B. Experimental Procedures

Step 1: Identify the tension-control knob, the instrument panel with electronic display, the bicycle pedal, etc. Notice that the instrument pedal is compact and small and can measure and display multiple variables such as speed, distance, time, calorie; etc. The reddish-brown knob on the instrument allows users to change the variables (parameters) on display by the instrument. The knob also allows the user to erase or zero the displayed values. Notice also that the tension-control knob has 7 x 360^0 of freedom, where n stands for the number of 360^0 rotation and in this case n = $7^1/_2$ rotations.

Step 2: Loosen the bike pedal by turning the tension-control knob anticlockwise until limit is reached. Note that clockwise moment tightens the bike pedal while anticlockwise moment loosens it. Mark a datum line on both the retaining and stationary parts of the knob to allow the determination of 360^0 rotation during the course of the experiment. Remember the knob has n = $7^1/_2$ rotation. Each of the complete rotation, n, constitutes an experiment set point, giving rise to $7^1/_2$ set points.

Step 3: At this set point, pedal the bike at constant speed say 18-20rpm (30m/s), maintaining the constant speed for the duration of 3 minutes timed or measured by you using the stop watch provided. On the other hand, note and record the values at 10 heart beats.

Step 4: Repeat step 3 for each of the set points, namely; 0^0 and 7 x 360^0, respectively, where n is represented by 1, 2, 3, 4, …, 7, respectively.

III. DATA PRESENTATION

Table 1: Measurements Obtained for Divers Ergonomic Variables

S/N	Set Point 'S' n x 360^0Tensioning	No. of Heart Beats 'N'	Time for the Heart Beats 'R' (Sec)	Time T_X(mins) for Stopwatch Measurement	Time Ty(mins) Measured by Bike's Time Scale	Energy 'E' (Calorie)
1	0 x 360^0	10	7.30	3.00	3.04	11.90
2	1 x 360^0	10	7.00	3.00	3.03	11.60
3	2 x 360^0	10	6.80	3.00	3.02	11.40
4	3 x 360^0	10	6.60	3.00	3.01	11.20
5	4 x 360^0	10	6.50	3.00	3.01	11.00
6	5 x 360^0	10	6.40	3.00	3.00	10.90
7	6 x 360^0	10	6.30	3.00	3.00	10.70
8	7 x 360^0	10	6.20	3.00	3.00	10.60

Table 2: Derivatives of Various Measurable Quantities I

S/N	Set Point 'S' n x 360^0 Tensioning	No. of Heart Beats 'N'	Time for the Heart Beats 'R' (Sec)	'E' Energy (Calorie)	Heart Beat per min (H_b)	'W' Energy per Time (Calorie/min)
1	0 x 360^0	10	7.30	11.90	82.20	3.97
2	1 x 360^0	10	7.00	11.60	85.70	3.87
3	2 x 360^0	10	6.80	11.40	88.24	3.80
4	3 x 360^0	10	6.60	11.20	90.91	3.73
5	4 x 360^0	10	6.50	11.00	92.31	3.67
6	5 x 360^0	10	6.40	10.90	93.75	3.63
7	6 x 360^0	10	6.30	10.70	95.24	3.57
8	7 x 360^0	10	6.20	10.60	96.77	3.53

Table 3: Derivatives of Various Measurable Quantities II

S/N	Set Point 'S' n x 360^0 Tensioning	Time T_y(mins) measured by Stopwatch	Time Tx(mins) measured by Bike's Time Scale	ΔT(mins) (Ty – Tx)	Log^{-1}(Ty – Tx) (mins)	$\frac{Ty - Tx}{Ty}$	$\frac{1}{Log}\left(\frac{Ty - Tx}{Ty}\right)$
1	0 x 360^0	3.00	3.04	-0.04	0.91	-0.0133	0.970
2	1 x 360^0	3.00	3.03	-0.03	0.93	-0.01	0.977
3	2 x 360^0	3.00	3.02	-0.02	0.95	$-6.67x10^{-3}$	0.985
4	3 x 360^0	3.00	3.01	-0.01	0.97	$-3.3x10^{-3}$	0.922
5	4 x 360^0	3.00	3.01	-0.01	0.97	$-3.3x10^{-3}$	0.922
6	5 x 360^0	3.00	3.00	0.00	1.00	0.00	1.00
7	6 x 360^0	3.00	3.00	0.00	1.00	0.00	1.00
8	7 x 360^0	3.00	3.00	0.00	1.00	0.00	1.00

