Using Equations of Energy for Frequencies of Vibration of Two Different Shells

Mohammad Reza Isvandzibaei

***Abstract*—Vibration of two different shells made of a functionally gradient material composed of stainless steel and nickel is presented. The aim this work is to study the natural frequency (Hz) and the effects clamped boundary conditions on the two different shells. The study is carried out using third order theory. The analysis is carried out using kinetic and potential energies. The governing equations of motion of functionally graded cylindrical shells are derived based on two different materials. Results are presented on the natural frequency and clamped boundary conditions.**

***Keywords*– Clamped, Kinetic and Potential**

I. INTRODUCTION

Shell structures are used as constructive components in many engineering applications. Shells are exposed to dynamic load and they can carry applied loads. Cylindrical shells are making with different materials for various applications in industry. The dynamic characteristic of structures has been reported by many researchers and dynamics behaviors, including vibration are reasons for the reputation these structures in engineering. Vibrations of cylindrical shells have been carried out extensively [1]–[5]. In all the above works, different thin shell theories were used. Vibration of cylindrical shells with ring support is considered by Loy and Lam [6].

The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan [7] as a means of preparing thermal barrier materials. In this study the kind of materials are stainless steel and nickel. There are many methods for fabrication functionally graded materials that these materials are graded in the thickness direction. The study is carried out using third order theory and the analysis is carried out using kinetic and potential energies. The boundary condition is clamped for both edge cylindrical shells. The influence of the different materials and the effect of the clamped boundary conditions on natural frequencies are discussed.

M.R. Isvandzibaei, Department of Mechanical Engineering, Andimeshk Branch, Islamic Azad University, Andimeshk, Iran, (Email: esvandzebaei@yahoo.com)

II. FUNCTIONALLY GRADED MATERIALS

For the cylindrical shell made of FGM the material properties such as the modulus of elasticity E, Poisson ratio ν and the mass density ρ are assumed to be functions of the volume fraction of the constituent materials when the coordinate axis across the shell thickness is denoted by z and measured from the shell's middle plane. The functional relations between E, ν and ρ with z for a stainless steel and nickel FGM shell are assumed as [8].

$$E = (E_1 - E_2)\left(\frac{2Z+h}{2h}\right)^N + E_2 \tag{1}$$

$$\nu = (\nu_1 - \nu_2)\left(\frac{2Z+h}{2h}\right)^N + \nu_2 \tag{2}$$

$$\rho = (\rho_1 - \rho_2)\left(\frac{2Z+h}{2h}\right)^N + \rho_2 \tag{3}$$

The third- order theory used in this study is based on the following displacement field:

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) + \alpha_3^2.\psi_1(\alpha_1,\alpha_2) + \alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) + \alpha_3^2.\psi_2(\alpha_1,\alpha_2) + \alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{4}$$

The stress-strain relations for a FGM cylindrical shell is expressed by:

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{21} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{Bmatrix} \quad (5)$$

III. ENERGY EQUATIONS

The strain energy of the FGM cylindrical shell is expressed as:

$$U=\frac{1}{2}\int_0^L\int_0^{2\pi}\{\varepsilon\}^T[S]\{\varepsilon\}\,Rd\theta\,dx \quad (6)$$

The kinetic energy of the FGM cylindrical shell is expressed as

$$T=\frac{1}{2}\int_0^L\int_0^{2\pi}\rho_T\{(\frac{\partial u}{\partial t})^2+(\frac{\partial v}{\partial t})^2+(\frac{\partial \omega}{\partial t})^2+ (\frac{\partial \psi_x}{\partial t})^2+(\frac{\partial \psi_\theta}{\partial t})^2\}Rd\theta\,dx \quad (7)$$

The displacement fields for a FG cylindrical shell and the displacement fields which satisfy these boundary conditions can be written as

$$\begin{aligned} u &= A\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\ \upsilon &= B\phi(x)\sin(n\theta)\cos(\omega t)\\ w &= C\phi(x)\cos(n\theta)\cos(\omega t)\\ \psi_x &= D\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\ \psi_\theta &= E\phi(x)\sin(n\theta)\cos(\omega t)\end{aligned} \quad (8)$$

where, A, B, C, D and E are the constants denoting the amplitudes of the vibrations in the x,θ and z directions. $\phi(x)$ is the axial function that satisfies the geometric boundary conditions. The axial function $\phi(x)$ is chosen as the beam function as

$$\phi(x)=\gamma_1\cosh(\frac{\lambda_m x}{L})+\gamma_2\cos(\frac{\lambda_m x}{L})-\zeta_m(\gamma_3\sinh(\frac{\lambda_m x}{L}) +\gamma_4\sin(\frac{\lambda_m x}{L})) \quad (9)$$

The boundary conditions for clamped that satisfy, both edge shell can be expressed as:

$$\phi(0)=\frac{\partial\phi(L)}{\partial x}=0 \quad (10)$$

To determine the frequency for FGM cylindrical shell the Ritz technique is used. The energy functional F is defined by the Lagrangian function as

$$F=U_{max}-T_{max} \quad (11)$$

The equation for solution this problem is obtained with powers of ω is obtained

$$\alpha_\circ\omega^{10}+\alpha_1\omega^8+\alpha_2\omega^6+\alpha_3\omega^4+\alpha_4\omega^2+\alpha_5=\circ \quad (12)$$

where $\alpha_i\,(i=0,1,2,3,4,5)$ are some constants. Eq. (12) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the FGM cylindrical shell based third order theory. The smallest of the five roots is the natural angular frequency studied in the present study.
The FGM cylindrical shell is composed of Nickel and Stainless steel are presented in table 1.

TABLE I
PROPERTIES OF MATERIALS

Coefficients	Stainless Steel			Nickel		
	E	ν	ρ	E	ν	ρ
P_0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

IV. RESULTS AND DISCUSSION

In this paper, studies are presented for a FGM cylindrical shell with energy equations is considered. In first FGM shell, nickel is on inner surface and stainless steel is on outer surface and second FGM shell, stainless steel is on inner surface and nickel is on outer surface. Tables II and III show variations of frequencies for first FGM shell and second FGM shell. The effect of the position of material on the frequencies both FGM shells are different for clamped boundary conditions. For first FGM shell, the frequencies decreased when N increased, and for second FGM shell, the frequencies increased when N increased.

TABLE II
VARIATION OF FREQUENCIES FOR FIRST FGM SHELL WITH CLAMPED BOUNDARY CONDITION
(m = 1, h / R=0.002, L / R=20)

n	N=0.7	N=1	N=5
1	12.32	12.21	11.91
2	6.29	6.19	6.12
3	6.05	5.94	5.86
4	8.73	8.58	8.29
5	11.48	11.37	11.25

TABLE III
VARIATION OF FREQUENCIES FOR FIRST FGM SHELL WITH CLAMPED BOUNDARY CONDITION
(m = 1, h / R=0.002, L / R=20)

n	N=0.7	N=1	N=5
1	13.45	13.89	14.11
2	8.36	8.52	8.83
3	8.18	7.91	7.73
4	10.60	10.37	10.11
5	12.49	12.28	12.14

V. CONCLUSION

This work presents a study on the vibration of two different FGM shells composed of stainless steel and nickel. In first FGM shell, nickel is on inner surface and stainless steel is on outer surface and second FGM shell, stainless steel is on inner surface and nickel is on outer surface. The boundary conditions is clamped for both edge. The study showed that the configurations of the materials is important and affect on the frequencies.

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M. R. Isvandzibaei, Department of Mechanical Engineering, Andimeshk Branch, Islamic Azad University, Andimeshk, Iran.