The Effect of Ellipsoidal Reference Datum on Projected Coordinates in Sudan

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***Abstract*– Clarke-1880 is the ellipsoidal datum on which Sudan maps are reduced to. Recently, Global Positioning System (GPS) receivers are widely used for coordinates determination in the country. Since GPS coordinates are reduced to World Geodetic System (WGS-84) then, differences in the coordinates of the same points may arise because of the difference in the reference datum used in each. This research work aims to compare between projected co-ordinates reduced to Clarke-1880 on Universal Transverse Mercator system (UTM) with the same points projected taking WGS-84 as a reference datum. Then, the effect of reference datum can be evaluated and judging its significance. Results proved that difference in northing is always larger than that in in easting, and the linear difference can be ignored in all quarter million maps and smaller.**

***Keywords*– Datum, Ellipsoid, Map Projection, GPS, WGS-84, Clark-1880 and Geoid**

I. INTRODUCTION

The shape of the earth was assumed to be spherical for long time in the past. Today, ellipsoid becomes the nearest mathematical shape that represent the earth. Survey observations can be reduced to one of the three geodetic reference surfaces; the earth surface, geoid, and the ellipsoid.
The earth's surface is the topographic surface of the earth. It is extremely irregular and not mathematically defined. It is approximately ellipsoidal in shape, the maximum departures from an ellipsoid being of the order of 8.5 km. The earth's surface is important because most of observations are made on it.

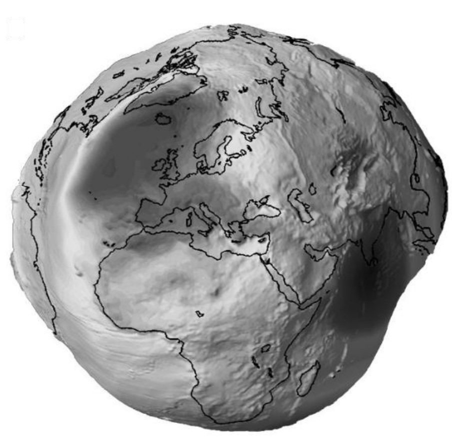

Figure 1: The earth's surface

The equipotential surface of the earth's attraction and rotation defines geoid. Geoid coincides, on average, with the mean sea level. The shape of the geoid is approximately ellipsoidal. In fact, a best fitting ellipsoid could be placed in such a way that the maximum departures from that ellipsoid would be about 110m.

The geoid is of fundamental importance in geodesy because many geodetic observations are related to it. For example, all survey instruments are set up with their primary axis along the local direction of gravity, which is perpendicular to the local equipotential surface and almost perpendicular to the geoid. Ellipsoid as shown in Figure 2 is simply an ellipse rotated about its minor axis. Mathematically the shape is an oblate spheroid. Ellipsoid is important in geodesy because it represent the nearest simple mathematical shape to the geoid so measurements can be reduced to it.

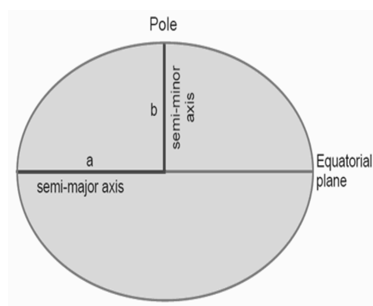

Figure 2: Ellipsoidal surface

The world geodetic system (WGS-84) ellipsoid today becomes an important ellipsoid. This is so, because it is the surface on which Global Positioning System (GPS) observations are reduced to. On the other hand, Clarke-1880 is one of the famous traditional reference ellipsoids or geodetic datum. This ellipsoid was widely used in different countries including Sudan.

II. MAPPING IN SUDAN

Maps or spatial data can generally be categorized into either planimetric or topographic maps. Planimetric maps show only the horizontal positions of features (X, Y). On the other hand, topographic maps show planimetric details and elevation information (X, Y, and Z).