Table 4: Table of Divers Measured and Derived Values

S/N	R(s)	W (Cal/min)	W^2 (Cal/min)^2	WR (Cal/sec)	Ty(min)	Tx(min)	ΔT(mins) Ty – Tx	Log^{-1} (Ty – Tx)	$Log^{-1}\left(\frac{Ty - Tx}{Ty}\right)$
1	0 x 360^0	3.97	15.76	28.98	3.00	3.04	-0.04	0.91	0.970
2	1 x 360^0	3.87	14.98	27.09	3.00	3.03	-0.03	0.93	0.977
3	2 x 360^0	3.80	14.44	25.84	3.02	3.02	-0.02	0.95	0.985
4	3 x 360^0	3.73	13.91	24.62	3.00	3.01	-0.01	0.97	0.992
5	4 x 360^0	3.67	13.47	23.86	3.00	3.01	-0.01	0.97	0.992
6	5 x 360^0	3.63	13.18	23.23	3.00	3.00	0.00	1.00	1.00
7	6 x 360^0	3.57	12.75	22.49	3.00	3.00	0.00	1.00	1.00
8	7 x 360^0	3.53	12.46	21.89	3.00	3.00	0.00	1.00	1.00

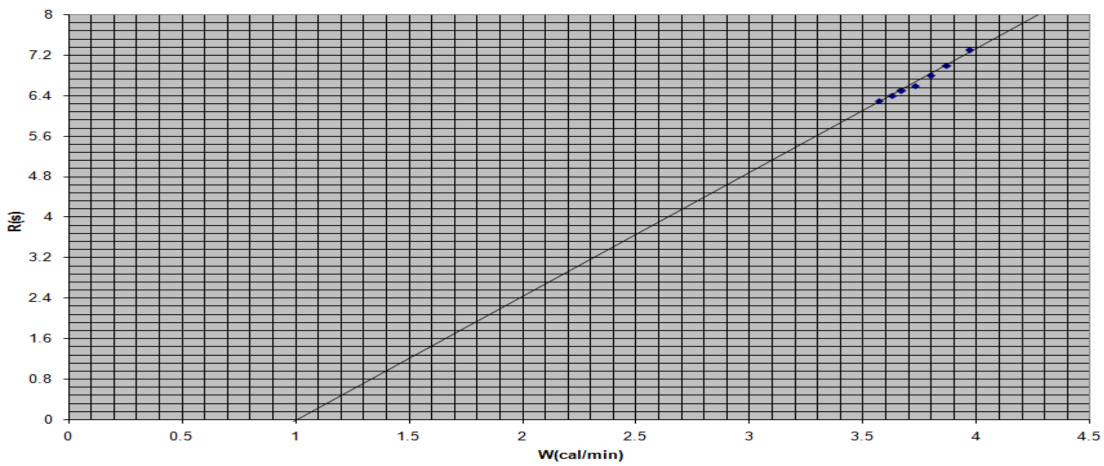

Fig. 1: Graph of R(s) versus W(Cal/min)

