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Evaluation of GPS in Orthometric Heights Determination in Khartoum State (Sudan)

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Abstract– **Levelling is the usual technique used to determine difference in height between ground points. Nowadays Global Positioning System (GPS) provides a quick modern technology to determine this difference. Because the datum surfaces are different in both techniques, different results may be expected. In this research work, difference in heights between numbers of points, in Khartoum State, were carried out using both ordinary levelling (in which Geoid is the reference datum) and GPS techniques (in which Ellipsoid is the reference datum). Results obtained showed that GPS data can be used directly to produce contour map up to 1:100,000 map scale with 25m interval. Results also assisted to determine the Geoid undulation in the area. Moreover, GPS data can be improved using EGM2008 Gravitational model, to be suitable for 1:25,000 contour map scales, with 6.25m contour interval.**

Keywords– GPS, Ordinary and Geodetic Levelling Orthometric Height and Geoid Undulation

I. INTRODUCTION

Number of quantities has to be measured in usual survey works. These quantities may be bearings, angles, or distances (horizontal, vertical or inclined). Reduction of theses values leads to determination of relative position of the points.

Levelling is the name given to the process of measuring the difference in elevation between two or more points. The heights of points relative to a chosen surface are known as reduced levels of these points. Reference surface is usually known as a datum.

In engineering surveying, leveling has many applications and is used in all stages in construction projects from the initial site survey through to the final setting out [2].

Levelling process can be carried out using different survey equipments and techniques. It may be precise; when precise optical or digital levels are used, or it may be ordinary; when using ordinary optical, automatic or digital levels. Moreover it can be carried out trigonometrically or barometrically.

II. ORDINARY AND GEODETIC LEVELLING

In short distances, difference in level between two points can easily be determined by setting a staff vertically in each point. Then, constructing a horizontal line using optical or digital level. Difference in the intersection points of the horizontal line with both staves produce the difference in level between these ground points Fig. 1. This technique is simply known as ordinary leveling [4].

In long distances, when geoid undulation of the earth has to be taken into account, direct levelling doesn't solve the problem and it will be more complicated. This is so because horizontal line does not solve the problem directly. Levelling technique taking the geoid undulation into account is usually known as geodetic levelling.

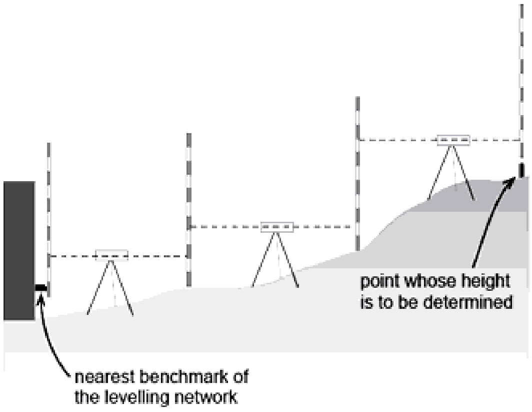

Fig. 1: Ordinary Levelling

III. GEOID AND ELLIPSOID

The ***Geoid*** is an equipotential of the gravity force surface located approximately at mean sea level, which is everywhere perpendicular to the direction of gravity. ***Orthometric levelling*** is the term used when using geoid as a reference datum. Because of variations in the earth mass distribution, the geoid has an irregular shape [3].

Regular shape can be approximated by the ***Ellipsoid***. Ellipsoid is a mathematical surface obtained by revolving an ellipse about its minor axis. Levelling based on taking the geoid as a reference datum is then termed ***Geodetic levelling***. The dimensions of the ellipse are selected to give a best fit of the ellipsoid to the geoid over a large area, and are based upon surveys made in the area.

A two-dimensional view which illustrates conceptually the geoid and ellipsoid is shown in Fig. 2. As illustrated, the geoid contains non uniform undulations, and is therefore not readily defined mathematically. Ellipsoids, which approximate the geoid and can be defined mathematically, are therefore used to compute positions of widely spaced points that are located.

