

# Detailed Harmonic Analysis for Pulse Width Modulation based AC to AC Matrix Converter

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**Abstract**—Converters are used in wide range of applications to save energy and attain desirable voltage. Matrix converter can convert three phase AC input to three phase AC output with variable voltage amplitude and frequency directly. It can be used as bidirectional power flow converter without any intermediate storage element. The objective of this proposed research is to minimize the harmonic losses to get maximum output voltage ratio, sinusoidal current, desired variable voltage amplitude and desired variable frequency. Pulse width modulation algorithm controls the input, output voltage and frequency independently. In this work, pulse width modulation based matrix converter will be designed to attain voltage ratio up to 1 and to reduce switching losses so that low total harmonic distortion could be achieved with sinusoidal waveforms of desired amplitude and frequency. Simulation environment will be created in Matlab.

**Keywords**—Matrix Converter, Pulse Width Modulation, Total Harmonic Distortion and Matlab

## I. INTRODUCTION

Converters are used in wide range of applications to save energy and attain desirable voltage. Matrix converter can convert three phase AC input to three phase AC output with variable voltage amplitude and frequency directly [1]. It can be used as bidirectional power flow converter without any intermediate storage element.

Harmonic voltage and current are produced due to non-linear loads. These harmonics produce high frequencies which cause poor quality of power supply and heating the equipment. Harmonics can be eliminated by selecting the suitable harmonic elimination algorithm [2], [3].

The matrix converter topologies are more attractive after the development of power switches like bipolar junction transistors, metal oxide silicon field effect transistors, and insulated gate bipolar junction transistors [4]. The development of first matrix converter was in 1980 [5], [6]. Venturini and Alesia, in 1980, proposed the converter using bidirectional switches and named it as matrix converter. They proposed modulation algorithm which is known as direct transfer function approach. The desired output voltages are achieved by the product of input voltages and modulation matrix. In 1983, Rodriguez introduced a different technique of fictitious DC links for matrix converter operation [7]. According to this technique, the output was switched between the most positive input line and the most negative input line. This technique is known as indirect transfer function approach

[8]. The control method of Matrix Converters was introduced by Braun in 1983 [9] and by Kastner and Rodriguez in 1985 [10]. Schauder and Neft proved experimentally in 1992 that high quality of input and output current can be produced by using matrix converter which comprise of only bidirectional switches [11]. Implementation and operation of bidirectional switches under normal condition with active filters are main focus of researches [12]-[15].

Stammler and Schonung are the pioneers of pulse width modulation. In 1964, they proposed that a sinusoidal signal was compared with reference signal to attain sinusoidal pulse width modulation [16]. Researches put a lot of efforts on PWM inverters because of its highest efficiency of DC bus utilization [17].

Pulse width modulation is a technique in which a message or a signal is encoded in such a way that it takes the form of a pulsating signal. Power supplied to the load can be controlled with the implementation of PWM. In PWM, the converter losses are low and switching is efficient so that maximum value of fundamental component of system frequency is attained [18], [19]. Ali Reza and Reza Ghazi proposed method of pulse width modulation (PWM) in which the triangular wave is created by integration of the reference signals. The basic two advantages are; the first is the triangular signal contains the information of the signal to be obtained in output and the second is that its amplitude is varied in proportion to the amplitude of the reference signal [20], [27], [28].

In this research work, pulse width modulation based matrix converter is developed to attain any desired frequency and amplitude of input and output so that any change in load or source side will not affect source or load respectively. Harmonic contents are greatly reduced by reducing switching losses. Hence, any frequency changing drives or speed changing devices can be connected to adaptive quality of supply with the implementation of matrix converter.

This paper consists of following sections. Section I consists of introduction and section II covers the matrix converter and its theory. Section III consists of proposed research methodology of PWM based matrix converter. Section IV covers the simulation results and Section V concludes the research.

