Polynomial Fitting of Geodynamic Data viz Lagrangian Interpolation

Ewumi T. Olubunmi
Department of Physics, Ekiti State University, Ekiti State, Nigeria

***Abstract*– Various external forces influence water movements in a homogenous sea. A quadratic polynomial fit has been obtained viz Lagrangian interpolation based on the geodynamic data associated with propagation of the topographic waves in a homogenous sea in this study.**

***Keywords*– Topographic Waves, Geostrophic, Quadratic Polynomial Fit and Homogenous Ocean**

I. INTRODUCTION

The ocean surface is an example of a complex wave motion formed by the action of wind. The displacement of a fluid particle from equilibrium position and action of a restoring (gravitational) force on the particle produces a wave like motion in the ocean called an internal wave.

The motion of ocean water is strongly influenced by the spatial variation in homogeneity of the wind field over the ocean surface and the topography of the ocean bottom. Topographic waves are modeled using a primitive-equation ocean model.

Various external forces influence water movements in a homogenous sea. These comprise major forces that maintain the ocean currents including air currents, the changes in atmosphere pressure at the surface of the sea and the periodic tide-generating astronomic forces. The changes in atmospheric pressure are transmitted through the entire mass of water down to the ocean bottom and this give rise to horizontal pressure differences and the formation gradient currents. The air currents result to two fold effects consisting of the tangential force of the ocean(wind stress) which produces a surface current transmitted by the effect of viscosity(turbulence) to the water layers waves also constitute water movements in the direction of the wind.

Internal forces arise from the vertical and horizontal disturbances of mass within the ocean. These differences in the mass distribution both in the horizontal and vertical directions are the consequences of changes in the heat content (temperature) and in the salinity.

A) Equation of Motion

The product of mass and acceleration equals the vector sum of forces as asserted by Newton's second law of motion. This statement is invariably called the equation of motion. The study of large scale ocean or atmospheric motions include the Coriolis force to be geophysically relevant, and once the Coriolis force is included in host of subtle and fascinating dynamical phenomena are possible(Albert, 1961,Batchelor, 1967, Pickard et al, 1975, Bascom, 1980).

The important forces which drive the large-scale motion are the force of gravity, the Coriolis force, pressure gradient force and frictional forces. The centrifugal force of earth's rotation is usually included in gravity. The three dimensional acceleration of a particle is described by the vector equation of motion, which contains the following terms:

Particle acceleration = Coriolis term + Pressure gradient term + Gravity terms + frictional term and expressed as;

$$\frac{dc}{dt} = (-2u \times c) + \left(\frac{-1}{\rho}\nabla P\right) + g + F \quad \ldots\ldots\ldots\ldots\ldots (1.0)$$

where dc/dt is the acceleration of a unit mass due to accumulated effects per unit mass of the Coriolis force $-2u \times c$, the pressure gradient force $-1/\rho\nabla P$, the force of gravity g and F, the generalized force due to frictional effects.

The above equation can be written as;

$$\frac{du}{dt} = (2\boldsymbol{\Omega}\boldsymbol{sin\Phi})\boldsymbol{v} - \frac{\mathbf{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial x}} + \boldsymbol{F_x} \quad \ldots\ldots (1.1)$$

$$\frac{du}{dt} = (2\boldsymbol{\Omega}\boldsymbol{sin\Phi})\boldsymbol{u} - \frac{\mathbf{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial x}} + \boldsymbol{F_y} \quad \ldots\ldots (1.2)$$

$$\frac{dw}{dt} = +g - \frac{\mathbf{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial z}} + \boldsymbol{F_z} \quad \ldots\ldots (1.3)$$

In the absence of sources or sinks of mass within the fluid, the condition of mass conservation is expressed by the Coriolis equation (Bryan et al., 1972, Holton, 1972, Kowalik et al., 1993);

$$\frac{\partial\rho}{\partial t} + \nabla.\rho u = 0 \quad \ldots\ldots (1.31)$$

The local increase of density with time must be balanced by a divergence of mass flux ρu, where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + u.\nabla \quad \ldots\ldots (1.31)\text{ b}$$

is the total derivative (often called the substantial derivative) with respect to time of any property following individual fluid elements.

