WGS84 to Adindan-Sudan Datum Transformation Manipulated by ITRF96

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Abstract– **The national geodetic network stations of Sudan were observed and computed using conventional triangulation techniques that related to the local datum (Adindan) and reduced by the parameters of Clarke 1880 ellipsoid. Nowadays that conventional technique has been replaced by satellite system/Global Positioning System (GPS) in which observations are reduced to the geocentric datum WGS84 reduced toWGS84 ellipsoid. Today, each survey project in the country has its own independent absolute base station that is not relatively connected to the other projects. This independent observation makes arise of problem of inconsistency between the coordinates of different projects in Sudan. Through this study work, the definition of the datum orientations and the transformation parameters between Adindan local datum and geocentric WGS84 datum were determined twice; Firstly, using the absolute GPS base stations that distributed over the country common to the national stations. Secondly, using the International Terrestrial Reference Frame 1996 (ITRF96). Results showed that the GPS coordinates relative to ITRF stations seem to be more consistent to the local datum than those of absolute GPS base stations. This result indicates that the ITRF observation is suitable for national network establishment than absolute GPS observations.**

Keywords– **Triangulation, GPS, Datum, Adindan, WGS84, Absolute Observations, ITRF and Network**

I. INTRODUCTION

Adindan datum is the historical local datum of Sudan that all triangulation and traverse network observations has subsequently been reduced to it. Adindan base terminal ZY was chosen as the origin of 22º 10' 7.1098" latitude (North) and 31º 29' 21.6079" longitude (East), with azimuth of 58º 14' 28.45" from the north to YY.ZY is now about 10 meters below the surface of Lake Nasser.

On the other hand, Clarke 1880 is that ellipsoid of a semi major axis of 6378249.145m, and293.465 reciprocal of the flattening (1/f).

During the initial development and testing phase of GPS, the Broadcast ephemerides and post-processed ephemerides computed by the U.S. Department of Defence have been given nominally in the WGS72 system [4]. In October 1985, both sets of ephemerides are computed in the new World Geodetic System 1984 (WGS84) of a semi major axis of 6378173m and 298.257223563resiprocal of the flattening. The new system incorporates an improved gravity field model, an origin much closer the geocentric, and an improved set of station coordinates. WGS84 defines global reference system the accuracy and consistency of which is more than adequate for most surveying applications [4].

WGS84coordinate system is a Conventional Terrestrial System (CTS) realized by modifying the Navy Navigation Satellite System (NNSS) or TRANSIT, Doppler Reference Frame (NSWC 9Z-2) in origin and scale, and rotating it to bring its reference meridian into coincidence with the Bureau International de 1'Heure (BIH)-defined zero meridian [3].

Since all existing maps and old survey information in Sudan are reduced to Adindan national datum, precise transformation relationship between the world geodetic system 1984 (WGS84) and the local geodetic datum need to be established.

II. THE INTERNATIONAL TERRESTRIAL REFERENCE SYSTEM (ITRS)

The most precise geodetic measuring techniques for long base-lines are at present Satellite Laser Ranging (SLR) and Very Long Base-Line Interferometer (VLBI). Both techniques guarantee a precision of 1-3 cm over distances up to about 5000 km. Global networks of up to 70 SLR-and up to 81 VLBI-stations respectively were observed for limited periods. Since 1987 a new International Earth Rotation Service (IERS) has been operating making use of SLR- and VLBI-results predominantly and producing every year a new global set of X, Y, Z-coordinates by combining various SLR- and VLBI-solution. The precise satellite laser ranging technique has led to a precise worldwide terrestrial coordinates system, called the International Terrestrial Reference System (ITRS). The ITRS is maintained by the IERS and the realization of the ITRS is the International Terrestrial Reference Frame (ITRF) [4].

The International Terrestrial Reference System (ITRS) is the recognized conceptual basis for forming coordinate frames based on the Geodetic Reference System 1980 (GRS80) which has the same geometric parameters as WGS 84. The International Earth Rotation and Reference systems Service (IERS) is responsible for developing detailed specifications for a concrete realization, and for the formation and maintenance of a practical International Terrestrial Reference Frame (ITRF).

III. PRECISE GPS SATELLITE EPHEMERIDES

The precise GPS satellite ephemerides are computed at Geodetic Surveying Division (GSD) from the data collected at the Canadian stations augmented by up to 24 globally distributed stations of the International GPS Geodynamic Service (IGS). They are available typically within 3 to 6 days following the observations. Based on IGS orbit comparisons, the CACS precise GPS satellite ephemerides precision is better than 15 centimeters (one sigma) in each coordinate. A rapid solution is computed after the end of the day using data available at the time. Normally data from 15 to 20 globally distributed stations are included in this solution and it is made available typically within 36 hours following the observations. Its accuracy is estimated at better than 30 cm (one sigma) in each coordinate component.