## II. MATRIX CONVERTERS

A device that changes the frequency of AC supply with better control is called matrix converter. It is a device which

can convert an AC supply of fixed voltage into an AC supply of variable amplitude and frequency directly. The matrix converter consists on small sizes, bidirectional power flow sinusoidal input current, and lack of bulky reactive elements [21].

Three phase AC to AC matrix converter has nine bi-directional switches arrangement in which any of three input phases can be attached to any output phases. Matrix converter input is connected to three phase voltage fed system and output is connected to three phase current fed system. The bidirectional switch is a fundamental component of matrix converter that is by force commutated. Therefore switches are used in controlled manner in matrix converter to generate a variable frequency output so that the operation at high frequency is possible. There are various types of matrix converters in which the components used can be reduced as per requirement and therefore the modulation technique is also less complex [22]-[26].

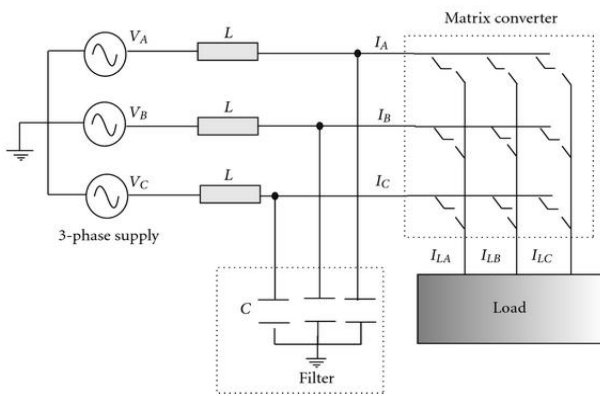


Fig. 1. Three Phase AC to AC Matrix Converter

Consider a three phase to three phase matrix converter consisting of 9 bi-directional switches that connect the input phases a, b, and c to the output phases A, B and C as shown in Fig. 1. The switching function of the switches  $S_{kj}(t)$  is defined as 0 when it is opened and 1 when it is closed as given by (1).

$$S_{kj}(t) = \begin{cases} 1 & \text{for } k \in \{a,b,c\} \text{ and } j \in \{A,B,C\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The input supply is a voltage source, short circuit of the input phases is not allowed, and open circuit of the load is not possible because an inductive load generates an overvoltage. Therefore, this constraint of the switches is expressed by (2).

$$S_{aj} + S_{bj} + S_{cj} = 1 \text{ for } j \in \{A,B,C\} \quad (2)$$

These switching combination modes are categorized into three groups as follows:

- Group 1 consists of switching combinations, where each output phase is connected to a different input phase.
- The second group consists of those switching combinations where only two output phases are shorted.
- The third group consists of those switching combinations where all the three output phases

shorted. This combination is called a zero output phase switching combination.

Matrix converter converts the three phase input of a given amplitude and frequency to three phase output of a fixed amplitude and frequency. Any desirable output frequency can be achieved by this converter. The three phase inputs of the converter are given by (3).

$$\begin{aligned} V_a &= V_i \cos \omega_i t \\ V_b &= V_i \cos(\omega_i t + \frac{2\pi}{3}) \\ V_c &= V_i \cos(\omega_i t + \frac{4\pi}{3}) \end{aligned} \quad (3)$$

The three phase outputs of the converter are given by (4).

$$\begin{aligned} V_A &= V_0 \cos(\omega_o t) \\ V_B &= V_0 \cos(\omega_o t + \frac{2\pi}{3}) \\ V_C &= V_0 \cos(\omega_o t + \frac{4\pi}{3}) \end{aligned} \quad (4)$$

The following matrix equations are the relationship of the input and output voltages given by (5). where  $M_{ij}$  is the duty cycle of switch  $S_{11}$  and so on.

$$\begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} V_A(t) \\ V_B(t) \\ V_C(t) \end{bmatrix} \quad (5)$$

The source and load voltages can be represented as vectors form defined by (6)

$$V_i = \begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix}; V_0 = \begin{bmatrix} V_A(t) \\ V_B(t) \\ V_C(t) \end{bmatrix} \quad (6)$$

