The Efficiency of the Financial Market by the Multi-Approximate Entropy: Case of the Tunisian Financial Market

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Abstract– **The object of this paper is to study the efficiency of the Tunisian financial market; we propose to study the efficiency of the market from the point of view of the informational entropy. The basic idea is that, rather than to be a concept that in the classical economics, the variations of efficiency of the market in the course of time and on horizons of time. Within this framework, the efficiency of the market is measured in terms of models contained in the sequence of the changes of price compared to the models in a random order In agreement with the ideas of financing of the evolution, the empirical results of the Tunindex index, observed over the period which is spread out of 2008 up to 2012, showed that the degree of efficiency of the market varies in time and depends on the scale of time In general, the Tunindex index is more efficient for short-term (approximately days) that for in the long run (approximately month and quarters) of the scales of time. In addition, the efficiency of the market presents a cyclic behavior with two periods dominant to knowing the period of the financial crisis 2008 and the events of 2011.**

Keywords– **Efficiency of the Financial Market, Multi Approximate Entropy and TUNINDEX Index**

I. INTRODUCTION

The theory of efficiency is certainly one of the most discussed theories in the economic or financial domain, in spite of the abundance of empirical work aiming at testing the assumption of efficiency, no clear conclusion seems to arise This absence of unanimous results is undoubtedly related to the importance which efficiency in the financial theory is Fama (1965) is at the origin of the first general standard of the concept of informational efficiency: A financial market is efficient if the prices integrate all information constantly available.

A market can be defined efficient in the informational direction if the prices of the credits negotiated on this market reflecting all information available instantaneously. The version of the weak form of the Hypothesis of Efficient Markets (EMH) was tested by rigorous empirical tests in different national stock markets since Samuelson (1965) presented the concept of random walk to the financial economy, the EMH is always in the controversy. The opinions of the researchers on the EMH changed during the 40 last years. Fama (1965 1970) among the researchers which has to study the EMH in several articles; Fama (1991) itself modified its opinion because of many anomalies announced after Fama (1970). However, most of the literature concentrated mainly in test if the stock market is efficient or inefficient for the period chosen, using statistical tests like the tests of correlation, the test of runs, the tests of variance ratio , the tests of unit root and the spectral analysis… Today nobody can recapitulate the discussion of the EMH with two or three lines. The debate between the founders of the EMH and behavioral is always followed and, the authors never believe, will not finish. Taking into account the possibility of finding deterministic chaos in the financial markets, or at least the fractal Brownian movement This article will be organized as follows In the first section, we will first of all present the review of the literature of the previous studies by indicating the importance of the approach the entropy in the analysis of specificities of the financial market, like a new method of measurement of efficiency of the financial market, Second, we present methodology, as well as the measurement of efficiency by the multi approximate entropy In the second section we will have the empirical results as well as interpretations following the study of the efficiency of the Tunisian financial market (TUNINDEX index (Tunis).

II. LITERATURE REVIEW

Into its article founder, Fama (1970) introduced the term "efficiency of the market" to indicate the role of information in the price formation. The Hypothesis of efficient markets (EMH) establishes that new information quickly and is correctly reflect in the course of security in force. Under the EMH, the conditions of arbitration are quickly eliminated by the action from the actors from the informed market. In other words, classical economic logic indicates that the money is not exerted against a marketing appropriateness given, the foreseeable surplus outputs must be reduced and finally eliminated (Stein, 2009). An important consequence of the EMH is that the changes of price must be unforeseeable if they fully integrate information and waiting of the great diversity of the actors of the market. In a market informational efficient, the dynamics of the prices must follow a random behavior of walk resulting from the action of the actors of the market try to benefit from the information is quickly built-in the market prices, by eliminating the possibilities from profit which woke up the interest of the trades. Consequently, no profits can be obtained from the marketing based on information because information on the trends of the prices is uniformly distributed; leaving only information of noise associated with the random price fluctuations. Among these ideas, an enormous corpus of scientific literature focused on the EMH showing that the prices follow a random behavior of walk by studying the foresee ability of the outputs of security on the basis of last variation of the prices. The reader is returned to the recent investigation conducted by Lim and Brooks (2011) for a detailed analysis of the principal contributions to the empirical analysis of the random assumption of walk.

A) The assumption of the adaptive markets

Recent investigations showed the existence of an important difference between the partisans and the EMH and the defenders of behavioral finance (Lim & Brooks, (2011); Malkiel, Mullainathan, and Stangle, (2005)). It is obvious that this variation comes owing to the fact that the gone efficiency is posed like a condition repeating spring Lo (2004) proposed that the two intellectual positions can be reconciled within an evolutionary framework where, rather than of the stat repeating spring, the gone efficiency is a characteristic which varies continuously in time and through the markets. In this respect, Lo (2004) proposed the new paradigm of the Hypothesis of efficient markets (EMH) in which the EMH can coexist with the ideas of the behavior in a manner intellectually coherent (Lim & Brooks, 2011). The idea subjacent with the EMH is that the markets behave like an evolutionary system in which the participants and the instruments interact and evolve dynamically according to the intrinsic rules of economic selection. Consequently, the financial agents have a competition and to evolve to survive Contrary to the classical sight, these agents equipped with a limited rationality (Simon, 1955), are conformed to decisions in a very dubious environment which deviates from the decisions in an optimal way (Farmer & Lo,(1999); Giglio, Matsushita, and DaSilva (2008) ; Lo, (2004 2005).

From this point of view, the efficiency of the market is not a question of all or nothing, but a reflection of the complex interaction of the environments of market (for example, socio-economic conditions) with the structure of the behavior of the participants in the market. Consequently, the efficiency of the market varies in the time and the whole of the markets. Departures of "full efficiency" are authorized that the dynamic market adapts to, for example, the evolution of the conditions and the external shocks. It made the point that, under the terms of the EMH, convergence towards the balance (which is in the middle of the EMH) neither is guaranteed, nor likely to intervene constantly (Lo, 2005).

B) Approach of the analysis of entropy

The existence or not of exploitable models in the trend of prices was the principal contribution to test the efficiency of the market. The combination of the analysis of the trends and the capacity of the specific agents to process the data should provide a framework for the testable catch the EMH

From a macroscopic point of view, the entropy can be related on the number and the diversity of the models and modifications which the system can very display for one a large number of trajectories. Although the entropy is contained in a time series, its request to analyze the temporal financial series was limited to a restricted number of the studies of research. It is obvious that Gulko (1999), firstly proposed the use of the concepts of entropy for has fine to study the financial time series by proving that the formalism maximum entropy, also called informational efficiency, makes the assumption driven gone efficient operationally testable.

Darbellay and Wuertz (2000) showed the utility of the concepts of entropy to characterize the financial series by showing that the covering advantage of the approach of entropy lies in its capacity to give an account of non-linear dependences. Pincus and Kalmans (2004) suggested that the algorithm of the approximate entropy is appropriate to analyze financial time series as it can be applied to very short sequences and can be used as marker of the stability of the market Recently, the concepts of entropy were used to quantify the efficiency of the stock market and foreign exchange market. Contrary to the previous approaches while concentrating on an answer repeating spring to the question of efficiency, the entropy can provide a degree report of the efficiency of a given market. Oh, Kim, and Eom (2007) used the indices of the world foreign exchange markets, in order to study the efficiency of the various exchange markets for the periods of the financial crises. It was noted that the markets with more important liquidities (for example, in Europe and North America) have an efficiency of market higher than those with a smaller liquidity Risso (2008) employed the concept of the entropy of Shannon on the dynamic symbolic system of stock indexes to prove that the probability of having an accident increases while the entropy decreases.

