A Computational Algorithm for the Numerical Solution of Non Homogenous Linear Ordinary Differential Equation with f(t)=exp(t).

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Abstract– **Differential equations are quite inevitable in describing, and explicitly delineating diverse kinds of biological and physical processes ranging from one form to the other.**
The analytic solution of a non homogenous differential equation with f(t)=exp(t) has been delineated in this work with presentation of a computational algorithm.

Keywords– **O.D.E, Non Homogenous O.D.E, Algorithm, Matrix Equation and Linearity**

I. INTRODUCTION

The matrix equation can be formulated to solve a differential equation subject to some initial conditions.

Diverse problems in physical and biological applications are built around some forms of differential equations which are expediently required to be solved in one instance or the other. A biological or physical process that can be faithfully described by this particular form of linear non homogenous differential equation can be well solved (Alli, Adewole, 2013).

A mathematical problem describing oscillations and coupled system can be formulated as an eigen- value equation viz an appropriate differential equation and solved explicitly for its eigen - values and eigen - vectors which could give an elaborate or explicit delineation of the physical system in consideration. The non-homogenous case of the linear differential equation for which f(x) is non zero is extremely important in diverse practical or physical applications, for instance a forced oscillation is under the influence of an external forcing frequency subject to the form of the external forcing frequency f(x).

Obviously, speaking vast number of natural and physical processes are describable by an explicit formulation of a rhobust system of differential equations. Appropriately solving the illustrative ordinary differential equation no doubt conspicuously gives a thorough and revealing delineation of the physical or natural system in consideration. A radio frequency and tuning to station of appropriate frequency entails an external forcing frequency signal source being in resonance with the radio circuit and this system can be well delineated by a formulation of a linear ordinary differential equation for an R-L-C series circuit and appropriate solution obtained is undoubtedly revealing.

Practically, the use of linear ordinary differential equations have enormously increased and even pervading biological applications and frequently encountered in chaotic systems as non linear forms, non linearity, solitons, etc.

II. DISCUSSION

A differential equation can be described as homogenous or non homogenous. Considering a system of linear ordinary differential equation of the form:

$$Ly(x) = f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

(Blackey, 1953, Erwin Kreyszig, 2013, Sokolnikoff, 1941, Tranter, 1966), where:

$$Ly(x) = a_0(x)\frac{d^n y(x)}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y(x)}{dx^{n-1}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_1(x)\frac{dy}{dx} + a_0(x)y. \ldots\ldots \text{(2)}$$

If f(x) is equivalent to zero i.e., f(x)=0, equation (2) is said to be homogenous equation.

If f(x) is not equal to zero i.e., f(x) ≠ 0, then the equation is said to be non-homogenous.

Considering an equation of the form:

$$\frac{d^2x}{dt^2} + x = 0 \text{ .... (3)}$$

The above equation can be expressed as a matrix equation in matrix notation (Tranter, 1966) and an algorithm that can be applied for subsequent numerical computation presented (Press, 1992) as follows:

Analytic Solution & Computational Algorithm

A computational algorithm engendered towards solution of a non-homogenous differential equation of the form:

$$\frac{d^2x}{dt^2} - b\frac{dx}{dt} = f(t) \text{ .... (4)}$$

which can be solved by applying the following formula:

$$x = e^{[A]t}x_0 + e^{[A]t}\int_0^t e^{-[A]t}\{f(t')\}dt' \text{ ....(5)}$$

$$x = A\{x_0\} + \{f(t)\} \text{ ....(5b)}$$

By setting A=$\begin{bmatrix}1 & 0\\0 & b\end{bmatrix}$ in equation (5) above, b=2 subject to the initial condition;$x(0) = 1$, $\frac{dx(0)}{dt} = -1$; and for a given function f(t)=e^{t}, the solution of the non-homogenous equation can be explicitly obtained, thus:

$$[x]=\begin{bmatrix}e^t & 0\\0 & e^{2t}\end{bmatrix}\begin{pmatrix}1\\-1\end{pmatrix} + \begin{bmatrix}e^t & 0\\0 & e^{2t}\end{bmatrix}\int_0^t \begin{bmatrix}e^{-t} & 0\\0 & e^{-2t}\end{bmatrix}\begin{pmatrix}e^t\\0\end{pmatrix}dt \text{ .... (6.1)}$$

$$= \begin{bmatrix}e^t\\-e^{2t}\end{bmatrix} + \begin{bmatrix}e^t & 0\\0 & e^{2t}\end{bmatrix}\int_0^t e^{-t}e^t dt$$

$$= \begin{bmatrix}e^t\\-e^{2t}\end{bmatrix} + \begin{bmatrix}e^t & 0\\0 & e^{2t}\end{bmatrix}\int_0^t dt$$

$$= \begin{bmatrix}e^t\\-e^{2t}\end{bmatrix} + \begin{bmatrix}e^t & 0\\0 & e^{2t}\end{bmatrix}\begin{bmatrix}t\\0\end{bmatrix}$$

$$= \begin{bmatrix}e^t\\-e^{2t}\end{bmatrix} + \begin{bmatrix}te^t\\0\end{bmatrix}$$

$$[x] = \begin{bmatrix}(1+t)e^t\\-e^{2t}\end{bmatrix} \text{ .... (6.2)}$$

Computational Algorithm

The analytic solution of the above non homogenous equation is presented and a computational algorithm for its numerical computation is presented subsequently:

```
EXTERNAL FUNC 2
DO 11  I = 2, 4, 6, 8
SS=0
T=1.0*I
B=2
CALL Q GAUSS (FUNC 2, 0, T, SS)
XX1=Exp(T) +SS*Exp(T)
XX2=-Exp(B*T)
XX3=(1+T)*Exp(T)
XX4=-Exp(B*T)
WRITE (*,*). T, XX1, XX2, XX3, XX4.
11 CONTINUE
STOP
END
FUNCTION FUNC 1 (X)
FUNC 1= Exp(X)
RETURN
END
FUNCTION FUNC 2 (X)
EXTERNAL FUNC 1
FUNC 2=Exp (-X)*FUNC 1 (X)
RETURN
END
```

III. CONCLUSION

A computational algorithm which can be implemented based on the analytic solution of a non homogenous equation subject to the force function f(t)=et has been presented here, however as an enthralling task, the force function could be modified or replaced elsewhere. This solution undoubtedly will be extremely useful in obtaining the explicit numerical solution of a related biological or physical process that can be described by this non homogenous linear differential equation.

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