The source and load currents can be represented as vectors form defined by (7)

$$I_i = \begin{bmatrix} I_a(t) \\ I_b(t) \\ I_c(t) \end{bmatrix}; I_0 = \begin{bmatrix} I_A(t) \\ I_B(t) \\ I_C(t) \end{bmatrix} \quad (7)$$

### III. METHODOLOGY OF PWM BASED MATRIX CONVERTER

Bidirectional switches are employed to connect an input phase to the output phase. It is supposed that the switches are ideal, i.e. no losses occur in switches. The three phase input supply is given by (8).

$$V_i(t) = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \cos \omega_i t \\ \cos(\omega_i t + \frac{2\pi}{3}) \\ \cos(\omega_i t + \frac{4\pi}{3}) \end{bmatrix} \quad (8)$$

A timing sequence is repeated in switching operation. This time  $T_s$  is reciprocal to the switching frequency  $f_s$  and is given by (9)

$$T_s = t_{Aa} + t_{Ba} + t_{Ca} = t_{Ab} + t_{Bb} + t_{Cb} = t_{Ac} + t_{Bc} + t_{Cc} = \frac{1}{f_s} \quad (9)$$

The switching frequency  $f_s$  is constant which means that the length of each sequence is same. The three phase output voltages in terms of switching time are given by (10)(11)(12):

$$V_a(t) = V_{im} \cos(\omega_m t) \frac{t_{Aa}}{T_s} + V_{im} \cos(\omega_m t + \frac{2\pi}{3}) \frac{t_{Ba}}{T_s} + V_{im} \cos(\omega_m t + \frac{4\pi}{3}) \frac{t_{Ca}}{T_s} \quad (10)$$

$$V_b(t) = V_{im} \cos(\omega_m t) \frac{t_{Ab}}{T_s} + V_{im} \cos(\omega_m t + \frac{2\pi}{3}) \frac{t_{Bb}}{T_s} + V_{im} \cos(\omega_m t + \frac{4\pi}{3}) \frac{t_{Cb}}{T_s} \quad (11)$$

$$V_c(t) = V_{im} \cos(\omega_m t) \frac{t_{Ac}}{T_s} + V_{im} \cos(\omega_m t + \frac{2\pi}{3}) \frac{t_{Bc}}{T_s} + V_{im} \cos(\omega_m t + \frac{4\pi}{3}) \frac{t_{Cc}}{T_s} \quad (12)$$

Now, if input frequency is increased from  $\omega_i$  to  $\omega_0$  then a modulating frequency  $\omega_m$  is required to be added to input frequency as  $\omega_0 = \omega_i + \omega_m$ . The switching sequences thus formed are:

$V_0$  at  $0^\circ$ , Switching sequence is given by (13)

$$\begin{aligned} t_{Aa} &= \frac{T_s}{3} \left( 1 + 2q \cos(\omega_m t + \theta) \right) \\ t_{Ba} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{2\pi}{3}\right) \right) \\ t_{Ca} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{4\pi}{3}\right) \right) \end{aligned} \quad (13)$$

$V_0$  at  $120^\circ$ , Switching sequence given by (14)

$$\begin{aligned} t_{Ab} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{4\pi}{3}\right) \right) \\ t_{Bb} &= \frac{T_s}{3} \left( 1 + 2q \cos(\omega_m t + \theta) \right) \\ t_{Cb} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{2\pi}{3}\right) \right) \end{aligned} \quad (14)$$

$V_0$  at  $240^\circ$ , Switching sequence given by (15)

$$\begin{aligned} t_{Ac} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{2\pi}{3}\right) \right) \\ t_{Bc} &= \frac{T_s}{3} \left( 1 + 2q \cos\left(\omega_m t + \theta - \frac{4\pi}{3}\right) \right) \\ t_{Cc} &= \frac{T_s}{3} \left( 1 + 2q \cos(\omega_m t + \theta) \right) \end{aligned} \quad (15)$$