Approximate entropy:

A direct application of the concepts of entropy requires the infinite availability of sets of data with an infinite precision and the exact resolution. It is not possible in practice because measurements of real systems are sampled with ε limited resolution and 1/Ts of finished sampling rate. To rectify this situation, the approximate statistical entropies were introduced (Pincus,1991) to quantify the regularity of the time finite series length approximate calculations of the entropy are conceptually simple and are based on the probability that the models of the temporal series which are similar and remain the same ones on the next additional comparisons. Consequently, the time series with large AE should have raised dubious fluctuations an algorithm for the calculation of entropy for the finished data can be described as follows. A series of times length N sampled with Ts time intervals:

$$(X_i) = (X_1; X_2; \ldots X_N), or\ X_i = X\ (t_o + iT_s) \quad (1)$$

It should be noted that the length N can be related to a scale of time τ = nts tow dimensional sequence vectors U (m) (I) =

{xi, xi +1,…, xi + M1}, and V (m) (J) = {xj, xj +1,…, X J + M-1}, I ≠ J, 1≤ I, J ≤ N-m +1, are selected. These vectors U (m)(I) and v (m) (J) are called similar if their distance from, v (I, J) = max {| U (i + k) - v (j + k) |: 0 ≤ k ≤M- 1}, is smaller than a specified tolerance

For each N-m +1 vectors U (m) (I), the number of similar vectors v (m) (J) is given by the measurement of their respective distance s, if Ni(m) is the number of vectors v (m) (J), similar to U(m) (I), the relative frequency to find a vector v (m)(J), which is similar to U (m) (I) within a ε of level of tolerance is given by:

$$C_i(m, \varepsilon, t) = ni^{(m)}/N - m \qquad (2)$$

Where Nm is the number of vectors v (m)(j) ≠ U (m) (i) which are potentially similar to U (m)(I). Then, one looks at the relative frequency of the logarithm of:

$$C_i(m, \varepsilon, t), i\ e\ , \varphi(m, \varepsilon, t) = 1\big/N - m + 1 \sum_{i=1}^{N-m+1} LnCi\,(m, \varepsilon, t) \qquad (3)$$

For N finished, the approximate entropy is estimated by the statistics:

$$E\ (m, \varepsilon, T) = 1/Ts\ [\varphi\ (m, \varepsilon, T) - \varphi\ (m+1, \varepsilon, T)] \qquad (4)$$

In this way, lower values of E (m, ε, τ)reflect more regular time series, while higher values are associated with-less foreseeable (more complex) - time series in the scale of time τ Effects of the tolerance ε, length N given and vector of dimension m in the exercise of calculation AE (Pincus, 1991), stable statistics were found for N>500, in addition, of the parameters m = 2 and ε =0 15σ, where σ is the standard deviation of the time series, are usually used in the applications.

C) Approximate multi-scale entropy

We can expect that the entropy is dependent on the scale, (one can count that the measurement of the entropy is dependent on the scale of time), which means that the signal's more dubious for certain scales of times and more irregular for others, so that a complete characterization of the entropy of the time series should consider the variability on a non-trivial range of scales. A scale of several methods AE comprises two stages:

i) The method allowing looking at representations of the dynamics of the system on various scales of time. For a given moment of the series X = {x1, x2…,xn}, and a procedure of low-pass filter LP (F)with a cut-off frequency F, to obtain time filtered series ASYF = LP (F) · X where Yf = {YF, 1, YF, 2,…,YF, NR}. The new time series Y F preserves the complexity of signal X for the frequencies lower than F, or of the scales of time higher to τ =1/F. Various operations of low-pass filtering LP(F) are available in the commercial software In our study, such as it is used in the technical analysis by the actors of the market to obtain long-term trends of the financial signals, low-pass filtering is carried out with filters of moving average In this way, the filtered signal is obtained:

$$Y_{f,i} = 1/n \sum_{j=1}^{n} X_{i+j-1} \qquad (5)$$

The scale of time is given by τ = Δ tn and F = 1/τ Here, Δt is sample period.

ii) The second method consists in each time quantifying the degree of irregularity of the Y_f series is carried out by using the procedure described above approximate entropy.

III. METHODOLOGY

In the continuation of a method to test the possible changes in the efficiency of the market, we had recourse to the basic practice of the professionals of the market, namely the search for exploitable models contained in a sequence of changes of price (Theodore, 1996) In this spirit, this work uses the concepts of information and methods of entropy to study the efficiency of the market.

The entropy is a basic concept used to quantify the disorder and the uncertainty of the dynamic systems, which can be related to the number and models of diversities of the modifications that the system, can display one of broadest trajectories (Pincus, 1991). The algorithm of approximate entropy (Pincus, 1991) is used in our study and implemented in a system of slipping window to estimate the content entropy of a time series a given length. The efficiency of the market given is defined in relative terms of the entropy of random sequences. More precisely, the efficiency of the index of the market is defined in function of the distance compared to entropy of reference which is calculated on the basis of Gaussian random sequence. Index TUNINDEX is regarded as a case of work to illustrate the applicability of the approach of entropy.

The results showed that the efficiency of the market does not evolve only in time, but also on horizons of time. The analysis of the efficiency of the market in terms of concepts of entropy for the variations of TUNINDEX showed the existence of with two different periods Interesting fact, the last cycle can be related to the appearance of great recessions.

IV. EMPIRICAL RESULTS FOR TUNINDEX INDEX

The previous methodology suggested by (Pincus, 1991) is applied to the index of TUNINDEX Data day laborers were obtained over the period 2008 to 2012.

A) Descriptive statistics of the variables

Table I provides the descriptive statistics, average, standard deviation, skweness, kurtosis and the value of the statistics of the test of normality by Jarque-Bera.

Table I: Descriptive statistics

Mean	0 000477
Median	0 000429
Maximum	0 041941
Minimum	-0 048805
Std Dev	0 006935
Skewness	-0 425346
Kurtosis	12 83835
Jarque-Bera	5026 181
Probability	0 000000
Sum	0 590664
Sum Sq Dev	0 059452
Observations	1237

Statistics provided by Eviews

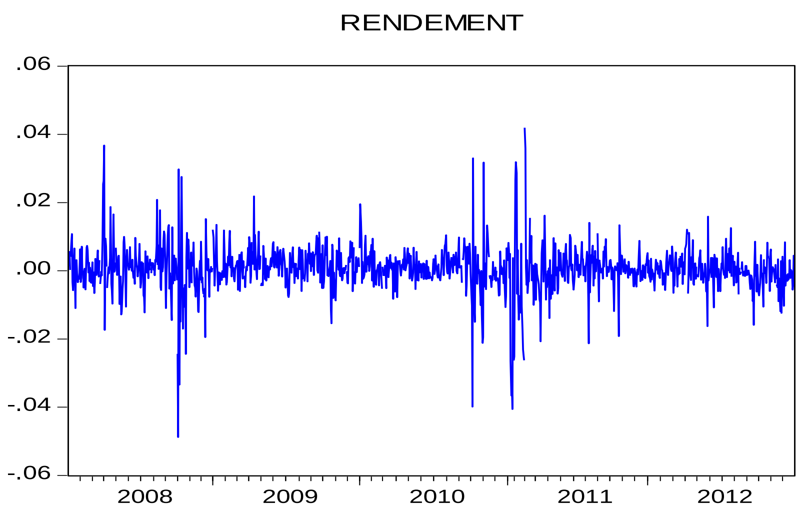

Fig. 1: Evolution of profitability

Let us recall that for a normal law, the skweness is worthless and the kurtosis is worth 3, this law being characterized by its symmetry compared to the average like by the weak probability of the extreme points. Under the worthless assumption of normality, statistics JB follow a law of two to two degrees of freedom.