Newton's law of motion for a fluid continuum takes the form,

$$\rho\frac{du}{dt} = -\nabla P + \rho\nabla\emptyset + \boldsymbol{\mathcal{F}}(\boldsymbol{u}) \quad \text{......................................(1.32)}$$

or that the mass per unit volume times the acceleration is equal to the sum of the pressure gradient force, the body force $\rho\boldsymbol{\nabla}\emptyset$ and the force $\boldsymbol{\mathcal{F}}$, where $\emptyset$ is the potential by which conservative body forces such as gravity can be represented. $\boldsymbol{\mathcal{F}}$ in principle is any non-conservative force, $\boldsymbol{\mathcal{F}}$ is the frictional force for in the fluid and for Newtonian fluids like air or water;

$$\mathcal{F} = \mu\nabla^2 u + {}^{\mu}/_{3}\,\nabla(\nabla.u) \quad \text{…………………………..(1.33)}$$

where μ is the molecular viscosity. This representation of $\boldsymbol{\mathcal{F}}$ is exact when μ in principle a function of dynamic variable is considered constant over the field of motion.

B) Geostrophic Motion

When the fluid flow is both unaccelerated and frictionless, its motion is said to be geostrophic, that is, "earth-turned", the only forces acting are the pressure gradient force, the Coriolis force and gravity. By setting equations; 1.1, 1.2 and 1.3 above to zero for the steady state, $\frac{du}{dt} = \frac{dv}{dt} = \frac{dw}{dt} = 0$ and considering frictionless case, we obtain the following equations;

$$(2\boldsymbol{\Omega sin\Phi})\boldsymbol{v} = \frac{\boldsymbol{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial x}} \quad \text{..(1.34)}$$

$$(2\boldsymbol{\Omega sin\Phi})\boldsymbol{u} = -\frac{\boldsymbol{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial y}} \quad \text{...(1.35)}$$

$$+g = \frac{\boldsymbol{1}}{\boldsymbol{\rho}}\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial z}} \quad \text{..(1.36)}$$

II. METHODOLOGY

Various external forces influences water movements in a homogenous sea. A quadratic polynomial fit has been obtained viz Lagrangian interpolation based on the geodynamic data associated with propagation of the topographic waves in a homogenous sea in this study.

The salient features of the pertinent equations were unveiled and the input parameters requisite for the computational task were stated.

A) Lagrangian Interpolation

Briefly, the Lagrangian polynomial interpolation formula is expressed as follows:

$\emptyset(x) \approx \sum_0^n L_i f(x)$, where L_i is the Lagrangrian and $f(x)$ is the corresponding data point to x, for $n + 1$ data points.

III. DISCUSSION

Many types of waves involving different physical factors exist in the ocean. An analogy could be made to an elementary spring-mass system, thus all waves must be associated with some kind of restoring force equivalent to an elementary spring-mass system or simple pendulum, as a result it is convenient to make a crude classification of ocean waves. The most prominent waves on the water surface are the surface gravity waves whose main restoring force is gravity.

The ocean generated waves comprises; long waves, deep-water waves, shallow water waves, episodic waves and internal waves. Long waves in the ocean may be caused by large falls in surface pressure associated with tropical cyclones or by seismic activity. Deep water –waves occur in water that is deeper than one-half of the wavelength. Shallow-water waves at a depth less than one-twentieth of the wavelength. Episodic waves occurs due to a combination of intersecting wave trains , changing depths and currents and are frequent near the continental shelf, in water about 200m deep. Internal waves occur anywhere within the water column.

Ocean waves are generated by wind and swells are waves not under the direct influence of wind because the waves have moved out of the active area under wind influence. Tsunamis which occur less frequently but very devastating are consequences of long-period oscillations due to large submarine earthquakes or landslides. Waves are also generated due to human activities such as explosions and ship motion within the same broad range of time scale.

A) Topographic Waves and Dynamics of Ocean Bottom

Small bottom irregularities can turn an otherwise steady geostrophic flow into slow moving waves. The dynamics of an ocean with bottom slope is elaborated here.