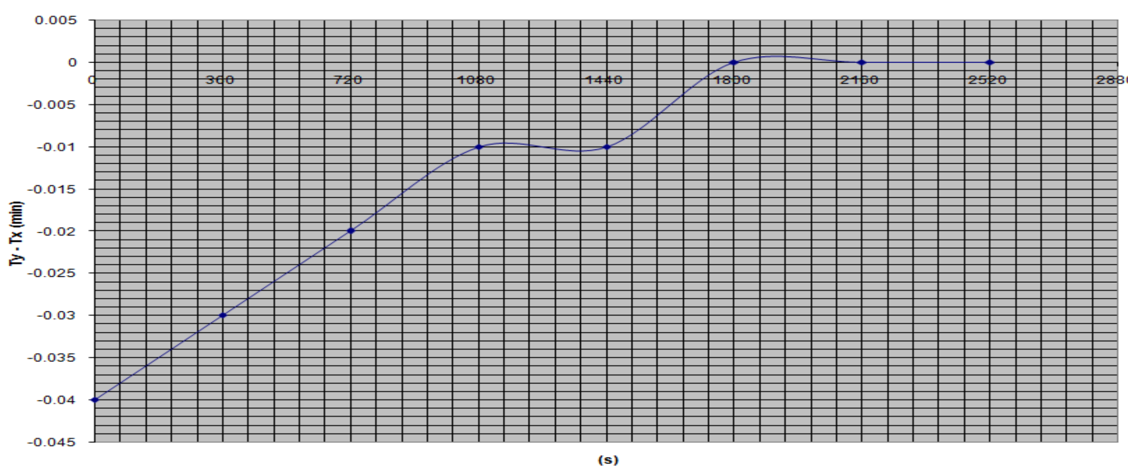

Fig. 2: Graph of $\Delta T(T_y\text{-}T_x)$ versus Setpoint(S)

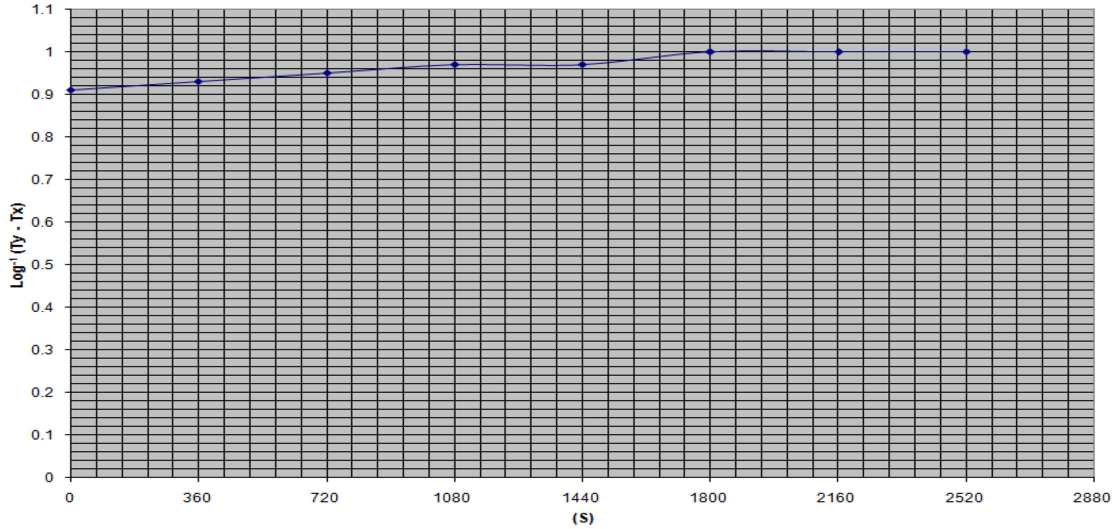

Fig. 3: Graph of $\text{Log}^{-1}(T_y - T_x)$ versus Setpoint (S)

