A Mimic SDE (formulation) for Asset Stock Interpretation based on Comparison viz a Derived Expression for the: "Mass of an Individual Fish Larvae in an Uncapped Rate Stochastic Process"

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***Abstract*– Stochastic variations are frequently encountered in diverse number of processes. A capped rate stochastic process could be described as bounded by some limit and thorough delineation of the various factors affecting the growth of fish larvae is highly essential, the stochastic process viz the Ito lemma has been inundated. Fluctuations or randomness are encountered in a number of practical applications in biology, physical sciences, atmospheric science and oceanography, finance, etc, which are driven by various processes such as the Wiener diffusion path, Poisson or Compound process, etc. In an earlier discussion an expression has been derived for the mass of an individual fish larvae, as an extension we deduced a mimic SDE expression potentially useful for delineation of asset stock price and volatility.**

***Keywords*– Asset Stock Price, Stochastic Process and Ito Lemma**

I. INTRODUCTION

Stochastic and deterministic processes are frequently encountered in vast number of daily applications including physical, chemical and biological, stock, etc. Precisely, our extension to asset stock interpretation based on comparison with an earlier discussion (Adewole et al, 2012) with a mimic expression constitute the major subject of this discussion.

The concept of stochastic events or process cannot be taken with levity as it spans across most aspects of life and applications. Various forms of variations are encountered in one or more processes, indeed in sciences, finance to mention a few; we cannot but mention stochastic process. A capped rate process though has a maximum or limit imposed but practically, there are some fluctuations that worth being considered.

In real sense, fluctuations or variations exist, which take different forms, usually random appearing as noise. For instance, growth of an individual is influenced by internal and external factors. Internal factors are basically, physiological and metabolic pathways. The external factors would include factors like prey, predator, which are biological and physical factors like temperature, humidity, pH, salinity, light intensity etc. In a growth model, an individual is expected to grow continuously, realistically speaking, there is a limit or maximum growth limit. If the change in mass is considered with time, initially the growth rate depends on the physiological and metabolic pathways such as rate of conversion of food to mass and amount of food conversion utilized. However, there is a maximum or capped growth limit the individual can attain.

An ideal tool for this work is the Cushing-Horwood model of larval fish growth (Cushing & Horwood, 1994). According to the growth/mortality hypothesis (Cushing and Horwood, 1994; Rice et al, 1993), larvae which grow quickly through a ''mortality window'' have a survival advantage over those that do not.

Focusing on the fish larvae recruitment in this study, the capped rate stochastic process has been considered and the mass change considered in consonance with physiological factor, metabolic cost and time with other relevant factors. Even when a process is capped, there is expedient need to incorporate some stochasticity thereby unveiling a stochastic pattern.

II. METHODOLOGY

The Ito interpretation was applied in a previous investigation in delineating the capped rate process in fish larvae recruitment with consideration of some stochastic variables determining the change in mass of individual fish larvae.

The model applied in a previous work is the Cushing-Horwood model (1994) with numerical treatment. The change in mass of the individual by this model is given by:

$$\Delta M = \min\left[(b(M)\, Z(M) - C(M) \times G(M)\right] \Delta t \qquad (1.0.0)$$

An analytic mimic SDE expression for interpretation of asset stock price and volatility based on comparison with the mass of individual larvae in an uncapped rate stochastic process has been deduced following a stochastic differential equation given by:

$$\Delta S = rS\Delta t + \sigma\sqrt{\Delta t}Z \qquad (1.0.0)\text{ b}$$

III. DISCUSSION

Many mathematical models which attempt to describe this process use continuous approximations specifically, an ordinary differential equation (ODE) is derived. There is an exigent need to incorporate stochastically in the ODE when uncertainty plays a significant role in the process, for example when prey are distributed patchly or the predator has a high mortality risk. However, the stochastic generalizations often rely on infinitesimally small time steps, not applicable to biological systems. Not including the "unpredictable" environment noise in fisheries models can lead (and has lead) to erroneous predictions of behavior of exploited stocks, and may have contributed to the deterioration of these stocks.