Modulation matrix is calculated by (16)

$$M(t) = \begin{bmatrix} 1+2q \cos \omega_m t & 1+2q \cos(\omega_m t - \frac{2\pi}{3}) & 1+2q \cos(\omega_m t - \frac{4\pi}{3}) \\ 1+2q \cos(\omega_m t - \frac{2\pi}{3}) & 1+2q \cos(\omega_m t - \frac{4\pi}{3}) & 1+2q \cos \omega_m t \\ 1+2q \cos(\omega_m t - \frac{4\pi}{3}) & 1+2q \cos \omega_m t & 1+2q \cos(\omega_m t - \frac{2\pi}{3}) \end{bmatrix} \quad (16)$$

With  $\omega_0 = \omega_i + \omega_m$ . The output voltage is calculated by substituting (8) and (16) in (17) and is given by (18).

$$V_o(t) = M(t) * V_i(t) \quad (17)$$

$$V_0(t) = \begin{bmatrix} 1+2q \cos \omega_m t & 1+2q \cos(\omega_m t - \frac{2\pi}{3}) & 1+2q \cos(\omega_m t - \frac{4\pi}{3}) \\ 1+2q \cos(\omega_m t - \frac{2\pi}{3}) & 1+2q \cos(\omega_m t - \frac{4\pi}{3}) & 1+2q \cos \omega_m t \\ 1+2q \cos(\omega_m t - \frac{4\pi}{3}) & 1+2q \cos \omega_m t & 1+2q \cos(\omega_m t - \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} \cos \omega_i t \\ \cos(\omega_i t + \frac{2\pi}{3}) \\ \cos(\omega_i t + \frac{4\pi}{3}) \end{bmatrix} \quad (18)$$

### IV. SIMULATION AND RESULTS

Matlab /Simulink are used to create simulation environment for PWM based matrix converter. A three phase supply is required whose amplitude and frequency can be varied. Modulation block takes the input voltages, value of input frequency, output frequency, and output voltages and produces the desired output voltages and the switching intervals for the nine bidirectional switches as shown in Fig. 2.

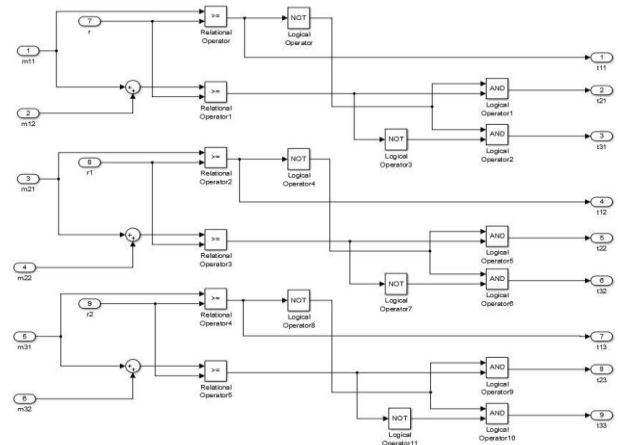


Fig. 2. This figure shows the logical operation taking place in Matrix Converters

Insulated gate bipolar junction transistors (IGBTs) of the matrix converter are deployed to generate a gating signal for PWM as shown in Fig. 3. This block takes the modulated switching intervals and compares them with a repeating sequence to produce the PWM signals of required frequency. Matrix converter block consists of nine bidirectional switches. Characteristics of on and off switches can be identified by applying the gating signal achieved from gate signal generator to switches.