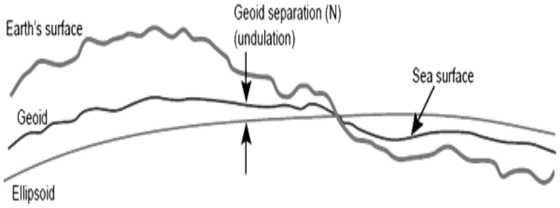

Fig. 2: Geoid and ellipsoid surfaces

Historically, ***Clarke1880*** ellipsoid was used as a reference datum -local datum- for long time in Sudan (Termed technically ***Adindan Sudan***). Recently, because of the wide spread use of global positioning system (***GPS***), world geodetic system (***WGS1984***) becomes an available alternative.

IV. GLOBAL POSITIONING SYSTEM

Global Positioning System (GPS) was developed by the (U.S.A) to determine coordinates of points. Global positioning system consists of three components; these are space segment, control segment and user segment. Every component has its essential role in improving the positioning accuracy [1].

In this system, ground control receivers monitor the signals transmitted from a set of space segment (satellites). The received signals are used to solve for the coordinates of the position where the receivers are located. The GPS satellites are configured to provide the users with capability of positioning fixing.

Ground receiver equipments consist of two major units, antenna and units of analysis. The antenna is designed to receive the waves that come from satellites. Unit of analysis is linked to Antenna to analyze the data received by Antenna.

Global Positioning System (GPS) provides number of advantages e.g., the comprehensive coverage during the 24-hour, comprehensive coverage of spatial locations of each hemisphere, does not need to monitor the use of direct and traditional methods, link all the points of the coordinates of a global uniform and provide a great deal of time in the work of the main connecting points of the major projects.

Because of its high precision measurements, its uses not confined to the process of identifying the exact locations of the vessels at sea and ground control point locations, but many geodetic applications and uses of the system became an active role to play effective applications in the areas of civil engineering such as civil engineering, surveying, aerial surveys, environmental engineering , engineering of airports, air navigation, geodetic, geophysical applications and modern systems such as geographic information systems (GIS) etc.

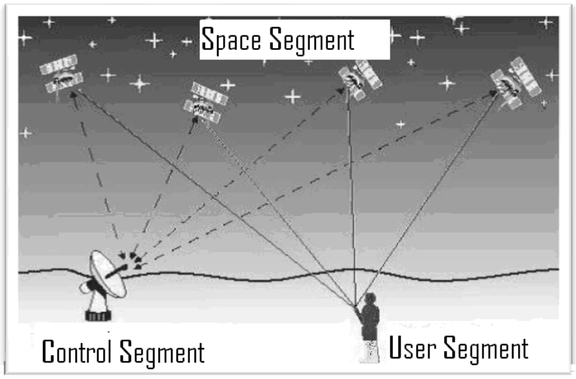

Fig. 3: GPS Components

V. EARTH GRAVITATIONAL MODEL (2008)

The official EGM2008 has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. The WGS 84 constants used to define the reference ellipsoid, and the associated normal gravity field, to which the geoid undulations are referenced are:

- a = 6378137.00m (semi-major axis of WGS 84 ellipsoid),
- f = 1/298.257223563 (flattening of WGS 84 ellipsoid),
- GM = 3.986004418 x 1014 m3s-2 (Product of the Earth's mass and the Gravitational Constant),
- ω = 7292115 x 10-11 radians/sec (Earth's angular velocity).

All synthesis software, coefficients, and pre-computed geoid grids, assume a tide free system, as far as permanent tide is concerned [6].

Note that the harmonic synthesis software applies a constant, zero-degree term of - 41 cm to all geoid undulations computed using EGM2008 with the height anomaly to geoid undulation correction model (also provided). Similarly, all pre-computed geoid undulations incorporate this constant zero-degree term. This term converts geoid undulations that are intrinsically referenced to an ideal mean-earth ellipsoid into undulations that are referenced to WGS 84. The value of - 41 cm derives from a mean-earth ellipsoid for which the estimated parameters in the Tide Free system are:

a = 6378136.58 m and 1/f = 298.257686.