We notice mainly that for the series of profitability, the worthless hypothesis of normality is rejected. We note initially that the coefficient of Kurtosis is very high (12 83835), largely higher than 3. This phenomenon of excess of kurtosis confirms well by the character strongly leptokurtic of series of stock exchange profitability. In second place, the coefficient of Skweness is different from 0, we notice that the coefficient of Skweness (0 425346) is negative for the series of profitability of index TUNINDEX this indicates that the distribution of series is spread out to wards the left. This illustrates the presence of asymmetry, which can be an indicator of nonlinearity since it is known that the Gaussian linear models are necessarily symmetrical. We note that the worthless assumption of normality is rejected for the studied series consequently; stock exchange profitability does not follow a normal law, which is a general characteristic of the financial series.

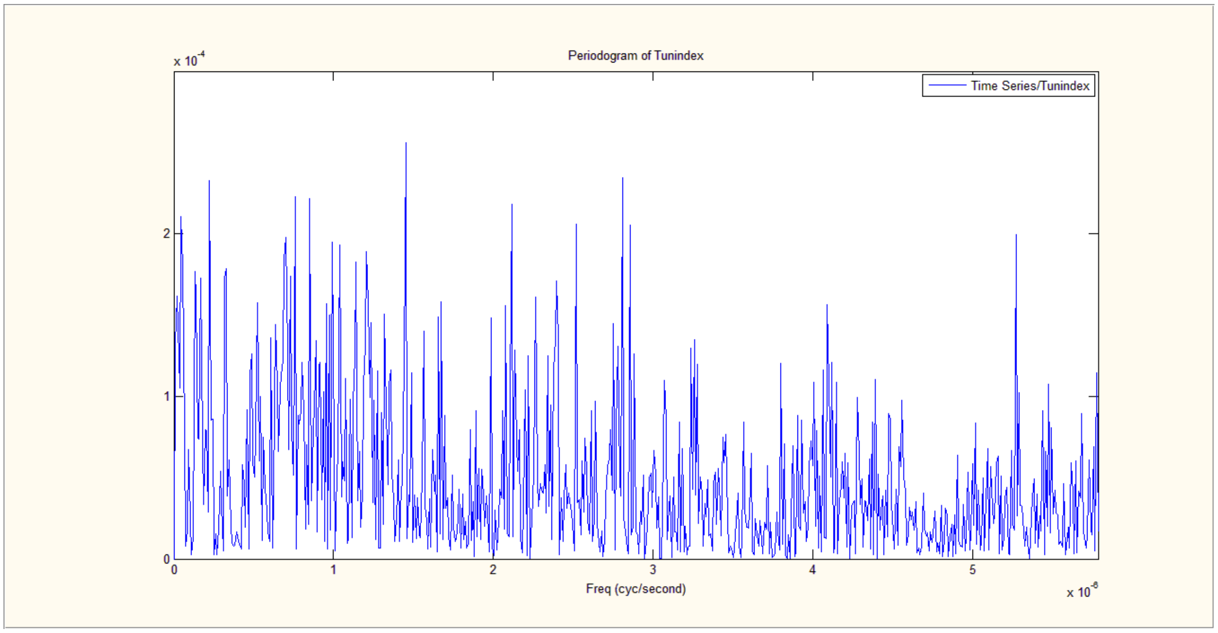

Fig. 2: Evolution of daily profitability by the period gram

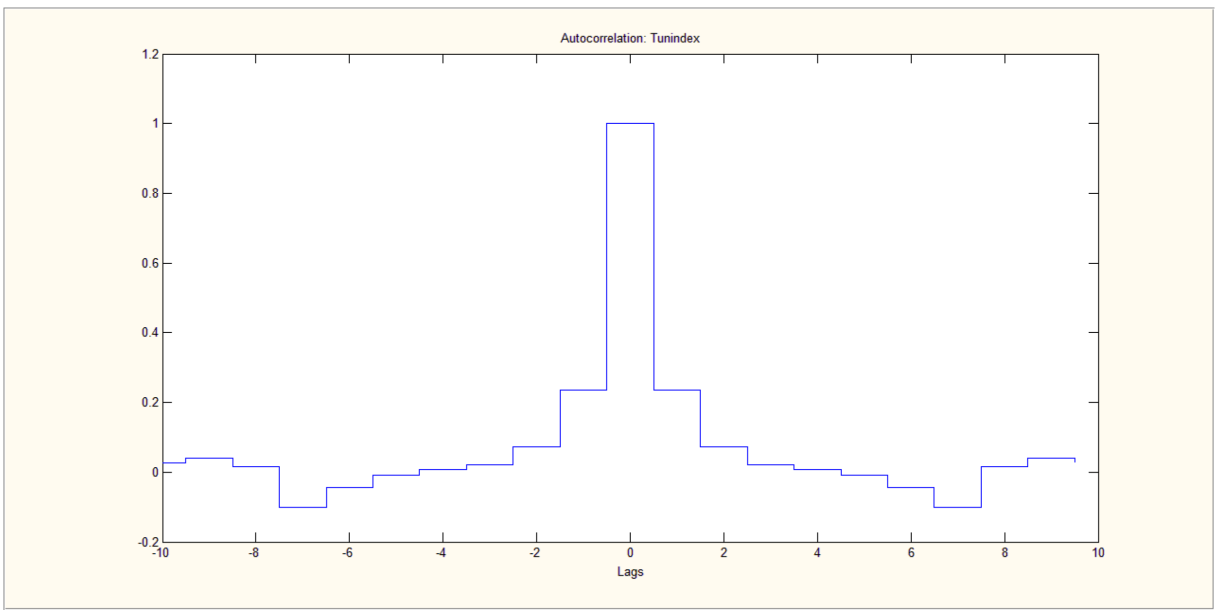

Fig. 3: The auto-correlation of the series of daily profitability

Two graphics the preceding ones present the evolution of the studied time series which presents has its tower the profitability day of the Tunindex index, it is thus, our series presents several peaks of inefficiency due to external and internal events through various indicators and various parameters of the national and international economic situation, such as the international financial crisis and the events of the Tunisian revolution which involve a deceleration of the financial activity of the Tunisian financial institutions. Other tests of normality are enough important to visualize the normality of our time series such as the test of Kolmogorov-Smirnov E and the test of Kozachenko and Leonenko:

*The test of Kolmogorov-Smirnov (KS): The utility of this test is to compare two frequency distributions and to test if those are significantly different used here to check if frequency distribution of a variable x suit normal distribution. The principle of this test (Test of Kolmogorov-Smirnov: (KS) summarizes hardly the data: he forgets simply the order in which were collected. The data define a law R called empirical law, noted P_n :

$$P_n\{a,b\} = \frac{1}{n}\sum_{i=1}^{n} 1_{(a,b)}(x_i) \quad (6)$$

For any interval (a, b),

(1.5) this law P_n is a discrete one, whose function of distribution (called empirical function of distribution) is noted F_n

$$F_n(t) = P_n\{(-\infty, t)\} = \frac{1}{n}\sum_{i=1}^{n} 1_{(-\infty,t)}(X_i \quad (7)$$