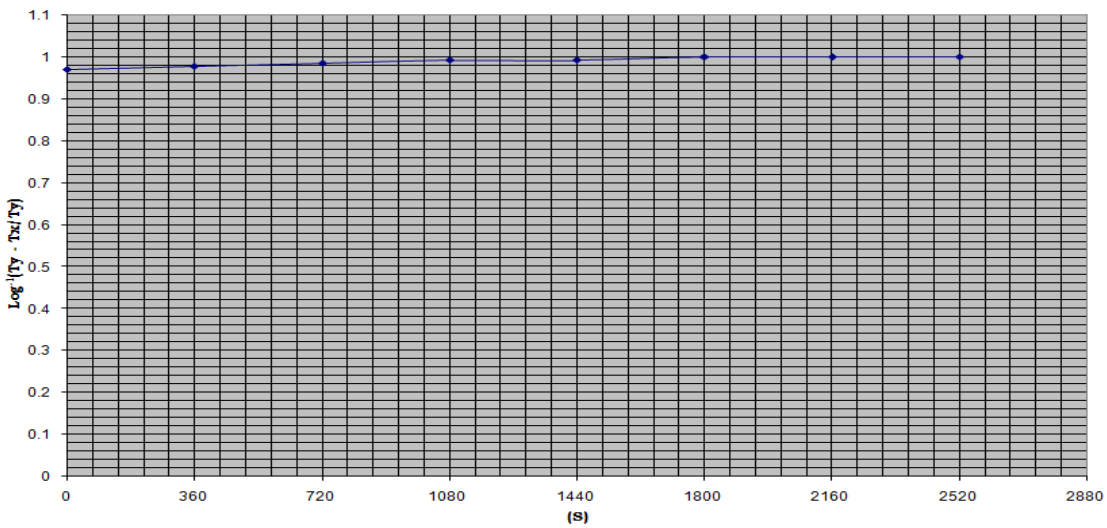

Fig. 4: Graph of $\text{Log}^{-1}[(T_y-T_x)/T_y]$ versus Setpoint (S)

A. Data Analysis and Application

Applying Composite Trapezoidal Rule modified with Romberg formula to the system [4]; (for Energy):

If $E = E(s)$

$$\int_a^b E\,ds = \frac{h}{2}\left[E_0 + 2\sum_{i-1}^{N-1}(E_i + E_N)\right] \qquad (9)$$

Where; h (step size) = (b-a)/N (10)

From the measured value; E_o = Initial value of E

h = (2520-0)/7 = 360.

Absolute E; $E_{abs} = \int_0^{2520} E\,ds = 360/2\,[11.90+2(66.8)+10.60]$

E_{abs} = 28098 Calorie = 28.098kCal

Applying Composite Trapezoidal Rule modified with Romberg formula to the system [4]; (for Heart Beat H_b):

$H_b = H_b(s)$ then; $\int_a^b H_b\, dS = h/2\, [H_b + 2\sum_{i-1}^{N-1} H_{bi} + E_N]$

Where; h= 360

$H_{b(abs)} = \int_0^{2520} E dS = 360/2\, [82.20+2(569.91)+96.77]$

$H_{b(abs)} = 395.37$ per min

Applying Regression Approach to the system [5] (for Energy);

E = E (s)

Let the approximating polynomial be given by: $E = \alpha_0 + \alpha_1 S + \alpha_2 S^2$

Define a variable $X = \begin{pmatrix} 1 & S_0 & S_0^2 \\ 1 & S_1 & S_1^2 \\ 1 & S_2 & S_2^2 \\ 1 & S_3 & S_3^2 \\ . & . & . \\ . & . & . \\ 1 & S_n & S_n^2 \end{pmatrix}$

Such that;

$E_i = [x]; \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} \qquad E_i = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 360 & 12960 \\ 1 & 720 & 518400 \\ 1 & 1080 & 1166400 \\ 1 & 1440 & 2073600 \\ 1 & 1800 & 3240000 \\ 1 & 2160 & 4665600 \\ 1 & 2520 & 6350400 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$

$\alpha_0 = 11.8788, \alpha_1 = -0.0007, \alpha_2 = 0.0000;$

Therefore; E= 11.87988 – 0.0007s.

$E_{abs} = \int_0^{2520} E dS = [11.8788s - 0.0007s^2/2] = 27711.936$Calorie

Applying Regression Approach to the system [5] (for Heartbeat H_b);

Let the approximating polynomial be given by;

$$H_b = \alpha_0 + \alpha_1 S + \alpha_2 S^2 \quad (11)$$

Define a variable x = $\begin{pmatrix} 1 & S_0 & S_0^2 \\ 1 & S_1 & S_1^2 \\ 1 & S_2 & S_2^2 \\ 1 & S_3 & S_3^2 \\ . & . & . \\ . & . & . \\ 1 & S_n & S_n^2 \end{pmatrix}$

Such that;

H_{bi}= [x] $\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$; H_{bi} = $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 360 & 12960 \\ 1 & 720 & 518400 \\ 1 & 1080 & 1166400 \\ 1 & 1440 & 2073600 \\ 1 & 1800 & 3240000 \\ 1 & 2160 & 4665600 \\ 1 & 2520 & 6350400 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$

α_0 = 96.5725, α_1 = -0.0019, α_2 = 0

Therefore; H_b= 96.5725 – 0.0019s

$H_{b(abs)} = \int_0^{2520} E dS = [96.5725s - 0.0019s^2/2] \Big/_0^{2520} = 3955.5$ per min