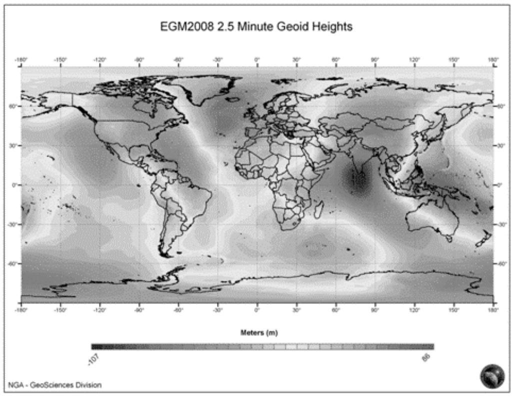

Fig. 4: Gravitational Model EGM2008

VI. MEASUREMENTS AND RESULTS

In this research work, ordinary levelling was carried out for 49 points of about one kilometer interval along 50 kilometers in Khartoum state (Sudan). Automatic level was used to do the job. Level line was started at a known benchmark and end at another one. Observations were reduced and adjusted. Orthometric heights (H) were obtained on Adindan Sudan reference datum as shown in Table 1 below.

Real Time GPS observations were carried out along the same line. Observations were processed and adjusted to compute ellipsoidal height (h) of all points. Results were obtained as illustrated in the table below.

Comparison between orthometric height of points and ellipsoidal heights of the same points can be carried out by computing the differences (N) between both results as shown in Table 1 below. These differences represent the geoid ellipsoid separation in the area.

Table 1: Geoid Ellipsoid separation (N) i.e. (Errors in GPS heights)

Point	Ellepoidal Height(h) (m)	Ortometric Hieght(H) (m)	N (m)
1	386.662	382.935	3.727
2	387.588	383.797	3.791
3	387.772	384.117	3.655
4	388.761	385.13	3.631
5	389.445	385.862	3.583
6	388.974	385.472	3.502
7	390.653	386.967	3.686
8	390.406	386.707	3.699
9	389.78	386.172	3.608
10	389.43	386.157	3.273
11	389.469	386.202	3.267
12	389.721	386.438	3.283
13	390.092	386.785	3.307
14	389.263	385.945	3.318
15	389.46	386.12	3.34
16	390.072	386.713	3.359
17	390.318	386.958	3.36
18	391.613	388.225	3.388
19	393.554	390.108	3.446
20	396.098	392.587	3.511
21	398.293	394.757	3.536
22	399.01	395.472	3.538
23	399.472	395.947	3.525
24	398.838	395.324	3.514
25	399.095	395.554	3.541
26	400.235	396.719	3.516
27	399.903	396.347	3.556
28	400.268	396.666	3.602
29	398.081	394.464	3.617
30	395.394	391.711	3.683
31	393.896	390.263	3.633
32	395.277	391.571	3.706
33	396.884	393.224	3.66
34	398.659	394.964	3.695
35	399.075	395.52	3.555
36	398.962	395.302	3.66
37	401.6	397.928	3.672
38	405.936	402.263	3.673
39	407.63	403.978	3.652
40	410.558	406.843	3.715
41	410.698	406.98	3.718
42	407.92	404.245	3.675
43	404.546	401.823	2.723
44	401.535	397.809	3.726
45	401.214	397.449	3.765
46	404.248	400.449	3.799
47	407.558	403.749	3.809
48	406.925	403.191	3.734
49	405.875	402.161	3.714
Average N = 3.564m			
Root Mean Square Error of N= 3.569m			

The average difference between orthometric heights and ellipsoidal heights in the study area was found to be 3.564m with 3.569m root mean square error.

Practically, ellipsoidal heights can be reduced to orthometric heights if geoid ellipsoid separation (N) is known.

In this work, EGM2008 geodetic model was used to compute geoid ellipsoid separation (undulation). Reduced geoid andulations for the observed points are listed in the following Table 2.

Table 2: Reduced EGM2008 geoid ellipsoid separation (N)