Historically, all topographic maps were produced using photogrammetry and land survey conventional techniques. Moreover, mapping in Sudan based on Adindan datum. "Adindan datum is the historical local datum of Sudan that all triangulation and traverse network observations have subsequently been reduced to it. Adindan base terminal ZY was chosen as the origin of 22º 10' 7.1098" latitude (North) and 31º 29' 21.6079" longitude (East), with azimuth of 58º 14' 28.45" from the north to YY.ZY is now about 10 meters below the surface of Lake Nasser[1]". This datum based on Clarke-1880 ellipsoid. On the other hand advantages provided by GPS make it an attractive tool to replace conventional land survey methods.

Most maps in Sudan were produced by Sudan survey authority. Theses maps are available in hard copies which are now being converted into digital form.

Maps in Sudan were produced in three scale levels and terms of time, these are:

i). 1:1000, 000 map series: there are about 16 map sheets covering the whole country. Sheets were produced in 1944 through 1975.
ii). 1:250, 000 map series: cover the whole country. Each sheet cover about 110×160km. Map sheets were produced in 1936 through 1976.
iii). 1:100, 000 map series: cover selected areas of the country. Only 220 sheets were produced in 1967, to 1983, each map includes about 30 layers.
iv). Therefore, it can said to be that large scale maps are still required to cover the country.

III. MAP PROJECTION

Map projection is a transformation of the three dimensional surface of the earth into a two dimensional surface of the map. i. e., the curved surface of the Earth is portrayed on a flat map. The transformation from a round surface to a plane cannot be accomplished ideally without distortion.

Projections can be divided into three types: Azimuthal (Zenithal) projections, Conical projections, and cylindrical projections.

Cylindrical Projections is a projection of the earth surface to a cylinder. This is similar to wrap a paper round a globe so as to touch it along the equator. The cylinder could also touch the globe along any other great circle.

Cylindrical projections can either be normal, transverse, or oblique, as shown in Figure 3:

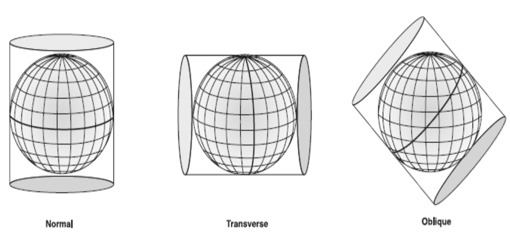

Figure 3: Types of cylindrical Projection

IV. THE MERCATOR'S PROJECTION

The Mercator projection is one of the most common cylindrical projections, in which the equator is usually set to be the line of tangency. Meridians are geometrically projected onto the cylindrical surface, and parallels are mathematically projected, producing grate circular angles of 90 degrees. The cylinder is cut along any meridian to produce the final cylindrical projection. The meridians are equally spaced, while the spacing between parallel lines of latitude increases toward the poles. One Characteristic of this projection is conformal - i. e. preserving shapes- and displays true direction along straight lines.

Mercator projection is properly the best known of all projections, because it is used for navigation purposes and also in nearly all atlases for maps of the world.

V. THE TRANSVERSE MERCATOR PROJECTION

The Universal Transverse Mercator (UTM) is simply a transverse Mercator projection to which specific parameters, such as central meridians, have been applied. In other words, the Universal Transverse Mercator (UTM) is a grid-based method of specifying locations on the surface of the Earth that is a practical application of a 2-dimensional Cartesian coordinate system.

The UTM divides the surface of Earth between 80°S and 84°N latitude into 60 zones, each 6° of longitude in width and centered over a meridian of longitude. Zone 1 is bounded by longitude 180° to 174° W and is centered on the 177th West meridian. Zone numbering increases in an eastward direction.

By using narrow zones of 6° (up to 800 km) in width, and reducing the scale factor along the central meridian by only 0.0004 to 0.9996 (a reduction of 1:2500), the amount of distortion is held below 1 part in 1,000 inside each zone. Distortion of scale increases to 1.0010 at the outer zone boundaries along the equator.

In each zone, the scale factor of the central meridian reduces the diameter of the transverse cylinder to produce a secant projection with two standard lines, or lines of true scale, located approximately 180 km on either side of, and approximately parallel to, the central meridian (ArcCos 0.9996 = 1.62° at the Equator). The scale factor is less than 1 inside these lines and greater than 1 outside of these lines, but the overall distortion of scale inside the entire zone is minimized.