Comparing the absolute values of Energy ‘E_{abs}’ and Heartbeat ‘$H_{b(abs)}$’ using both composite Trapezoidal Rule and Regression Approach, we observed that the absolute values are approximately equal. Since the time is constant for all set-point, the pedalling for all set point remains 180 secs. Using the table of values, the regression equation is computed to obtain the slope and intercept.

$$R = mW + R_o$$

Where; R = heart rate,m = slope,W = work rate, R_o = Intercept

Slope, $m = \frac{[N(\Sigma WR) - (\Sigma W)(\Sigma R)]}{[(N\Sigma W^2) - (\Sigma W)^2]}$ Where; N = 8

$$= \frac{(8x198) - (29.77x53.1)}{(8x110.95) - (29.77)^2} = 2.4$$

Intercept, $R_o = \frac{[N(\Sigma R)(\Sigma W^2)] - [(\Sigma R)(\Sigma WR]}{[(N(\Sigma W^2) - (\Sigma W)^2]}$

$$= \frac{(53.1x110.95) - (29.77x198)}{(8x110.95) - (29.77)^2} = -2.24$$

From the graph of R(s) versus W(Cal/min) as in fig.1 above;Slope, m = 2.5, andIntercept, R_o = -2.32. Therefore, the regression line equation will be;R = 2.4W – 2.24

B. Discussion

Fig.1 shows a linear graph. This means that the heart rate increases linearly with the amount of work done. This is because the graph indicates that as heart rate increases, the energy spent also increases. The energy spent (E) is the amount of calorie used to the time interval (3 minutes).This energy in the experiment increases linearly and positively with the time rate. Thus, it can be said that the heart rate is a direct measure of energy spent during the process of doing work.

Fig. 2, Fig. 3 and Fig. 4 are similar in shape but the points in the graph of $(T_y - T_x)$ against 'S' are most widely spaced followed by that of $Log^{-1}(T_y - T_x)$ against 'S'. Also the points in the graph of $Log^{-1}\frac{Ty - Tx}{Ty}$ are more tightly packed together.

Another major difference, is that the value of the graph of $(T_y$-$T_x)$ versus 'S' lies in the negative side of the graph. The values of $(T_y$-$T_x)$ are almost all negative only for the three zero's that are present at the bottom of the values but in the case of $Log^{-1}(T_y$-$T_x)$ versus 'S', and $Log^{-1}\frac{Ty - Tx}{Ty}$ versus 'S' the values lie in the positive sides.

From the graphs, it can be seen that the relationship between $(T_y$-$T_x)$, $Log^{-1}(T_y$-$T_x)$ and $Log^{-1}\frac{Ty - Tx}{Ty}$ with 'S' is non-linear.

One reason, for the difference in value of T_y and T_x from 360^0 to 1440^0 set-point is due to skipping between the pulley and the bicycle belt. This is also an indication that there is a minimum tension in the belt and that the time calibration will be the same as that on the stop watch. This is also an indication that a minimum effort is also required. Other factors that may bring about this inconsistency are the inefficiencies of the rider of the bicycle and the stop watch operator. Also, the state of the bicycle ergometer may be responsible for some inconsistency.

IV. APPLYING A THERMODYNAMIC ENSEMBLE

The system described above is an open isothermal system thus the grand canonical ensemble or macro canonical ensemble is implied. Since the second law of thermodynamics applies to open isothermal systems, the grand canonical ensemble also describes an open isothermal system. In grand canonical ensemble μ, V and T are fixed [6], [7].

A grand canonical ensemble (macro canonical ensemble) is an imaginary collection of model system put together to mirror the calculation probability distribution of microscopic state of a given physical system which is being maintained in a given microscopic state. Since an open system does not keep a constant energy, the total amount of energy in the system will fluctuate. Thus, the system can access only those of its macro-state that corresponds to a given value of the energy, E. The equation for the grand canonical ensemble can be represented as [8];

$$PV = K_B T \, ln \, (\Omega(E)) \qquad (12)$$

Since V and T are constant, and assumed unity. Representing PV as the absolute Heartbeat $(H_{b,abs})$ and $(\Omega(E))$ as the absolute Energy, (E_{abs}) supplied to the bicycle.