It is a function in staircase, increasing, which jumps of $1/n$ in each point of the sample .The statistics of D_n Kolmogorov-Smirnov are defined by:

$$D_n \equiv \sqrt{n}\, sup_{X\in R}\,(F_n(x) - F_x) \quad (8)$$

The test of Kolmogorov-Smirnov rejects the worthless assumption if D_n is too large, We now will study some aspects of the D_n statistics:

i). Under assumption worthless (i.e., if $X_{i\sim i\,i\,d}\, P)$, the law of D_n is free, it does not depend on P(provided that P has a density), it depends only on N

ii). The calculation ofD_n is reduced to a sorting of the sample, the evaluation of F on the points of the sample and in search of a maximum in a table of values $2n + 1$

iii). If the worthless assumption by is not checked, for example if the $X_{i\sim i\,i\,d}\, P'$ with P≠P' then for any M>0 $P'\{D_n > M\} \to 1$, the test is thus increasing

Under the convention $X_1, \dots X_n$ which indicates the version sorted in order ascending of $X_1, \dots X_n)$ (, the random variables $X_{(i)}$ are called the statistics of order of the sample. According to the definition of D_n, this can be calculated effectively starting from the sample sorted in ascending order (,$X_1, \dots X_n)$

$$D_n = \sqrt{n}_{0\le i\le n} \max\max\left(\left|\frac{i}{n} - F(X_{(i)})\right|, \left(\left|\frac{i}{n} - F(X_{(i+1)})\right|\right.\right. \quad (9)$$

With the convention $x_{(0)} = -\infty$

To conclude, the test of K-S is one of the nonparametric tests of adequacy most current (with the khi² but this last loses more information). It makes it possible to compare a distribution observed with another, or a known distribution of standard law of probability. In particular, this test gives a good indication of adjustment to a normal law (it however is modified in this precise case and becomes test of Lilliefors). Moreover, it adapts on ordinal scales and it is of this fact very much used in the market studies. Its principal defect is not to be very effective in the tails of distribution. The Test of normality of Kolmogorov-Smirnov for a sample rests on the maximum difference between the cumulative distribution of the sample and the cumulative distribution which is tested. If the statistics D are significant, we must reject the assumption according to which the respective distribution is normal. In most software, the values of probability which are deferred result from the tables of Massey (1951); these values of probability are correct when the average and the standard deviation of the normal distribution known a priori and are not considered starting from the data. However, these parameters are generally calculated starting from the true data. In this case, the test of normality implies a conditional assumption complexes (“Which is the probability of obtaining statistics D of this importance or who is higher, knowing to him that the average and the standard deviation are calculated starting from the data”), and makes it possible to interpret the probabilities of Lilliefors (Lilliefors,1967).

In our study the function of distribution, as well as the maximum and minimal difference are presented as follows:

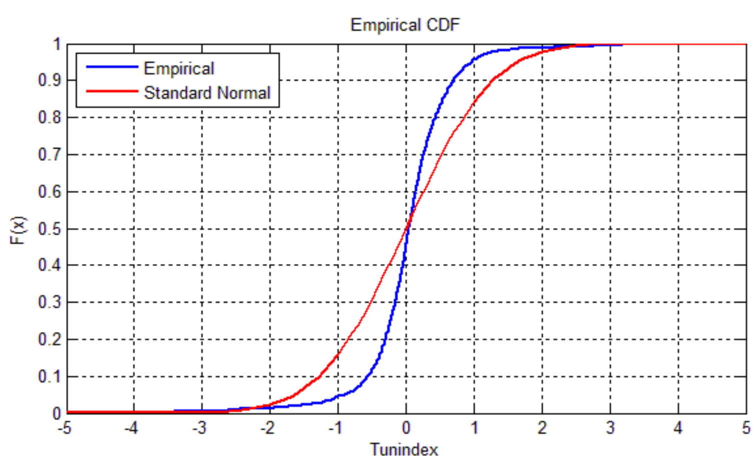

Fig. 4: The empirical function of distribution F

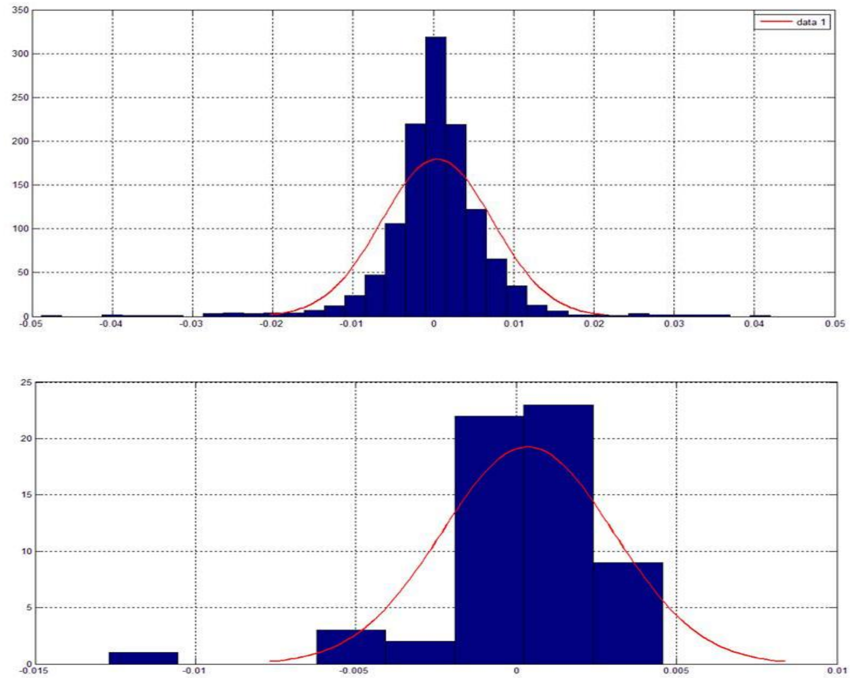

Fig. 5: Histogram of the distribution

The Fig. 4 presents the empirical function of distribution F (in blue), as well as the two distributions maximum and minimal which follow the standard normal law, the goal of our test is to compare the relative frequency distribution cumulated of a variable observed with the theoretical distribution which this variable would have if it were distributed normally. One seeks the class where the difference between the theoretical distribution and the distribution observed is largest, and one checks if this variations significantly large, if the assumption of H0 normality: normal distribution can be rejected with the threshold considered.

It is possible to visualize the form of the distribution of the data to be analyzed compare the form of this histogram with a curve representing a normal law (parameters of this conclude with normality from the data but can give an idea of the type of subjacent law: normal law, law of Cauchy or law of Student if the distribution seems symmetrical, lognormal law, law gamma, law of Weibull, exponential law or law beta if the distribution is asymmetrical declaration of the assumptions:

H0: The distribution of the variable… follows abnormal law

$$F_{rel}(x_i) = F_{rel\ th}(x_i) \quad \forall x_i \qquad (10)$$

H1: The distribution of the variable… does not follow the normal law

$F_{rel}\ (X_i) \neq F_{rel\ th}\ (X_i)$ for at least one value of X_i

We use the statistics of Kolmogorov –Smirnov where:

$$D_{obs} = \max\left(\left|F_{rel}(x_i) - F_{rel\ th}(x_i)\right|\right) \quad \forall x_i \qquad (11)$$

Such as the rule of decision, we reject H0 with the threshold α =0,05 if D observed > Dα (D theoretical) where:

- for α = 0.05: D0.05 = 0.895/*S*
- for α = 0.01: D0.01 = 1.035/*S* where:

$$S = \sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}} \qquad (12)$$

N being the number of individuals (and not of classes). In our case the sample is large, it is higher than 35, from where $D_\alpha = \frac{1\ 36}{\sqrt{n}}$ According to the table of corrected values according to Liliefors (1967) and Stephens (1974) at Legendre and Legendre (1998):

$$D_\alpha = \frac{1\ 36}{\sqrt{1237}} = 0\ 0385 \qquad (13)$$

According to calculation carried out, with the threshold 5%, with the breaking value which is worth 0 0385 have with the values of D observed according to the two distances (maximum and minimal):

D max (1) =0 1986> with p-value=5 1390e-43

D min (2) =0 1458> with p-value=1 0169e-23

These two actual values are higher than the value criticizes D, from where the worthless assumption is rejected, from where our series does not follow the normal law.