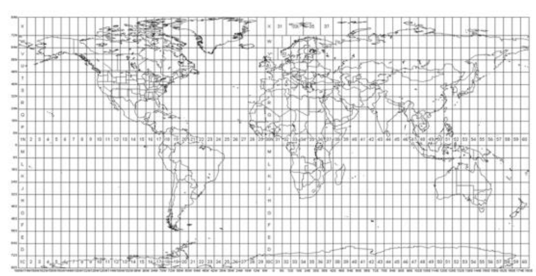

Figure 4: UTM Zones

VI. MEASUREMENTS AND RESULTS

Global Positioning System (GPS) reference ellipsoid is the World Geodetic System-84 (WGS-84), while CLARKE-1880 is the local ellipsoid used in Sudan. In this research a comparison was carried out between the projected coordinates for both systems on Universal Transverse Mercator (UTM) in Sudan, to find the exact difference between them. Evaluation of these differences can also assist to estimate mapping scales where differences can be ignored.

Numbers of well distributed points covering the whole country were used. Coordinates of these points were known in both geodetic systems; Clark-1880 and WGS-1984 as shown in Table 1 and Table 2.

Table 1: Point’s coordinates in CLARKE 1880

Point	Latitude	Longitude	H (m)
214	16° 10′ 30.625″	32° 35′ 55.768″	479.8
2007	15° 44′ 19.168″	32° 30′ 20.294″	451.68
4102	15° 31′ 28.504″	32° 29′ 36.012″	383.38
457	19° 30′ 05.118″	37° 15′ 37.181″	7.45
732	10° 59′ 52.692″	27° 50′ 01.493″	435.23
2458	10° 11′ 41.982″	28° 41′ 05.212″	409.55
2459	10° 11′ 30.317″	28° 31′ 59.576″	411.19
483	21° 42′ 00.724″	36° 52′ 12.736″	25.431
484	21° 51′ 26.540″	36° 49′ 50.123″	113.96
602	21° 47′ 21.778″	31° 28′ 01.615″	290.9
78	13° 03′ 24.061″	30° 20′ 55.902″	783.7
890	11° 48′ 41.420″	34° 28′ 50.204″	503.55
409	15° 29′ 21.214″	36° 25′ 36.616″	571.81
21	19° 02′ 29.974″	30° 16′ 25.315″	373.5
2601	15° 36′ 39.174″	32° 32′ 17.884″	410.06
709	11° 06′ 32.616″	29° 45′ 08.918″	921.01
EL ENEINA	13° 29′ 1.4062″	22° 28′ 2.8752″	815.37
DAMAZIN	11° 47′ 28.3297″	34° 20′ 13.2726″	479.05
NYALA	12° 03′ 21.2414″	24° 57′ 12.5339″	659.22

Table 2: Point’s coordinates in WGS-84

Point	Latitude	Longitude	H (m)
214	16° 10′ 32.34″	32° 35′ 58.17″	496
2007	15° 44′ 20.75″	32° 30′ 22.44″	444
4102	15° 31′ 30.36″	32° 29′ 38.597″	388
457	19° 30′ 05.60″	37° 15′ 39.62″	-2
732	10° 59′ 56.025″	27° 50′ 03.324″	432.669
2458	10° 11′ 45.404″	28° 41′ 07.116″	404.271
2459	10° 11′ 33.740″	28° 32′ 01.490″	405.865
483	21° 42′ 00.993″	36° 52′ 15.082″	34.018
484	21° 51′ 26.680″	36° 49′ 52.83″	130
602	21° 47′ 21.971″	31° 28′ 03.942″	304.275
78	13° 03′ 26.655″	30° 20′ 58.152″	800.304
890	11° 48′ 44.379″	34° 28′ 53.024″	516.89
409	15° 29′ 23.072″	36° 25′ 39.49″	581.268
21	19° 02′ 31.563″	30° 16′ 27.754″	380.293
2601	15° 36′ 40.998″	32° 32′ 20.378″	411.027
709	11° 06′ 35.907″	29° 45′ 11.105″	931.466
EL GENEINA	13° 29′ 03.919″	22° 28′ 04.4906″	806.49
DAMAZIN	11° 47′ 31.2593″	34° 20′ 15.8903″	480.67
NYALA	12° 03′ 24.1644″	24° 57′ 14.3598″	650.46

Figure 1 shows a map representing the distribution of the reference points over the country.