For simplicity a homogenous ocean is considered in a domain with periodic boundaries in y and a weak uniform bottom slope in the x direction as delineated by pertinent equations below:

Emphasizing the vertically integrated continuity equation:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu)\frac{\partial}{\partial y}(hv) = 0 \quad \text{……………....,(1.4)}$$

By substituting $h(x, y, z) = H_0 - \alpha x + \boldsymbol{\eta}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t})$ into equation (1.4) above gives;

$$\frac{\partial h}{\partial y} = \frac{\partial \eta}{\partial y} \quad \text{………………………………… (1.5)}$$

$$\frac{\partial}{\partial x}(hu) = u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} \quad \text{…………..………(1.6)}$$

$$\frac{\partial}{\partial x}(hv) = v\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} \quad \text{…………………,(1.7)}$$

$$\frac{\partial h}{\partial x} = -\alpha + \frac{\partial \eta}{\partial x} \quad \text{…………………………(1.8)}$$

$$\frac{\partial h}{\partial t} = \frac{\partial \eta}{\partial t} \quad \text{……………………………….(1.9)}$$

By substituting equation (1.5) into equation (1.4) yields;

$$\frac{\partial \eta}{\partial t} + \left(u\frac{\partial h}{\partial x} + v\frac{\partial \eta}{\partial y}\right) + \left(H_0 - \alpha x + \eta\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \alpha_0 u = 0 \quad \text{…………(2.0)}$$

From linear theory and requirement of a gentle slope, the continuity equation is written as follows:

$$\frac{\partial\eta}{\partial t}+(H_0)(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})-\alpha_0 u=0 \quad\text{(2.1)}$$

The corresponding linear vertically integrated momentum equations are:

$$\frac{\partial u}{\partial t}-fv=-g\frac{\partial\eta}{\partial x} \quad\text{(2.2)}$$

$$\frac{\partial v}{\partial t}-fu=-g\frac{\partial\eta}{\partial x} \quad\text{(2.3)}$$

The extra term $\alpha_0 u$ in the continuity equation, related to the bottom slope will allow the existence of slow waves similar to the planetary waves (Phillips,1963,Schulman E.E,1975, Pedlosky,1984) due to the variation of the Coriolis parameter. This system contains both small and large terms. The large ones (terms including f, g and H_0) comprise the otherwise steady geostrophic dynamics. In the presence of the small term $\alpha_0 u$, the time derivatives come into play, but are still expected to be small. Thus based on this smallness, we can take as a small approximation, the geostrophic balance:

$$u=\left(\frac{g}{f}\right)\frac{\partial\eta}{\partial y},\quad v=(\frac{g}{f})\frac{\partial\eta}{\partial x} \quad\text{(2.4)}$$

By substituting equations (2.4) in the small time derivatives of equations (2.2) and (2.3), we obtain:

$$\frac{\partial u}{\partial t}=-(\frac{g}{f})\frac{\partial^2\eta}{\partial y\partial t} \quad\text{(2.5)}$$

$$\frac{\partial v}{\partial t}=-(\frac{g}{f})\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(2.6)}$$

From equation (2.3);

$$fu=-g\frac{\partial\eta}{\partial y}-\frac{\partial v}{\partial t} \quad\text{(2.7)}$$

$$u=-\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{1}{f}\frac{\partial v}{\partial t} \quad\text{(2.8)}$$

$$u=-\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{1}{f}(\frac{g}{f})\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(2.9)}$$

$$u=-\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{g}{f^2}\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(3.0)}$$

Similarly for v;

$$fv=\frac{\partial u}{\partial t}+\frac{\partial\eta}{\partial x} \quad\text{(3.1)}$$

$$v=\frac{1}{f}\frac{\partial u}{\partial t}+\frac{g}{f}\frac{\partial\eta}{\partial x} \quad\text{(3.2)}$$

$$v=\frac{1}{f}(-\frac{g}{f})\frac{\partial^2\eta}{\partial x\partial t}+\frac{g}{f}\frac{\partial\eta}{\partial x} \quad\text{(3.3)}$$

$$v=\frac{g}{f}\frac{\partial\eta}{\partial x}-\frac{g}{f^2}\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(3.4)}$$