Our distribution is different from the normal because the values of D max and D min are higher than the breaking value

* The test of Kozachenko and Leonenko:

The tests of normality make it possible to check real data so follow a normal law or not. The tests of normality are typical cases of the tests of adequacy (or goodness of fit tests, tests allowing to compare distributions), applied to a normal law. These tests take an important place in statistics. Indeed, many tests suppose the normality of the distributions to be applicabl. In any rigor, it is essential to check normality before using the tests However, of many tests are sufficiently robust to be usable even if the distributions deviate from the normal law. The principle of the Test of normality of Kozachenko-Leonenko is to present an estimate of the entropy, the Algorithm of Kozachenko-Leonenko (UCK) [KLA (1987)], which founds the estimate of the theoretical entropy of the distance, with nearest close to a given order N, by comparing the digital results (actual values) with the theoretical values in two different situations: independent and dependent variables, we examine the distributions to know the field of definition of each distribution. This test which is proposed by Kozachenko and Leonenko, based on the study of the estimators who present the distance nearest close to the simple points, these estimators are useful for the evaluation of the random entropies of vectors, this test establishes asymptotic skew and the coherence of the estimators proposed, for certain distributions standards, the estimators proposed are applied in order to estimate the entropy which can be characterized by random multidimensional vectors, which described it by random un-dimensional vectors.

Kozachenko and Leonenko (1987), proposed the estimator according to Hn of the function H(F) given by:

$$H_n = \frac{P}{n}\sum_{i=1}^{n} Ln\,\rho_i + \left\{\frac{\pi^{\frac{P}{2}}}{\tau\left(\frac{p}{2}+1\right)}\right\} + \gamma + Ln\,(n-1) \qquad (14)$$

Since the random variable X is absolutely continuous, the first distance from nearest close, (the most been worth) are supposed being the positive small numbers because of the presence of N factors in the expression of Hn, given by the previous equation, the small fluctuations in most been worth will relatively involve higher fluctuations in the values of Hn more consequent, the valid objection by the use of the Hn estimator is that it can be used in practice only if small the p-values is recorded with the utmost precision, which is not often the case. The difficulty can be prevented if one builds an estimator on the basis of more (second or more) close distances. According to the test of Kozachenko and Leonenko (1987), and according to the table of the commune distributions (RMSE: Root Mean Squar Error), the value of H is of order 0 8685 i.e., the field of definition and the normal standard law, whereas the histogram of the approximate entropy multi presents the value of 0 8650 .For a given financial time series, the idea to use the multi approximate entropy is to provide a quantitative estimate of the efficiency of the market. The approach consists in comparing the models of entropy of the financial time series with the model of entropy for a random sequence. For this purpose, the model of multi entropy is calculated for a sufficiently large overall number of random sequences of N=1238size Thereafter, an envelope of entropy is calculated as being the area ranging between the minimal value and the maximum value for each scale of time.

The approximate entropy multi is calculated by according to the following function:

$Z = X + iY$ And $Z^2 = |X^2| + |Y^2|$ and y: constant (15)

Where, X= centers real comprised the financial time series and Y= secondary axis and i= complex number:

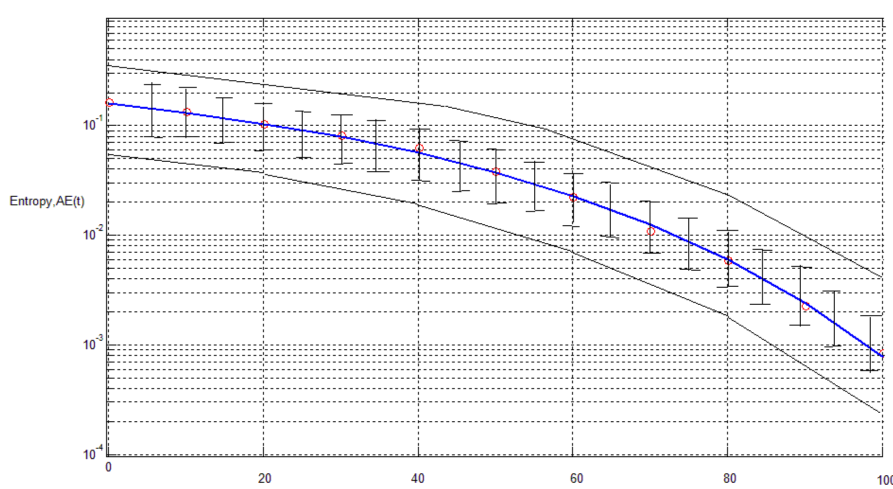

Fig. 6: The envelope of entropy

Fig. 6, watch the envelope of entropy calculated starting from a random sequence of N= 1238 observations. The average and the standard deviation of the standard samples of the entropy are also represented. In theory, the envelope in the following Fig. 7 contains all the possible diagrams of entropy for random sequences of size N= 1238. This envelope entropy will be used as reference to compare the models of entropy of the real financial series of the same size NR. In this way, it will be known as that the market is efficient at 100% if the model of the corresponding entropy is contained in the reference entropy. If the reason entropy is the envelope below, the market is only partially efficient). In this case, an index of efficiency will be given to the approach of the level of entropy of the lower limit of the index of reference of entropy. It is, for a time given τ and a scale of time T given which makes it possible to determine the elasticity or the trajectory of the curve of envelope, as well as the effect of the complex distribution, efficiency market ME (T, τ) is given by:

$$ME\ (T, T) = ((E\ (T, T)) / (B\_\ (T, T)) * 100) \quad (16)$$

Where E (T, τ) is the entropy and B (T, τ) is the lower limit of envelope, the minimum of envelope. Such as the coefficient B is determined by the following equation:

$$Y = \left({}^{1}/_{4}\right) * X^{i} \quad (17)$$

For the entropy followed by the evolution of the efficiency of the market, the approximate entropy procedure described above was implemented with a size well defined of the window According to the study of Lo (2004, 2005), in the evolutionary markets, it is necessary in the course of time to estimate the variations of the efficiency of the market.

B) Measurement of informational efficiency according to the multi-approximate entropy

The measurement of efficiency is calculated in two stages; initially the measurement of the multi-approximate entropy by function "APEN". Secondly, the approximate entropy multi is used to measure the amount of information contained in the series of output of TUNINDEX.

The use of the analysis of time series symbolic system, enables us to codify sets of data in time series of code of some figures distinct. The problem of the analysis symbolic system is that there is no formal manner to define the partitions of time series. However, in our case, we are interested by the combinations of negative and positive profitability of stock market. Previous methodology is used in this section, the study of the index of TUNINDEX in order to find some facts on the variations of the efficiency of the market. Time series for TUNINDEX have obtained with a daily frequency starting from January 2008 until December 2012. To avoid artifacts in the estimates of entropy in the face of a deterioration of index TUNINDEX, declarations were standardized as follows:

$$R_t = [dlog(I)_t - dlog(I_{t-1})]/dlog(I_t) \quad (18)$$

Where test the value of the index at the moment T for the years (2008, 2009, 2010, 2011 and 2012) following the Great Depression, the Rt output varies in the band [- 0,005, +0,005], with an average of 2 36 × 10-5. For NR = 1238 observations.