Point	Latitude	Longitude	EGM2008(N)
1	15° 35' 28.28098" N	32° 36' 56.64292" E	2.387
2	15° 35' 28.48429" N	32° 37' 30.22032" E	2.381
3	15° 35' 28.52582" N	32° 38' 03.79984" E	2.376
4	15° 35' 28.56041" N	32° 38' 37.37892" E	2.370
5	15° 35' 28.59265" N	32° 39' 10.95841" E	2.364
6	15° 35' 28.62923" N	32° 39' 44.53721" E	2.358
7	15° 35' 42.33917" N	32° 40' 05.03774" E	2.364
8	15° 36' 14.88522" N	32° 40' 04.98496" E	2.387
9	15° 36' 45.64159" N	32° 40' 04.91679" E	2.409
10	15° 37' 17.46176" N	32° 40' 04.95775" E	2.431
11	15° 37' 50.00769" N	32° 40' 04.91674" E	2.453
12	15° 38' 22.55383" N	32° 40' 04.88659" E	2.474
13	15° 38' 55.09950" N	32° 40' 04.89251" E	2.495
14	15° 39' 25.18699" N	32° 40' 16.93145" E	2.512
15	15° 39' 57.21745" N	32° 40' 22.77125" E	2.531
16	15° 40' 29.30227" N	32° 40' 28.40930" E	2.550
17	15° 41' 01.39351" N	32° 40' 34.00758" E	2.568
18	15° 41' 33.48313" N	32° 40' 39.61126" E	2.586
19	15° 42' 05.53982" N	32° 40' 45.41510" E	2.604
20	15° 42' 37.65666" N	32° 40' 50.80197" E	2.621
21	15° 43' 09.84494" N	32° 40' 55.77394" E	2.637
22	15° 43' 42.22209" N	32° 40' 56.77734" E	2.655
23	15° 44' 14.25138" N	32° 40' 50.82525" E	2.672
24	15° 44' 46.26347" N	32° 40' 44.76618" E	2.690
25	15° 45' 18.27536" N	32° 40' 38.70319" E	2.716
26	15° 45' 50.28918" N	32° 40' 32.65072" E	2.732
27	15° 46' 22.30340" N	32° 40' 26.60186" E	2.546
28	15° 47' 58.34820" N	32° 40' 08.46181" E	2.790
29	15° 48' 30.36066" N	32° 40' 02.40077" E	2.802
30	15° 49' 02.37182" N	32° 39' 56.33266" E	2.815
31	15° 49' 34.38611" N	32° 39' 50.28272" E	2.827
32	15° 50' 06.40397" N	32° 39' 44.25410" E	2.838
33	15° 50' 38.42119" N	32° 39' 38.21771" E	2.848
34	15° 51' 10.44067" N	32° 39' 32.19469" E	2.857
35	15° 51' 42.45744" N	32° 39' 26.15935" E	2.866
36	15° 52' 14.47629" N	32° 39' 20.13635" E	2.874
37	15° 52' 46.49807" N	32° 39' 14.12822" E	2.882
38	15° 53' 18.51532" N	32° 39' 08.09388" E	2.888
39	15° 53' 50.68458" N	32° 39' 04.20829" E	2.894
40	15° 54' 22.31993" N	32° 39' 12.06631" E	2.898
41	15° 54' 53.89724" N	32° 39' 20.20674" E	2.902
42	15° 55' 25.47769" N	32° 39' 28.33516" E	2.877
43	15° 55' 57.06144" N	32° 39' 36.44949" E	2.881
44	15° 56' 28.64441" N	32° 39' 44.56889" E	2.884
45	15° 57' 00.22799" N	32° 39' 52.68608" E	2.887
46	15° 57' 31.80978" N	32° 40' 00.81011" E	2.891
47	15° 58' 03.39026" N	32° 40' 08.93919" E	2.894
48	15° 58' 34.97430" N	32° 40' 17.05577" E	2.898
49	15° 59' 06.56038" N	32° 40' 25.16390" E	2.901

From the results obtained above, average separation can be computed to be 2.673m.

Ortometric heights are then calculated by adding geodetic heights (H) to geoid separation (N) using the following equation:

$$H = h + N \qquad (1)$$

Where,

H represents orthometric height,
h represents Ellipsoidal height and
N represents geoid ellipsoid undulation.

Computed Orhtometric heights obtained are listed as demonstrated in Table 3.

Finally, the actual orthometric heights were compared with those orthometric reduced from GPS observations by computing geoid ellipsoid separation (undulation) the error difference. The following Table 3 illustrates results.