By replacement of the component in the continuity equation (2.1) yields a single equation for $\boldsymbol{\eta}$ as follows:

$$u=-\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{g}{f^2}\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(3.5)}$$

$$u=-\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{g}{f^2}\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(3.6)}$$

$$\frac{\partial u}{\partial x}=-\frac{g}{f}\frac{\partial^2\eta}{\partial x\partial y}-\frac{g}{f^2}(\frac{\partial^2\eta}{\partial^2 x}) \quad\text{(3.7)}$$

$$v=\frac{g}{f}\frac{\partial\eta}{\partial y}-\frac{g}{f^2}\frac{\partial^2\eta}{\partial x\partial t} \quad\text{(3.8)}$$

$$\frac{\partial v}{\partial y}=\frac{g}{f}\frac{\partial^2\eta}{\partial x\partial y}-\frac{g}{f}\frac{\partial}{\partial t}(\frac{\partial^2\eta}{\partial x\partial t}) \quad\text{(3.9)}$$

By substituting equations (3.7), (3.4) and (2.9) into equation (2.1) yields:

$$\frac{\partial\eta}{\partial t}-R^2\frac{\partial}{\partial t}\nabla^2\eta+\frac{\alpha_0 g}{f}\frac{\partial\eta}{\partial y} \quad\text{(3.91)}$$

where $R=\frac{\sqrt{gH_0}}{f}$. This is the Rossby radius.

The solution of equation (3.91) gives:

$$\eta=Acos(lx+my-\omega t) \quad\text{(3.92)}$$

$$\frac{\partial\eta}{\partial x}=-Alsin(lx+my-\omega t) \quad\text{(3.93)}$$

$$\frac{\partial\eta}{\partial y}=-Amsin(lx+my-\omega t) \quad\text{(3.94)}$$

$$\frac{\partial\eta}{\partial t}=A\omega sin(lx+my-\omega t) \quad\text{(3.96)}$$

$$\frac{\partial^2\eta}{\partial x^2}=-Al^2cos(lx+my-\omega t) \quad\text{(3.97)}$$

$$\frac{\partial^2\eta}{\partial y^2}=-Am^2cos(lx+my-\omega t) \quad\text{(3.98)}$$

Substitution of equations (3.93) – (3.98) into equation (3.91) above gives the dispersion relation expressed as:

$$\omega=\frac{\alpha_0\ g}{f}\frac{m}{(1+R^2(l^2+m^2))} \quad\text{(3.99)}$$

These waves exist on their own due to the existence of the bottom slope α_0, hence they are called topographic waves. Without the presence of the bottom slope α_0, the flow would be steady and geostrophic.

Tsunamis are most likely to occur in ocean basins that are tectonically active. The Pacific ocean, ringed by crustal faults and volcanic activity is the birth place of most tsunamis. They have also appeared in the Carribean sea, which is bounded by an active island system and in the Mediterranean sea as well. It is less clear why tsunamis cause more damage in some localities than others. Difference from the source, local refraction effects, and focusing of the source pulse are all important. It is also thought that a wide continental shelf serves both as a wave reflector sending much of the energy through friction along the bottom. Shallow shelves can also trap wave. Incidents of significant tsunamis damage are rare in regions with wide continental shelves for whatever reasons.

IV. POLYNOMIAL FIT

Applying the Lagrangian interpolation, a quadratic polynomial fit has been obtained. The quadratic polynomial fit obtained using a few geodynamic data from table 4.0 below is as follows:

$C \approx 0.0154\eta^2 + 0.6430\eta - 0.5050$. Definitely, the ocean wave velocity can be obtained at any ocean depth.

Input:

The values of the bottom slope α and bottom friction factor can be obtained, the results for α = 0, r= 0 and n = 180 are listed in the Table 4:

Table 4: Results for α = 0, r= 0 and n = 180

C (m/s)	η (m)
3.00	2.40
4.86	4.80
8.96	9.60
10.61	12.00
14.82	17.00

C represents the resultant ocean wave velocity, η is the surface ocean depth.

The values of the bottom slope $\boldsymbol{\alpha}$ and bottom friction can be altered, so that $\boldsymbol{\alpha}$ becomes 1×10^{-3} and bottom friction r is now 0.0003 respectively.