It should be noted that entropy TUNINDEX is contained in the envelope of reference or below the lower limit of envelope such as the two limits of this curve the bases errors are named. In the first case, the efficiency of the market is of 100%, if the model of the corresponding entropy is contained in the reference entropy or the envelope, in this case the market is regarded as efficient with 100% if in the contrary case, if the reason entropy is below of the envelope where the model of the entropy exceeds the envelope; the market is only partially efficient. In this last case, efficiency is lower than 100% as a reason for entropy which does not answer a random sequence. This curve of envelope explains the effect of the distribution of the index Tunindex, with the top of this curve, the index reflects the efficiency of the financial market, in the contrary case, the Tunindex index does not reflect the efficiency of the financial market.

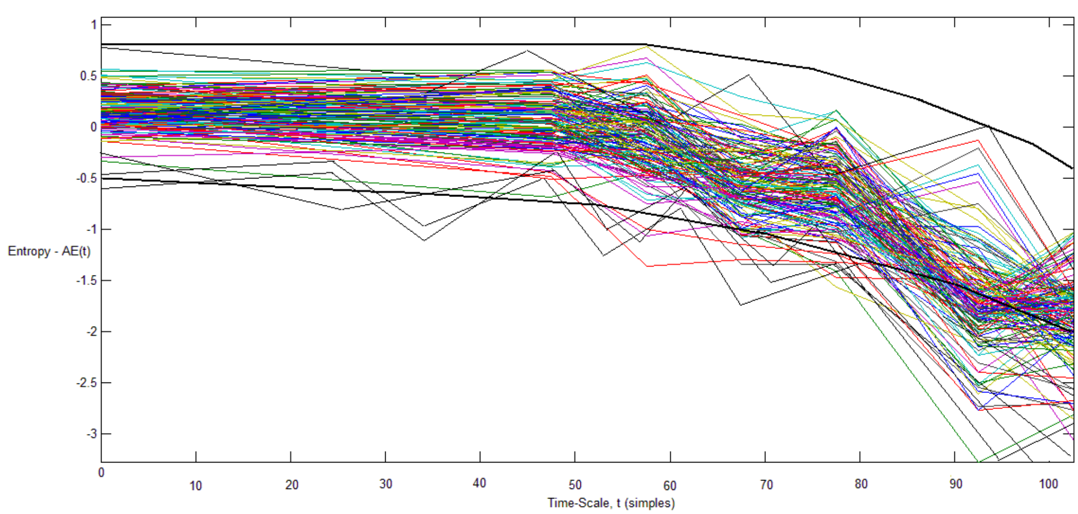

Fig. 7: Present the models of entropy for the scale from 1 to 100 working days compared to the random fluctuations of reference in the Fig. (6))

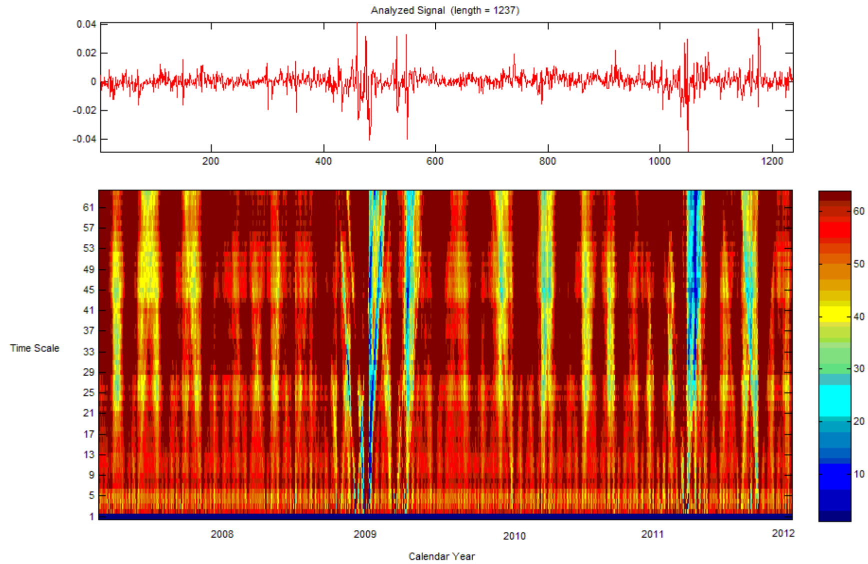

Fig. 8: The efficiency of the market according to time and the scale of time

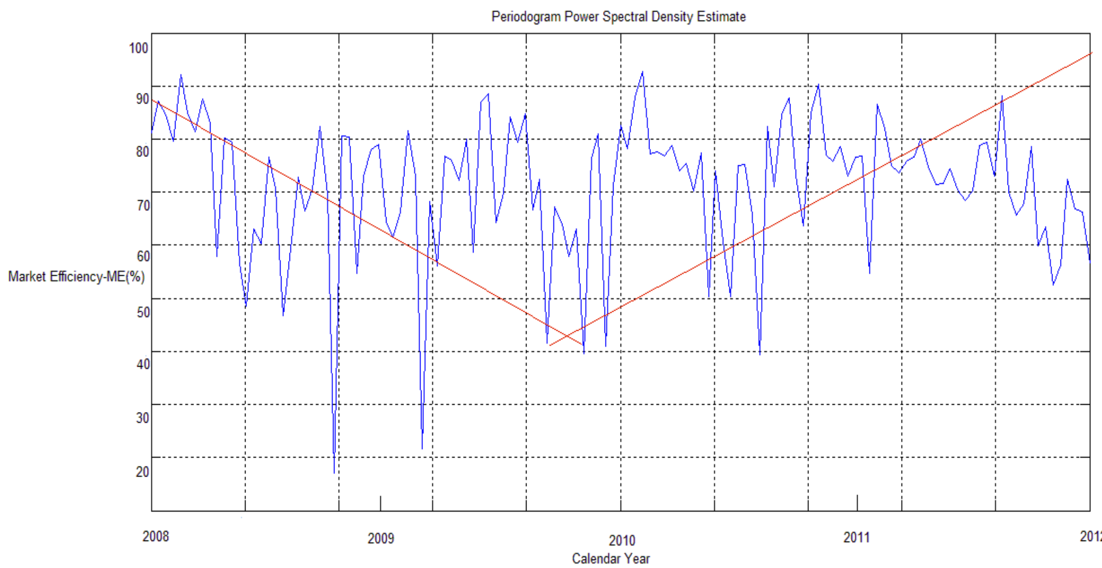

Fig. 9: Presents the behavior of the efficiency of a scale of time

As it already detected by Ito and Sugiyama (2009), the efficiency of the market present of the important variations in the course of time with variations from 40 to 100% over the examined period. In general, the market shows all its efficiency for scales of time going up to 20days B (approximately a month) .However, the important departures of 100% are observed for higher scales of time, which indicates that horizons of times higher than one month calendar, the Tunisian market is not completely efficient. In this way, Figure 8 shows that the efficiency of the market varies in the course of the time and also on horizons of time. These results suggest that the TUNINDEX cannot be described by a random behavior of walk, because it contains a reduced number of models of entropy according to time and deadlines .For the small scales of time, in the scale from1 to 10 B-days (approximately half of the calendar month), the market is close to being entirely (100%) efficient and this behavior did not change over the last years. They is, short-term, the Tunisian financial market reflected by Tunindex is unforeseeable that the reasons for change of price are similar to those of a purely random sequence. However, the variations of important efficiencies can be found for the great scales of time.