Deterministic models of recruitment can provide important insights into fish population dynamics in the face of exploitation (Fogarty, 1993). However, because the key natural phenomena are inherently stochastic, deterministic models can be argued to be inappropriate for qualifying recruitment. Rather, stochastic models should be constructed to arrive at recruitment probability (Pitchford and Brindley, 2001) and investigate recruitment variability (Forgarty, 1991, 1993).

Two naïve approaches to stochastic process comprises, the standard Weiner process and Poisson process. Where M(t) is defined as a diffusion process with constant drift:

$$dM(t) = \lambda dt + \sigma dW(t) \qquad (2.0.0)$$

W(t) is a standard Weiner process (Karlin, 1981, Grimmett and Stirzaker, 2001, Whitt, 2002). Another approach:

$$dM(t) = dN_{\lambda}(t) \qquad (2.0.1)$$

Uses a Poisson noise process (Feller, 1950). As in equation 2.0.0, M(t) also has expectation λt in equation 2.0.1. These two stochastic processes could be applied in principle to generalize equation (1.0) except where capped rate models are concerned, which could generate no leading results (James et al, 2005).

A) Further Discussion (Stochastic Differential Equation)

A typical stochastic differential equation is of the form:

$$dX_t = \mu(X_t, t)\, dt + \sigma(X_t,t)dB_t \qquad (3.0.0)$$

where B denotes a Weiner process (standard Brownian motion).

For functions fεN, the Ito integral is:

$$F[f](\omega)= \int_S^T f(t,\omega)dB_t\,(\omega) \qquad (3.0.1)$$

In integral form, the equation is:

$$X_{t+s} - X_t = \int_t^{t+s} \mu\,(X_{\mu}, u)du + \int_t^{t+s} \sigma(X_u, u)dBu \qquad (3.0.2)$$

For functions fεN, the Ito integral is:

$$F[f](\omega)= \int_S^T f(t,\omega)dB_t\,(\omega) \qquad (3.0.3)$$

where B_t is 1-dimensional Brownian motion.

The Stratonovich integral, is an alternative to the Ito integral. Unlike the Ito calculus, the chain rule of ordinary calculus applies to Stratonovich stochastic integrals and the two can be converted viz the other for convenience as demanded.

A comparison of Ito and Stratonovich integral could be done.

The white noise equation:

$$\frac{dx}{dt} = b(t, X_t) + \sigma(t, X_t)W_t \qquad (3.0.4)$$

has the solution X_t given as:

$$X_t = X_o + \int_o^t b(s,x)ds + \int_o^t \sigma(s, X_s)dB_s \qquad (3.0.5)$$

Conversion between the Ito and Stratonovich integrals may be performed using the formula:

$$\int_o^T \sigma(X_t) o dW_t = \frac{1}{2}\int_o^T \sigma^1(X_t)dt + \int_o^T \sigma(X_t)dW_t \qquad (3.0.5)\text{ (b)}$$

Where X is some process, σ is a continuously differentiable function with derivative, σ^1 and the last is an Ito integral.

W(t) is used in deriving a stochastic differential equation. The change in mass of the fish larva is considered in the current study. Each individual larva hatches with mass, M_o and grows according to the equation:

$$\Delta M = \min\,((f_1(M)\, Z(M) - f_2(M);\, G(M))\,\Delta t \qquad (3.0.6)$$

Where $f_1(M)$ is the larva's efficiency, at converting food into biomass, $f_2(M)$ is the metabolic cost. Equation (3.0.6) is no longer an ODE and becomes stochastic differential equations, SDE when the prey contact rate Z is defined as a random variable representing a heterogeneous food supply. With the defining function of the rate of change is not smooth in the minimum function of equation (3.0.6), Z follows a Gaussian distribution. SDEs are solved analytically and computationally or numerically.

Having adapted the change in mass for fish larva recruitment viz the population growth model, we deduced an expression previously stated in comparison with the Ito interpretation for the individual mass, M_o to grow and becomes M viz:

$$M = M_o \exp\left[r - \frac{1}{2}\alpha^2\right]t + \alpha W_t] \text{ viz integration}$$

of ordinary calculus.