Therefore; $H_{b,abs} = K_B \, ln \, (E_{abs})$ (13)

Since the number of particles of the system cannot be easily fixed, then it is convenient to use the grand canonical ensemble. The energy of the system does not keep a constant value. The system can access only those of its macro-state that corresponds to a given value of 'E' of the energy, since the total amount of energy in the system will fluctuate. The equation for the grand canonical ensemble can be represented as [7], [9];

$$PV = \frac{KT}{n} ln(\mu) \qquad (14)$$

Where, $ln(\mu)$ is known as fugacity

$$E = KT^2 \frac{\partial}{\partial T} ln\acute{z}(\mu, V, T) \qquad (15)$$

Therefore; $E = KT^2 ln\mu$ (16)

But; $\mu = KT \, ln\acute{z}$ (17)

$$\Rightarrow ln\acute{z} = \mu / KT \qquad (18)$$

Thermodynamics potentials can be obtained through linear combination of above quantities

$$F = N\mu - PV = -KT \, ln(\acute{z}/_{\acute{z}^N}) \qquad (19)$$

Note: The system under study conforms to Grand canonical ensemble $(N_V T - ensemble)$

$$\beta = {}^1/_{KT} \qquad (20)$$

A comparison of these equations with the theoretical equations of the system under study gives absolute/integrated values of Energy, E_{abs}, and Heartbeat, H_b which aids in the analysisof the efficacy of the system. The partition function can be used to find the expected (integrated) value of any microscopic property of the system, which can be related to microscopic variables. Since there is a variation between H_b and E, a constant of proportionality must define the relationship. This proportionality constant, K_B is gotten from the application of thermodynamics ensemble as seen in eqn.13.

From eqn.13;$K_B = \frac{H_{b,abs}}{\ln(E_{abs})} = \frac{90.64}{2.4126} = 37.5694 =$ *Constant of proportionality.*

Where, K_B describes the constant that relates H_b and E.

It could be noted that all relations established in this research work are valid only for 3.00 Time, T_x(mins) measured by stop watch and between 3.00 and 3.04 Time T_y(mins) measured by Bikes time scale. This is so because, it is the range at which the functionality of Human Energy can be guaranteed.

V. CONCLUSION

This research work shows that there is a relationship between the rate of energy expenditure and heartbeat rate. The higher the energy expenditure, the higher the heartbeat recorded. This is the reason the graph of fig.1is linear. It therefore could be said that the energy expenditure is directly proportional to the rate of heartbeat. There is a minimum tension required in the bicycle belt for its time scale to correspond with that of the stop watch. There is always a critical time for each tension in the belt for this synchronism to obtain when riding at a constant speed. Since there is exchange of energy and interaction between the system and the object (human), an open isothermal system is implied. This necessitated the use of Grand Canonical Ensemble to obtain absolute values for the variables. This experiment therefore is recommended for students who are studying courses in human factor engineering and analytical thermodynamics.

REFERENCES

[1] Le Blane, J.A., (1957);Use of Heart Rate as an Index of Work Output, Applied Physiology 10, pp.275-280

[2] Edward H. Smith, Mechanical Engineering Reference Book, 2nd Ed., pp.15-25.

[3] Juha,Javanainen, (2008);Phys-5500, Statistical Mechanics, p.46.

[4] M.K. Jain, S.R.K. Iyengar and R.K. Jain, (2010); Numerical Methods for Scientific and Engineering computation, 5th Ed., pp.386-393

[5] Engene, A. Avallone, Standard Hand Book for Mechanical Engineering, 4th Ed., pp.92-99

[6] Nash, Leonard K., (1974); Elements of Statistical Thermodynamics, Dover PublicationsInc.

[7] Kittel and Kroemer, (1968); Thermal Physics, 2nd Ed., pp.49.

[8] Schrodiner, Erwin, (1946); Statistical Thermodynamics, Dover Publications Inc.

[9] L.D. Landau and E.M.Lifshitz, (1996); “Statistical Physics”, 3rd Ed., Part 1, Butterworth-Heinemann, Oxford

[10] Suggs, C.W. and Splinter, W.E., (1961); Time and Energy Analysis of Agricultural Tanks, Trans ASAE 1(1), pp.50-52.

[11] George, L. Oguejeiofor, ‘Manual on Bicycle Ergometer’ Mechanical Engineering Department, NnamdiAzikiwe University, Awka, Nigeria, Unpublished.