A direct visual inspection of the behavior of efficiency for scales of time raised to the Fig. 9 indicates that the total efficiency of the market. The Tunisian stock market became more efficient in the last decades. In fact, efficiency shows a negative trend during the period 2007-2009, followed by a positive trend during subsequent years (from 2009 to 2011). This analysis based on data leads us to consider the existence of a rupture in the trend of long-term efficiency, which is done by adjusting the data of Fig. 9 by means of the part linear function of the form:

$$ME(T) = \begin{cases} a_1 t + b_1 \ to\ t < t_{br} \\ a_2 t + b_2 \ to\ t > t_{br} \end{cases} \quad (19)$$

Where TBR (TBR: THE REST POINT or the place of rupture), it is the place of rest or the point of change of the evolution of the series of output of the index tunindex. The results are also presented in the Fig. 9, showing that the optimal rupture is around the year 2010. Before this pause, the long-term trend is negative, then after the pause of the long-term trend is positive.

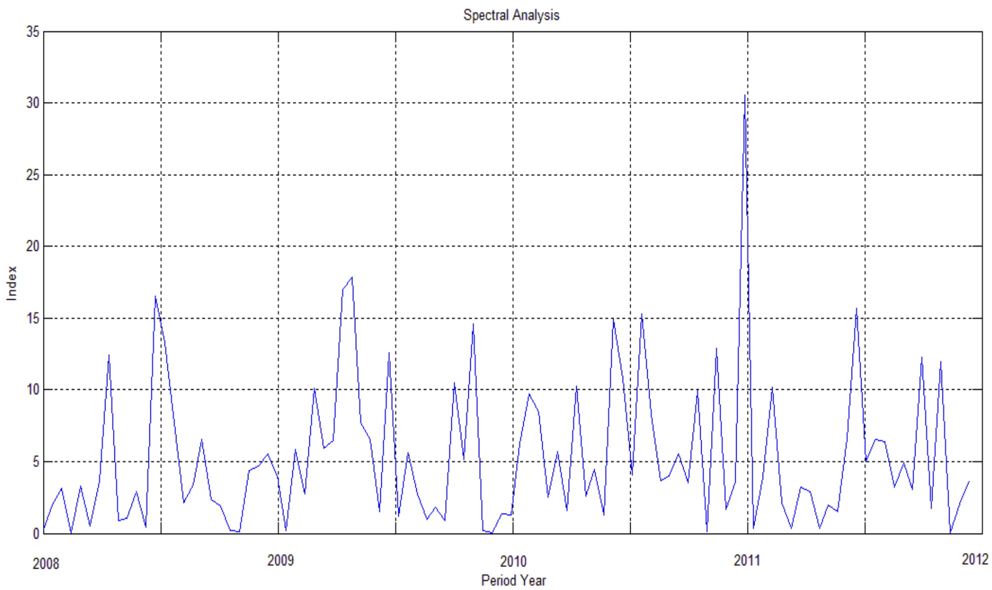

Fig. 10: Presents the behavior of the efficiency of a scale of time, by the spectral analysis

This figure presents the evolution of the index of the Tunisian financial market over the study period (2008-2012).A first spectral analysis is carried out with a non-parametric method adapted to any type of signals and whose behavior, as well as the performances (standardized variance, resolution, skew), are calculable and foreseeable: Welch-WOSA (balanced and realized period gram) or Blackman-Turkey (balanced correlogram). In the case of the signal of gears, being given the support of the function of autocorrelation and the strong signal reporting noise, a simple period gram with a window of Hanning taken on the whole of the signal is used as starting point. This analysis is followed of a spectral interpretation. The spectrum obtained particularly, rich and consists of more several peaks distributed regularly on the unit of the band. The spectrum also has a strong dynamics. We note the presence of 3 harmonic principal peaks between them and a greater concentration of peaks around their frequencies.

In order to better characterize the various spectral lines; in particular with regard to their amplitude, their phase and their possible damping, the method of Prony by deflation was also put in work on each sub-band. This operation is particularly interesting in the sub-bands around the 3 principal lines where the difference in amplitude between the principal line and the edge lines .Apart from the estimate of a spectrum of other types of analyses, can be considered as for example a search for cycle stationnarity, or a search for harmonics: several series of harmonics are found forming edge lines around the 3 Principal lines. The presence of the various harmonics is checked by a spectral analysis, once spectral interpretation is regarded as complete, it is compared with information a priori known of the expert of the field, which then makes it possible to validate interpretation or contrary to carrying out new analyses. In the case of the signal of gears, interpretation obtained is in conformity from the point of view of the expert of the field and does not require new analysis. A look ahead using specific criteria consists in evaluating certain properties of the signal such as the respect of the condition of Shannon, the stationnarity, the Gaussian assumption which are necessary for a good interpretation of the put spectral analyses concerned. It also consists in analyzing the autocorrelation of the signal. Within particular the estimate starting from objective criteria of its periodicity, of its support and the signal report on noise. Then an approach is built starting from a succession of analyses, leading to the detection and the identification of the various spectral reasons. Each stage consists of an interpretation of the spectrum considered known as rough, then of a comparative interpretation with the previous spectral estimates. A stage is followed of a point hinge which updates spectral interpretation determines the gaps and chooses the following stage of them Interpretations are facilitated by the installation of objective criteria:

significant peaks compared to the variance of the estimator, follow-up of peaks between various analyses.

Finally of the complementary analyses are considered according to the conclusions obtained and knowledge of the expert of the field: search for harmonics, study of the cycle-stationnarity. The generalization of such scenarios must make it possible to refer the various approaches according to the various spectral configurations met compared to the base of signals ASPECT (acoustic and mechanical vibratory). In the long term, these approaches of analysis could be automated by proposing criteria based on the theoretical properties of the estimators.

C. Interpretation of the results

These results prove that the economic recessions have a significant impact in the operation of the Tunisian stock market. To tackle this question, a short outline or an explanation about the impact of the financial crisis in 2008 and the events in 2011 on the financial situation in Tunisia: Tunisia which is a small country with a small financial market is not in the direct field of propagation wave of crisis left the U S A but will undergo like same the effects indirect of the crisis and this, in connection with: A risk of panic which would lead the investors of wallet in search of liquidity to liquidate without understanding their credits even in the markets of the emerging countries. Increase in the credit charges with its effects on the debt servicing.

However Tunisia has the chance to have a healthy macroeconomic management of its economy, with a scientist balances between liberty of action for private initiative and a role of referee present of the State. Moreover, the Tunisian authorities had wisdom and the inspiration of taking during the last years several actions and reforms of the banking system which prove strong useful today for better resisting the crisis. We can explain the disturbance of the Tunisian financial system by internal reasons where the financial situation and economic Tunisian master key currently by its worst crisis, because the shock was important and strong, such as several sectors of the economic and financial activity were blocked during this Jasmine revolution in 2011, of the problems of tourism, such as the stop of several hotels a fall of the foreign exchange revenues involves and consequently a fall on the level as of reserves what causes the stop of several production units of phosphate cost the national company more than 9 MD per day of stop, the social and trade-union claims disturbed the operation of several companies and public corporations as deprived, weighed on the profitability of the companies and their competitiveness. This situation will have a significant impact on direct foreign investments (IDF) in Tunisia and on the competitiveness of the Tunisian companies, if this is not accompanied by an improvement by the productivity, generally low

The blocking of the economic activity, the insecurity and political instability had a great impact on the economic activity and financial. These internal and external elements were linked to plunge the country in a difficult crisis and whose exit is not clear. What justifies the existence of the peaks of inefficiency on the evolution of the output of the index of the Tunisian financial market.