Beardsley et al. (1975, Bö et al, 1992) in Bryan (1963) integrated the vorticity equation for a rectangular, flat bottomed ocean in the absence of bottom friction, i.e., he set r=o and $\boldsymbol{\eta_\beta}$=0 in:

$$\frac{1}{\beta}\left\{\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\left[\nabla^2\psi + \eta_\beta\right] - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\left[\nabla^2\psi + \eta_\beta\right]\right\} + \frac{\partial\psi}{\partial x} = curl\tau - \frac{\mu}{\beta}\nabla^2 + \frac{\nabla^4}{\beta}\frac{\psi}{Re} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.4)$$

The interior velocity was scaled as in:

$$U = \frac{\tau_0}{\rho D \beta_0 L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.5)$$

and the calculations were done for most part with a fixed value of;

$$\beta^{-1} = \frac{U}{\beta_0 L^2} = 1.28 \times 10^{-3} \quad \ldots\ldots\ldots\ldots\ldots\ldots..(4.6)$$

while the Reynold's number : $Re = \frac{UL}{A_L}$ …………(4.7)

was varied from 5 to 60. Beyond Re=60, only unsteady solutions were observed since,

$$\frac{\delta_1}{\delta_M} = \left(\frac{Re}{\beta^{1/2}}\right)^{1/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.8)$$

The experiments ranged over values of ${}^{\delta_M}/_{\delta_l}$ from about 0.56 at Re=5 to 1.29 for Re=60. In the later limit, the boundary layer is certainly strongly, non-linear but friction is nearly as important. Indeed the limit $\frac{\delta_M}{\delta_l} \ll 1$ may be largely irrelevant for the steady pattern for experiments indicate that in that limit the steady solutions are undoubtedly unstable and unrealizable in practice.

A very interesting numerical experiment was carried out by Veronis (1966). He set A_H=0 and so relaxed the condition of no slip boundary of a rectangular basin and considered only bottom friction as a dissipation mechanism. In this case, in which the ocean basin has a flat bottom, the relevant measure of non-linearity in the western boundary current region is:

$$\frac{\delta_1}{\delta_s} = \frac{\beta^{1/2}}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\ldots(4.9)$$

Veroni's calculation ranged from $\frac{\delta_1}{\delta_s}$ of 2×10^{-3} to a maximum of 8.

V. CONCLUSION

The resultant ocean velocities and corresponding surface ocean depths are presented in table 4.0 from this investigation. A quadratic polynomial fit has been obtained viz Lagrangian interpolation based on the geodynamic data associated with the propagation of the topographic waves in a homogenous sea in this study.

The values of α and r are zero respectively in this investigation, which determine the variational trend of the ocean wave velocity with the surface ocean depth. The bottom slope had it not present in the dispersion relation previously listed (3.91) would have implied steady and geostrophic flow. Because, the waves exit on their own due to the existence of the bottom slope, they are called topographic waves.

REFERENCES

[1]. Albert, Defant, 1961. Physical Oceanography. Pergamon press, 1,728.

[2]. Bascom.W, 1980. Waves and beaches. The dynamics of the ocean survey, Doubledy Garden City, N.J., 336.

[3]. Batchelor G.K, 1967. An introduction to fluid dynamics. Cambridge University Press, 615.

[4]. Bö. C.W, Budch R.G, 1992. Eddy dynamics in a primitive equation model: Sensitivity to horizontal resolution and friction. J. Phys. Oceanography, 79-89.

[5]. Beardsley R.C, Robbins K, 1975. The sliced cylinder Laboratory model of the wind-driven ocean circulation Part I. Steady forcing and topographic Rossby wave instability. J. Fluid Mech., 6927-40.

[6]. Bryan K, 1963. A numerical investigation of a non-linear model of a wind driven ocean. J. Atmos. Sci., 20:594-606.

[7]. Bryan, Kirk and Cox, Micheal D, 1972. An approximate equation of state for numerical model of ocean circulation. J. Physical Ocean, 510-514.

[8]. Holton .JR, 1972. An introduction to dynamic meterology, Nature Press, 319.

[9]. Kowalik Z, Murty T.S, 1993. Numerical modeling of ocean dynamics, 481.
[10]. Pickard G.L, Pond S, 1983. Introductory dynamical oceanography, Pergamon press, New York, 329.
[11]. Pedlosky J, 1984. The equations for geostrophic motion in the ocean. J. Phys. Oceanogr., 448-456.
[12]. Schulman E.E, 1975. A study of topographic effects: in Numerical models of Ocean Circulation. Nat. Acad. Sci., 147-165.
[13]. Veronis G, 1966. Wind-driven ocean circulation, part 2. Numerical solutions of the non-linear problem. Deep-sea Res., 13:31-55.