This degradation over the precise solution has in general minor impact on the positioning accuracy for most GPS users. When retrieving ephemeris files via the Bulletin Board Service (CGBBS) the best solution available at the time will be provided. GPS satellite ephemerides are provided as daily files (0:00 10 23:45 GPS time) in the international accepted NGS-SP3 format which contains X, Y, Z satellite positions and clock corrections at 15-minute intervals. They are available in the NAD83 (CSRS) reference frame as well as in the International Terrestrial Reference Frame (ITRF).

IV. TRANSFORMATION

Transformation between coordinate systems is routinely carried out in surveying. If the coordinates are given for a number of stations common to both coordinate systems, the transformation parameters can be estimated.

When measurement are carried out using GPS receivers there is usually a need for transformation from the GPS measured coordinates to the local coordinate system of the particular country i.e. the results obtained from GPS need to be transformed into the local coordinate system.

The transformation of three-dimensional coordinate systems for the purpose of transforming geodetic datum's has been given much attention, in particular since geodetic satellite techniques made it possible to relate local geodetic datums to a geocentric datum. Three-dimensional transformations are more suitable for use with satellite positioning for a number of reasons. They are typically global in concept, they enable solution for height as well as horizontal position, and they are mathematically rigorous. The complete three-dimensional transformation involves seven parameters that relate Cartesian co-ordinates in the two systems. There are three-translation parameters to relate the origins of the two systems (ΔX, ΔY, ΔZ) three rotation parameters, one around each of the coordinate ($RX=\varepsilon$, $RY=\psi$, and $RZ=\omega$) to relate the orientation of the two systems and one scale parameter (ΔL) to account for any difference in scale between the two systems. Considering the geodetic system defining a Cartesian co-ordinate system (*X,Y,Z*) where the origin is at the centre of the reference ellipsoid. The parameters of this ellipsoid will be the equatorial radius (*a*) and the flattening (*f*). Knowing *X,Y,Z, a* and *f* it can be compute the geodetic coordinates (ϕ, λ, h) of points in of the "old" system. It can then consider a new Cartesian co-ordinate system *X1, Y1,Z1* whose origin may be (for example) at the centre of mass of the earth. Given the parameters *a1* and *f1* of a new reference ellipsoid it can compute the geodetic co-ordinates ($\phi 1$, $\lambda 1$, $h1$) of points in the new system.

There are a number of ways of defining the relationship between one reference system and another. The choice of the most appropriate network transformation model is influenced by:

- i). The extend of the area in which transformation going to be applied,
- ii). The presence of distortion in either of reference systems,
- iii). The dimensions of the reference systems (two or three dimension), and
- iv). The accuracy requirements.

Number of models that can be used to transform coordinates from one system to another. These transformation models may include:

A) Bursa-Wolf Similarity Transformation Model

Similarity transformation model is one of the most commonly used transformation methods in surveying that preserves the shape, so angles are not changed, but lengths of lines and the position of points may be changed. It assumes that there are no systematic distortions with in either networks.

The general similarity transformation is given by:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = S_F R \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} \quad .....(1)$$

Where,

X2,Y2,Z2 are the Cartesian coordinates in coordinate system *2,*

*X1,Y1,Z*1 are the Cartesian coordinates in coordinate system *1,*

TX,TY,TZ are the translation termsof the origin,

R is an orthogonal rotation matrix, and Δs is the differential scale factor**.**

There are seven parameters which are usually associated with a similarity transformation; three rotation angles, three translation components and one scale factor. If the rotations are small, as is expected when both coordinate systems refer to the same CTRS, then the equation is approximately linear and the order of rotations is unimportant.

The similarity transformation is popular due to:

- i). The small number of parameters involved,
- ii). The simplicity of the model, which is more easily implemented into software and
- iii). The fact that it is adequate for relating two coordinate systems which are homogeneous (no local distortion in scalar orientation).

One disadvantage of the seven parameters similarity transformation method is that both networks are assumed to have only linear distortions (excluding shear components). Often older terrestrial networks do have non-linear distortions because of the adjustment and survey methodologies employed.