In our study, for a quarter of the scales of time, efficiency of the market in the Fig. 10 presents a combination of small and great fluctuations. Certain cyclist in the behavior of these fluctuations is apparent. This is easily observable in the description of the color scale in the Fig. 8 two in the high beach of scale. To inquire into the presence of cyclic dynamics, the linear trend per pieces was removed and the trend of the time series was studied by spectral analysis of Fourier. The results are presented on Figure 10 where two dominant spectral components are located overall the period of 2008 up to 2012. The periods of these spectral components are not at all surprising that their existence in the economic dynamic was discussed previously. A short description of these cycles realizes as follows: it first cycle. One can make the point that the vague court spectral component is related to the cycles of inventory of Kitchin. Discovered by Kitchin (1923), it is obvious that these cycles were present in the dynamics of the financial activity along the 20th century and the years 2000. The origin of the Kitchin cycle is intrinsic with delays in the information management of the marketing companies. In fact, the companies react to the improvement of the commercial situation thanks to the increase in the production through the full employment of the fixed assets of measurement. Consequently, in a certain period of time (energy of a few months to two years), the market is flooded with products of which the quantity becomes little by little excessive. The decreases in the demand, the prices drop, the goods and produced services accumulate in the inventories, which inform the entrepreneurs of the need for reducing the production. However, this process takes a certain time. It takes a certain time for the information which supply largely exceeds to reach to the businessmen. Moreover, it takes entrepreneurs time to check this information and to decide to reduce the production, a certain time is also necessary to materialize this decision (they are the shifts which generates the Kitchin cycles).

Others turns of relevant time is the shift between the materialization of the decision mentioned above (causing fixed assets to be worked well below their level of full employment) and the reduction in the excessive quantities of products accumulated in the inventories. However, after this reduction has place, one can observe the conditions of a new phase of growth of the request, the prices, the production, etc (Korotayev and Tsirel, 2010). In certain cases, the first cycle is related to the appearance of small and average economic recessions, which can be attenuated by means of the cyclic inventory adapted meter and of the monetary policies. The results on the two last figures suggest that at the time of economic difficulties of the financial market their complexity like a consequence of the reduction of activity of the public inversion loses and deprived and diversified. In this way, the first cycle in the long-term entropy can be regarded as an indicator of the economic activity probably related to the handling of raw material stocks it 5 years second cycle. The appearance of the fall of the efficiency of the interior market of a 5 years cycle is striking. Fig. 11 shows the efficiency of the market for scales of time when it is easy to observe the appearance of great reductions in the efficiency of the interior market on the 5 years cycles (from 2008 to 2102). The falls of efficiency of the market between 2008 and 2012 coincident with the economic recession in the United States which a dominating effect on the international scale. In particular, the reductions in the efficiency of the market in 2008 correspond to the greatest recessions of post-war period, with a GDP decreases by 45 (of billion dollars). This suggests that the 5 years cycle is related to an increasing risk of great economic and financial international recessions. Interesting fact, although the fall of the efficiency of the market 2008 was confused with the financial crisis “subprime” it is obvious that the Tunisian financial system reflects in index TUNINDEX is an indicator of the state of the economic activity and financial, the private investors anticipate the periods of economic recovery and deceleration Extract of the efficiency of the market for one month scale of time in the Fig. 11.

Although the liberalization of the world financial system increased the long-term trend of the efficiency of the market into 90% on average, the economic recessions seem to be longer. What an indirect impact on cycle TUNINDEX this last can reflect a cycle of Kuznets 2008 until to 2012 the cycle could be related to demographic or real cycles. It should be noted that long-term cycles appear at the end of five years (cycles of Kitchin), which suggests the presence of mechanisms of resonance in the socio-economic system which amplify the harmful effects of the short-term cycles. In other words, it is obvious that the financial system undergoes a major cycle perhaps induced by resonance mechanisms which are unable to dissipate the negative affects which are accumulated, via the delays in the information processing, after five years.

IV. CONCLUSION

In this work, one proposed a new methodology in order to measure the informational efficiency, used generally in the information theory. Under this methodology, the measure of efficiency is made in two stages; of edge the measurement of the multi-approximate entropy by “APEN” function Secondly, the multi approximate entropy is used to measure the amount of information contained in the series of output of TUNINDEX. We used the method of the entropy in the form of a variable structure in time of efficiency of the market. The idea behind the methods of entropy is to quantify the diversity of the models of change of price compared to that of a chance sequence (random sequence). The approach becomes a suitable framework to quantify the efficiency of the market which they got results are in conformity with the empirical obviousness. In fact, our results are in conformity with the precedents obtained with methods based on models (for example, auto regression models), that the Tunisian stock market became most efficient around 2010. Interesting fact, our results showed that, during last years, the behavior efficiency of the Tunisian market is strongly affected by cyclic dynamics with dominant periods.

When the entropy is calculated, a curve of envelope entropy will be used like reference to compare the models of entropy of the real series of output, size (N=1238), according to this curve, the market is efficient to 100% if the model of the corresponding entropy is contained in the reference entropy. If the reason entropy is the envelope below, the market' is that partially efficient. We present again the models of the entropy compared to the random fluctuations of reference, when the entropy is measured, this entropy is applied for measured efficiency. The behavior efficiency of the Tunisian financial market is presented according to the time and of the scale of time (A runs and has long run) and by the analysis spectral. The methods of the approximate entropy multi approximate the evolution of the efficiency of the Tunisian financial market as well as the change during the study period. The empirical results show that this approach of statistical physics is useful, allowing more refined classification of dynamics of stock market. They noted that, for the stock markets, the temporal correlations are the principal factor of the inefficiency. The study carried out establishes that the Tunisian financial market tend towards efficiency without reaching it completely, Thus we notice that the market passes by periods of inefficiency because of the external events and in terns who touch the Tunisian financial market generates beams of inefficiencies, but the market will find the manner of turning over to efficiency.

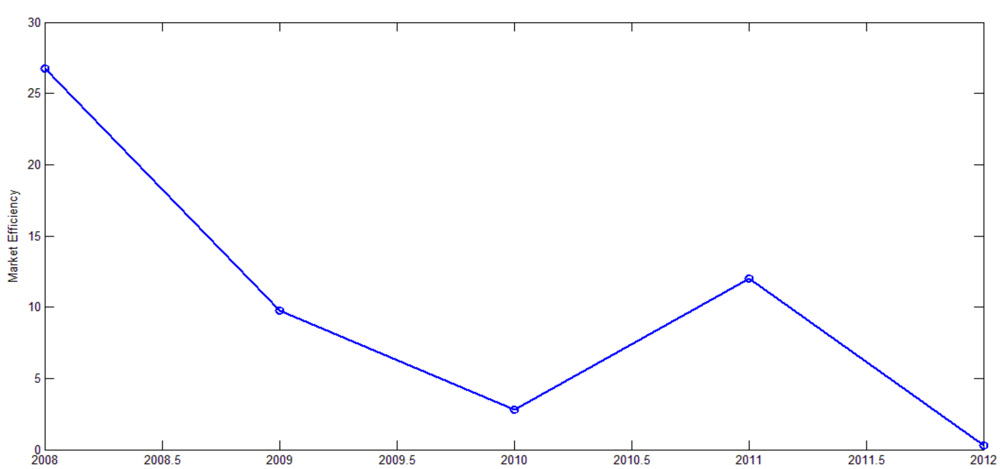

Fig. 11: Presents the duration and efficiency of the market during 5 years drops cycles. It should be noted a negative correlation between the duration of the economic recession and the effective fall of the markets

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