IV. FURTHER DISCUSSION

A) Stochastic Differential Equation for Asset Stock Price

Write up:

$$\Delta S = rS\Delta t + \sigma\sqrt{\Delta t}Z \quad (4.0.0)$$

is an SDE i.e., stochastic differential equation. Stochastic differential equations are no doubt encountered in diverse applications in biology and physical sciences, climate and oceanography, etc. Finance is an inevitable area of application, for instance asset or stock price can be delineated extensively by applying an SDE, stochastic differential equations follow a random pattern, obviously the SDE includes a noise or random variable or term. Noise term takes different definitions among; White noise, Poisson and Compound noise, etc, which has been previously discussed in our discussion.

The asset or stock price follows a random trajectory obvious from the stochastic differential equation (4.0.0). It is proposed and therefore inevitable that the asset or stock price can be extensively delineated following a stochastic formulation. In the real practical scenario or market as frequently experienced in finance, there abound various forms of fluctuations from different variables or market factors, the asset or stock price does not practically follow a definite pattern or fixed predetermined law. Thus, these variations would best be explained by a stochastic differential equation.

S equivalently S(t) denotes the asset or stock price, r is the interest rate, □ is the volatility, Δt is the time step or partition, Z is a stochastic random variable or noise.

Analogous to the method adopted in derivation of the expression or formula for the mass of a fish larva in an uncapped rate stochastic process, the SDE (4.0.0) can be solved explicitly and thus obtain the following expression:

$$S = S_0 \exp\left(-\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right) \quad (4.0.1)$$

where W_t defines a White noise or equivalently some noise or random variable(Higham,2004, Hull, 2005).

With equation (4.0.1), the asset or stock price can be delineated and thus sketch its path or trajectory.

Various pricing options comprises the European option and American option also the Asian , these can be explained a stochastic formula given the variables among the asset price(S), interest rate(r) or risk-free interest rate, expiration date(T), the exercise price (K), etc.

B) Pricing Options- Concise discussion

European Option Pricing:

For example, a European call option has a payoff max (S(T)-X,0) at expiry. Assuming a log-normal process, S has the form:

$$S = S_0 e^{\left(r - \frac{1}{2}\sigma^2\right)T} + \sigma\sqrt{T}Z) \quad (4.0.2)$$

where Z is a standard normal N(0,1) random variable. Thus the call option can be valued by sampling S(T) if Z is generated.

C) American Option Pricing

Formulation:

A general class of American pricing options can be formulated by specifying a Markov process$\{X(t), 0 \leq t \leq T\}$ representing relevant financial variables such as an underlying asset price, an option payoff h(X(t)) at time t, an instantaneous short rate process$\{r(t), 0 \leq t \leq T\}$, and a class of admissible stopping times $\boldsymbol{\mathcal{T}}$ with values in $[0, T]$.

The American option pricing formulation is to find the optimal expected discounted payoff $supE^{\sup E}_{\tau\epsilon\mathcal{T}}[e^{-\int_0^{\mathcal{T}} r(u)du}h(X(\mathcal{T})].$

It is implicit that the expectation is taken with respect to the risk -neutral measure. In this course we assume that the short rate constant, $r(t) = r$, a non-negative constant for, $0 \leq t \leq T$.

For example, if the option can be only be exercised at times; $0 < t_1 < t_2 \ldots < t_m = T$ (this type of option is often called (Bermudan option), then the value of an American put can be written as;

$supE_{i=1,\ldots,m}[e^{-rt_i}(K - S_i)^+]$.

where K is the exercise price ,S_i is the underlying asset price $S(t_i)$, r is risk-free interest rate.

More details on option pricing can be found elsewhere. However, we have given a detailed insight into the stochastic formulation of asset or stock price with requisite factors or variables mentioned.

V. CONCLUSION

The growth model plausibly, could be assumed to be capped. However, the attainment of capped rate stochastic situation might not be sufficient practically due to some stochastic or random fluctuations in real life processes, thus consideration of an uncapped rate situation becomes expedient. The stochastic variations in uncapped rate situation and the Wiener driven noise are practically inevitable.

However, in a previous discussion (Adewole et al, 2012), considering an Ito Interpretation, an analytical expression has been derived for determining the mass of an individual fish larvae in an uncapped rate stochastic. Based on observation and consideration of salient features and parameters, a mimic stochastic differential equation has been deduced for interpretation of asset stock price and volatility in comparison or analogue to the uncapped rate stochastic process in fish larvae recruitment.

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