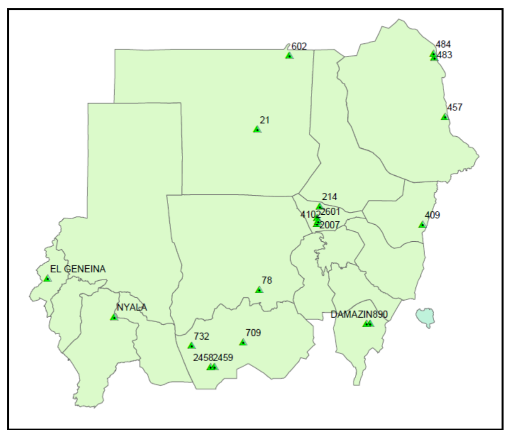

Figure 5: Distribution of reference points

The two sets of the reference points were projected utilizing Geocal software package on the UTM. Table 3 represents results obtained.

Table 3: Projected points on UTM

Point	CLARKE-1880		WGS-84	
	E (m)	N (m)	E (m)	N (m)
214	457113.9	1788196.7	457186.3	1788406.6
2007	447037.4	1739935.9	447102.5	1740137.9
4102	445663.1	1716261.4	445741.5	1716469.8
457	317428.8	2157049.9	317504.4	2157249.7
732	591087.2	1215771.6	591140.7	1215983.9
2458	684561.9	1127350.2	684615.8	1127557.7
2459	667956.2	1126909.3	668010.7	1127116.8

483	279658.9	2400955.5	279732.0	2401166.0
484	275803.7	2418416.5	275887.2	2418624.0
602	341524.0	2410098.0	341594.8	2410307.1
78	212473.7	1444778.6	212548.2	1444986.7
890	661308.9	1306019.6	661390.6	1306228.4
409	223918.7	1713942.8	224011.1	1714150.2
21	213036.0	2107483.7	213114.8	2107713.3
2601	450506.1	1725795.3	450581.5	1726003.5
709	800693.8	1229319.8	800753.3	1229532.5
EL GEN EINA	658838.9	1490971.1	658883.8	1491181.8
DAM AZIN	645672.9	1303695.4	645748.8	1303903.1
NYA LA	277215.9	1333446.4 9	277276.2	1333655.7

The differences (**δ**) between the projected points on UTM from both ellipsoidal reference datum were computed in Easting and Northing for each point. Results are listed as shown in Table 4:

Table 4: Difference between projected coordinates for each point

Point	δE (m)	δN (m)	Linear Difference (m)
214	-72.360	-209.940	222.060
2007	-65.120	-201.980	212.218
4102	-78.310	-208.460	222.684
457	-75.590	-199.740	213.565
732	-53.500	-212.310	218.947
2458	-53.840	-207.500	214.371
2459	-54.510	-207.470	214.511
483	-73.070	-210.500	222.822
484	-83.440	-207.570	223.713
602	-70.880	-209.130	220.815
78	-74.530	-208.180	221.119
890	-81.680	-208.860	224.264
409	-92.330	-207.370	226.996
21	-78.830	-229.600	242.756
2601	-75.450	-208.260	221.506
709	-59.570	-212.610	220.798
EL GENEINA	44.850	210.610	215.333
DAMAZIN	75.910	207.680	221.118
NYALA	60.350	209.210	217.741

Differences between both projected coordinates were plotted, in contour map and in 3-D maps. Figure 6 and Figure 7 represent the results.

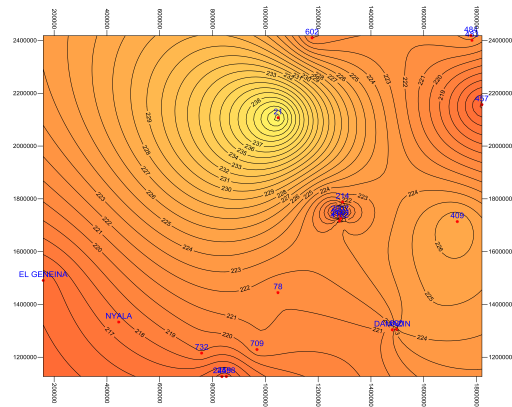

Figure 6: Contour map representing the differences in the whole country

Also 3-D map was created to represent the differences in a model, as shown in Figure 7.