Table 3: Difference between reduced and actual orthometric heights (errors)

Point	Computed Ortometric Hieght(H) h+N (m)	Actual Ortometric Hieght(H) (m)	Difference (m)
1	384.2753	382.935	1.340
2	385.2066	383.797	1.410
3	385.3961	384.117	1.279
4	386.3908	385.13	1.261
5	387.0807	385.862	1.219
6	386.6159	385.472	1.144
7	388.2888	386.967	1.322
8	388.0186	386.707	1.312
9	387.371	386.172	1.199
10	386.9992	386.157	0.842
11	387.0162	386.202	0.814
12	387.2467	386.438	0.809
13	387.5966	386.785	0.812
14	386.7506	385.945	0.806
15	386.9286	386.12	0.809
16	387.522	386.713	0.810
17	387.7497	386.958	0.792
18	389.0269	388.225	0.802
19	390.9505	390.108	0.843
20	393.4774	392.587	0.890
21	395.6556	394.757	0.899
22	396.3554	395.472	0.883
23	396.7997	395.947	0.853
24	396.1483	395.324	0.824
25	396.3791	395.554	0.825
26	397.5028	396.719	0.784
27	397.3569	396.347	1.010
28	397.4776	396.666	0.812
29	395.2786	394.464	0.815
30	392.5786	391.711	0.868
31	391.0691	390.263	0.807
32	392.4393	391.571	0.868
33	394.0361	393.224	0.812
34	395.8017	394.964	0.838

35	**396.2089**	**395.52**	**0.689**
36	**396.0878**	**395.302**	**0.786**
37	**398.7183**	**397.928**	**0.790**
38	**403.0476**	**402.263**	**0.785**
39	**404.7357**	**403.978**	**0.758**
40	**407.6596**	**406.843**	**0.817**
41	**407.7961**	**406.98**	**0.816**
42	**405.0428**	**404.245**	**0.798**
43	**401.6655**	**401.823**	**-0.158**
44	**398.6507**	**397.809**	**0.842**
45	**398.3268**	**397.449**	**0.878**
46	**401.3574**	**400.449**	**0.908**
47	**404.6642**	**403.749**	**0.915**
48	**404.0275**	**403.191**	**0.837**
49	**402.9739**	**402.161**	**0.813**
Average Difference = 0.891m			
Root Mean Square Error of Difference = 0922m			

Note that, the accuracy estimation based on the criteria of the root mean squire error RMSE, which can be computed as:

$$RMSE=\sqrt{\frac{\sum(\bar{x}-x)^2}{n}} \qquad (2)$$

Where,

x is the actual quantity,
x is the measured quantity, and
n is the number of quantities.

The root mean square errors of the measured points was computed and tabulated in Table 2.

From Table 3 above the mean difference was found to be 0.891m, where, 0.922m root mean square error was obtained.

Fig. 5 below show diagrammatically observed points distributed along the chainage line against heights.

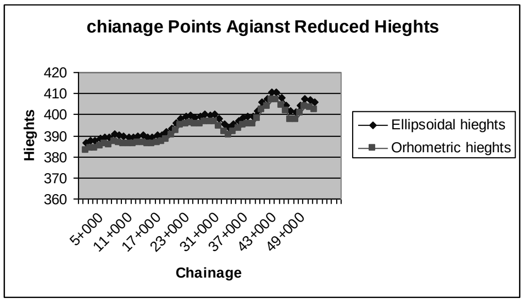

Fig. 5: Orthometric and Ellipsoidal heights

VII. CONCLUSION

Regarding the study area, and by referring to observation carried out and results obtained above, it can directly conclude that:

i. In the study area, error of about 3.569m in orthometric heights can be expected when utilizing GPS directly in levelling applications. This result leads to ability of using GPS directly in levelling observation to produce maps at scale of about 1:80,000 (and smaller) with 25m contour interval, according to Sudan map specification.
ii. Orthometric heights can be reduced from GPS observation by obtaining the geoid ellipsoid separation (geoid undulation) with aid of one of available geoid models. This can easily be done by subtracting resultant undulations from GPS heights.
iii. Applying EGM2008 geoid model to GPS observation in Khartoum state can successfully produce results estimated with 0.922m accuracy in orthometric height. This accuracy in height can be used to produce contour maps at scales 1:18,000 and smaller according to Sudan authority specification. These maps can be plotted with 2m interval.

VIII. RECOMMENDATIONS

This research work is oriented to study the accuracy of GPS in height determination in Khartoum state. Also to evaluate the improvements in heights that can be obtained when using EGM2008 geodetic model in geoid ellipsoid separation. Further studies can be oriented to:

i. Integrating number of local observed data to improve locally the EGM geoid model so that accuracy of GPS in height determination can be improved.
ii. Accurate determination of geoid ellipsoid model in the area.
iii. Evaluation of GPS in difference in height determination.

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