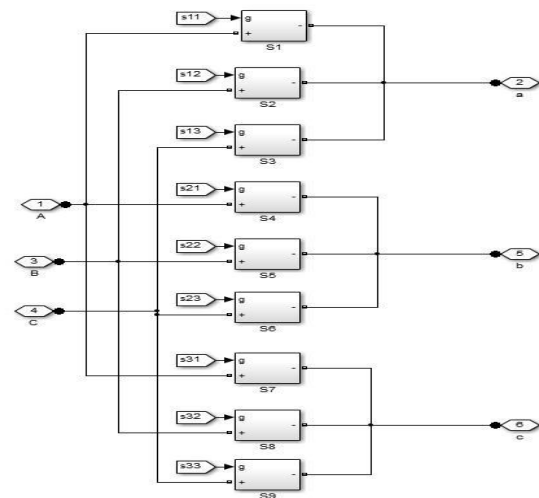


Fig. 3. Matrix Converter

The results of simulation performed in Matlab/Simulink consist of four categories.

**A. Fixed Input Frequency and Variable Output Frequency**

In a first category, input frequency is made fixed at 50 Hz and any desired output frequency is achieved at 10 Hz and 100 Hz as shown in Fig. 4 and Fig. 6. The FFT analysis of 10 Hz and 100 Hz frequency waveform are showed in Fig. 5 and Fig. 7.

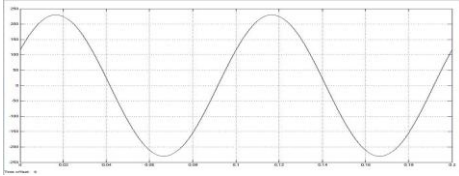


Fig. 4. 10 Hz output Frequency Waveform

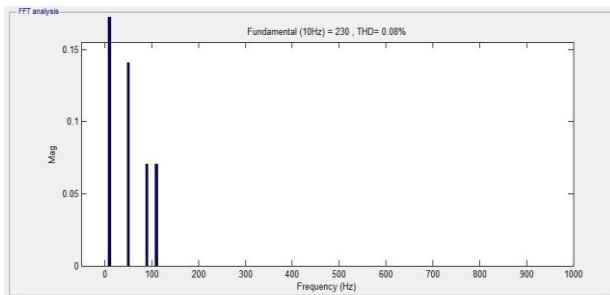


Fig. 5. FFT Analysis of 10 Hz Signal

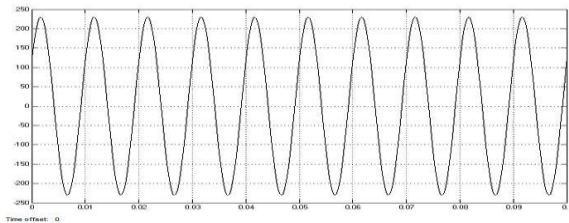


Fig. 6. 100 Hz Output Frequency Waveform

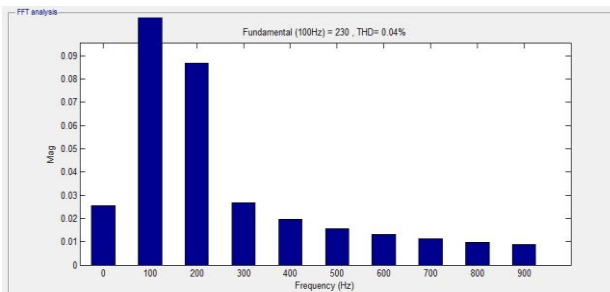


Fig. 7. FFT Analysis of 100 Hz Frequency Waveform

**B. Variable Input Frequency and Fixed Output Frequency**

In a second category, output frequency is made fixed at 50 Hz and any desired input frequency is achieved at 10 Hz and 100 Hz as shown in Fig. 8 and Fig. 10.

The FFT analysis of 10 Hz and 100 Hz frequency waveform are showed in Fig. 9 and Fig. 11.