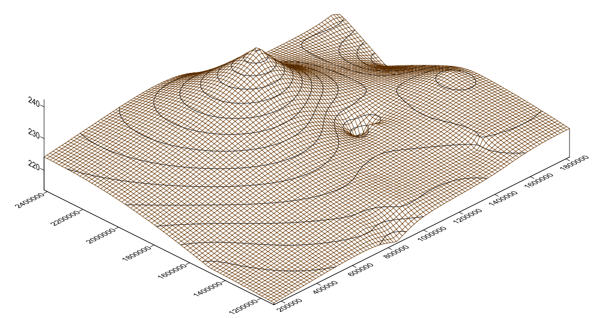

Figure 7: 3-D map representing the differences

Analyzing results obtained above it can be noted that 242.756m is the maximum linear error where the minimum was found to be 212.218m with 6.490m Root Mean Square Error (RMSE). Summary of the results obtained are listed in Table 5:

Table 5: Statistical analysis of the results

Statistical Measure	Difference in Easting (δE) (m)	Difference in Northing (δN) (m)	Linear Difference (m)
Maximum	92.330	229.600	242.756
Minimum	44.850	199.740	212.218
Average	69.691	209.315	220.912
RMSE	11.975	5.619	6.490

In order to study the general effect of the reference datum on the projected coordinates on UTM, analysis was carried out in one zone. Zone 36 was divided every 0.5 degree into 13 longitude and latitudes were divided from 0˚ to 25˚ every 5 degrees. These values were projected on UTM from both WGS-84 system and CLARKE-1880. Differences between the two sets of projected coordinates were computed. Results were then statistically analyzed as summarized in Table 6:

Table 6: Summary of Results for latitudes in zone-36

Latitude	Max Linear difference (m)	Min Linear difference (m)	Average (m)
0	218.090	219.165	218.501
5	220.166	220.778	220.412
10	220.899	221.283	221.118
15	220.867	221.292	221.032
20	220.242	220.719	220.506
25	219.721	220.021	219.870

Figure 8 represents a three-dimensional map demonstrating linear differences in zone-36. It can be noted that differences are increased towards the north.

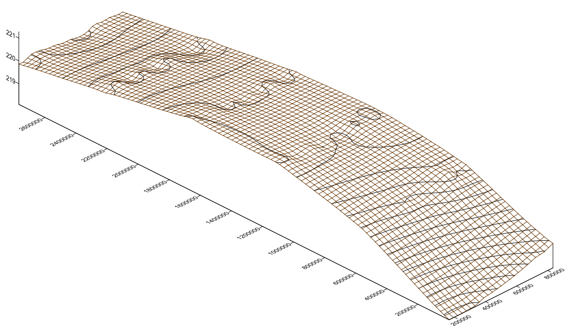

Figure 5: 3-D map of linear differences in zone-36

VII. CONCLUSION

Differences between the projected co-ordinates of the same points from both WGS-84 and CLARKE-1880 systems onto UTM may be referred to difference in size, shape and orientation of the two ellipsoids. Form the measurement carried out and results obtained in this research to study this effect it can be concluded with the following:

- The maximum difference in easting was found to be 44.850m where the minimum was 92.330m with 69.691m average. On the other hand these values were 199.740m, 229.60m, and 209.315m in northing successively.
- The linear differences of the projected points in the study points were 212.218m minimum and 242.756m maximum with the average of 220.912m.
- The difference in northing is always larger than that in easting. This may be referred to the characteristic of UTM projection in which the distortion is increasing as far as it goes away from the equator towards the poles.
- In general analysis of zone 36 it was found that minimum linear difference is 74.295m at latitude 0 and the maximum of 77.375m at the same latitude. Where the minimum linear difference at latitude 0 and maximum is 221.292m at latitude 15.
- From the result of linear differences it was found to be that the maximum value is not more than 250m thus, this difference can be neglected in quarter million maps scales and less, i.e. the coordinates can directly be projected on UTM without need to transform from one system to another i.e., CLARKE-1880 to WGS-84 or vice versa.

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