The seven parameters similarity model is known in geodesy as *Bursa-Wolf model* and takes the same form as the general similarity transformation of the equation[2].One problem with Bursa-Wolf model is that the adjusted parameters are highly correlated when the network of points used to determine the parameters covers only a small portion of the earth.

B) Molodenskii-Badekas Model

This model removes the high correlations that may exist between the model parameters [1] by relating them to the centred of the network (Fig. 1) by the following equation:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \bar{X}_1 \\ \bar{Y}_1 \\ \bar{Z}_1 \end{bmatrix} + \begin{bmatrix} T'_X \\ T'_Y \\ T'_Z \end{bmatrix} + \cdot_F R \begin{bmatrix} X_1 - \bar{X}_1 \\ Y_1 - \bar{Y}_1 \\ Z_1 - \bar{Z}_1 \end{bmatrix} \quad .....(2)$$

Where,

$\bar{X}_1 = \sum X_{1i}$ /n is the centered X-coordinate for the points in coordinate system *1*,

$\bar{Y}_1 = \sum Y_{1i}$ /n is centered Y-coordinate for the points in coordinate system 1,

$\bar{Z}_1 = \sum Z_{1i}$ /n is the centered-Z coordinate for the points in coordinate system 1.

T'_X, T'_Y, T'_Z are the Molodenskii-Badekas translations terms

Remaining terms are as defined before.

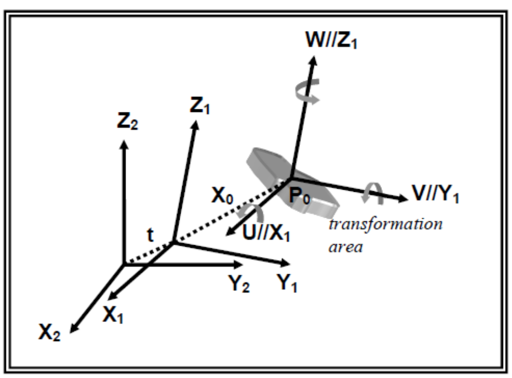

Fig. 1: Molodenskii-Badekas model

This model gives the same answers for the baseline length and angles of the survey network, and for the scale and rotation parameters, as the Bursa-Wolf model [4]. However, the translation parameters are different and have higher a posterior precision.

The seven parameters for either the Bursa-Wolf or the Molodenskii models can be solved for in a least-squares adjustment. The adjustment uses points with co-ordinates in both the satellite and local datum along with their estimated variances and derives least squares estimates of the seven transformation parameters to fit the differences between the two sets of co-ordinates. The reliability of these derived parameters is usually expressed in terms of standard deviation or variances.When transformation parameters from the Molodenskii-Badekas model are to be applied to transform coordinates of points, it is essential to know what values were used for. When working with global network of points the Molodensky-Badekas model has centroid coordinates that equal the centre of the ellipsoid and therefore reduces to the Bursa-Wolf model.

V. MEASUREMENT AND RESULTS

The purpose of this study is to determine the translation parameters between Adindan (Sudan) local datum and WGS84 Global Positioning System (GPS) datum using two sets of observation. Absolute observations and ITRF observations were carried out to number of points on adindan datum. FORTRAN computer program was developed to handle Molodensky-Badekas transformation model. The program follows the combined least squares adjustment technique. Input data consist of the ellipsoid coordinates of the two systems common to satellite (GPS) and the Adindan datum. The output is the Cartesian (X,Y,Z) coordinates and the transformation parameters between the two systems with their standard deviation and residuals.

In the first test, number of eight control point of adindan datum were selected as shown in Table 1:

Table 1: Adindan cartesian coordinates

Point	*X1(m)*	*Y1(m)*	*Z1(m)*
1	5209207.500	3040808.244	2067652.171
2	5147519.189	3535228.068	1296985.925
3	5736045.479	2359030.970	1487557.109
4	4947291.159	3651035.949	1692331.703
5	5435162.800	3106763.337	1220913.297
6	5140106.403	3190427.735	2014760.964
7	5363495.551	3140312.750	1431563.703
8	5054181.363	3093217.667	2352682.277

These points were obsereved using GPS receivers in static mode through an hour of time interval or more, to solve the error obtained by the Selective Availability (SA) so as to have a good approximation of the WGS84 ellipsoid for the observed values. Results were found to be as listed in Table 2:

Applying these two sets of data common to both adindan datum and WGS84 datum to the transformation program developed above, transformation parameters of Adinan and absolute GPS network with the residuals of the coordinates are shown in Table 3 and Table 4 here under, respectively:

Table 2: Absolute GPS observations

Point	*X1(m)*	*Y1(m)*	*Z1(m)*
1	5209041.458	3040793.907	2067879.387
2	5147364.011	3535225.056	1297194.386
3	5735898.784	2359026.520	1487764.835
4	4947129.107	3651022.835	1692538.866
5	5435015.297	3106755.475	1221124.731
6	5139953.836	3190419.032	2014974.149
7	5363350.485	3140306.366	1431773.229
8	5054028.281	3093202.358	2352892.189

Table 3: Absolute GPS to Adindan Transformation Parameters

Parameters	Value
ΔX (m)	- 155.618 ± 2.462
ΔY (m)	- 10.609 ± 2.462
ΔZ (m)	211.472 ± 2.462
ΔL (ppm)	9.525 ± 5.161
RX × 102 sec	- 6.12 ±2.00
RY × 102 sec	2.16 ± 2.09
RZ× 102 sec	- 5.30 ± 3.82
σ20	48.4862

Table 4: Residual values of datum transformation between absolute GPS stations and Adindan coordinates

Point	*Vx(m)*	*Vy(m)*	*Vz(m)*
1	-5.1360	-9.4967	9.5756
2	-5.3730	2.2373	2.2433
3	-2.3199	1.7361	-5.9819
4	-1.4291	3.2747	-4.0881
5	11.7827	-3.8413	-8.6219
6	-4.2345	-8.1676	1.9480
7	7.1991	6.2267	5.5953
8	-0.4893	8.0306	-0.6702

In the second test, again eight points were observed again using GPS recievers and adjusted to the precise ephemeris with reference datum ITRF96 and reference ellipsoid GRS80 which has the same Semi-major axis and reciprocal flattening as WGS84 ellipsoid. Table 5 shows the ITRF96 geocentric cartesian coordinates for the 8 points.

Table 5**:** ITRF96GPS observations

Point	*X1(m)*	*Y1(m)*	*Z1(m)*
1	5209051.179	3040794.994	2067858.390
2	5147351.580	3535213.493	1297189.697
3	5735898.786	2359026.520	1487764.835
4	4947124.563	3651017.896	1692537.238
5	5435006.649	3106749.715	1221120.126
6	5139946.519	3190413.669	2014966.880
7	5363337.027	3140299.956	1431770.658
8	5054033.322	3093199.747	2352881.182

Again coordinates of the common points related to adindan datums and its ITRF96 observed coordinates have been used as an input data to compute the transformation parameters between ITRF96 and Adindan datum. Table 6 shows the datum transformation parameters where Table 7 shows the residuals.

Table 6: ITRF GPS to Adindan Transformation Parameters

Parameters	Value
ΔX (m)	- 157.4773 ± 1.154
ΔY (m)	- 13.5910 ± 1.154
ΔZ (m)	205.2319 ± 1.154
ΔL (ppm)	-2.5534 ± 2.0036
RX × 102 sec	-1.748 ± 0.726
RY × 102 sec	-1.158 ± 0.969
RZ× 102 sec	-6.709 ± 0.988
σ20	10.464

Table 7: Residual values of datum transformation between ITRF96 and Adindan coordinates

Point	*Vx (m)*	*Vy (m)*	*Vz (m)*
1	1.9970	-2.7325	-1.3126
2	1.7485	-0.0782	0.9001
3	1.8636	2.8437	0.2760
4	0.3603	-2.5468	-1.7904
5	-2.7039	5.7928	-0.8076
6	2.7773	-3.3302	-1.3789
7	-0.0954	2.5208	-1.4039
8	-5.9472	-2.4697	5.5172

VI. CONCLUSIONS

According to the measurement carried out and results obtained above for transformation parameter between the local datum of Adindan Sudan and both absolute and ITRF GPS observations, it can be concluded with:

- ITRF96 transformation parameters shows small standard deviation and small varienc σ20.
- Residuals of the absolute GPS stations are large compared to the residuals of ITRF96 transformed coordinates.
- Results also showed good consistancy for the ITRF96 and very poor consistancy for absolute GPS coordinates.
- The datum transformation parameters between WGS84 and Adindan datum using ITRF96 coordinates were more consistent and precise than the parameters obtained by the static absolute GPS coordinates which were processed using the broad cast ephemeris.
- The coordinates of the stations obseved relative to the ITRF are suitable for the establishment of the national (Sudan) GPS network.
- Densifying national first order network stations can be carried out by observing GPS network relative to ITRF so it can be transformed to Adindan sudan local datum.

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