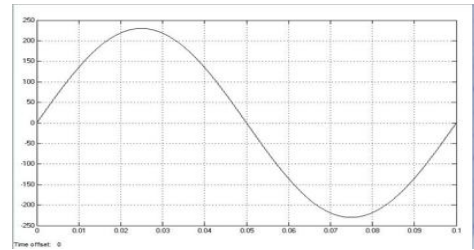


Fig. 8. 10Hz Input Frequency Waveform

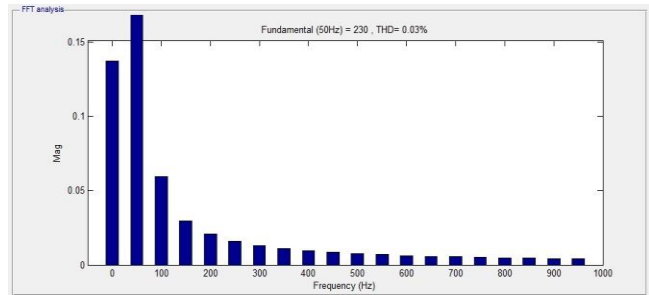


Fig. 9. FFT Analysis of 10 Hz Input Frequency

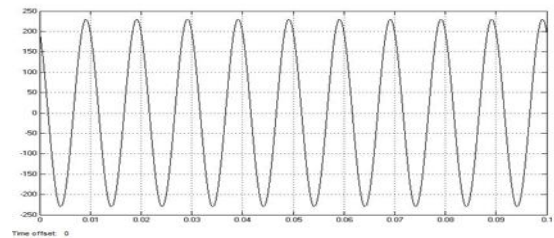


Fig. 10. 100 Hz Input Frequency Waveform

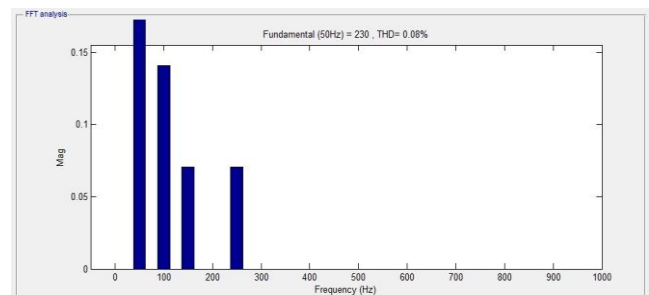


Fig. 11. FFT Analysis of 100 Hz Input Frequency

**C. Fixed Input Voltage and Variable Voltage**

In third category, input voltage is made fixed at 220V and any desired output voltage can be achieved at 110v and 500v as shown in Fig. 12 and Fig. 13.

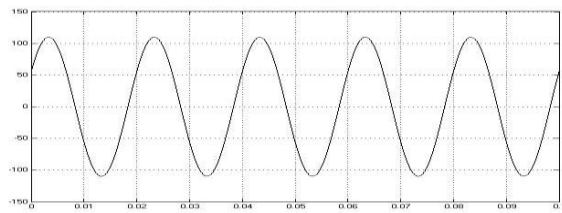


Fig. 12. 110 Volts Output Voltage Waveform

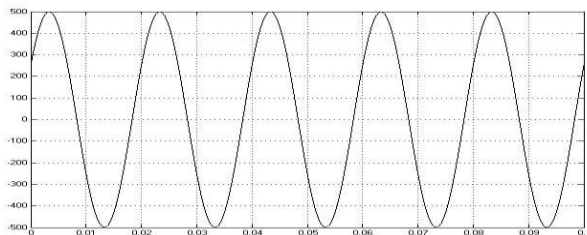


Fig. 13. 500 Volts Output Voltage Waveform

#### D. Variable Input Voltage and Fixed Output Voltage

In fourth category, output voltage is made fixed at 220V and any desired input voltage can be achieved at 110v and 500v as shown in Fig. 14 and Fig. 15.

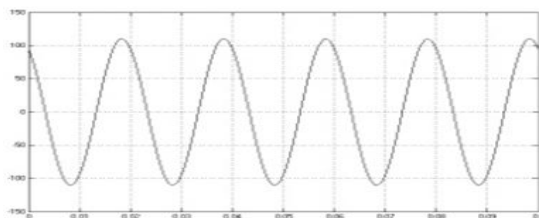


Fig. 14. 110 Volts Input Voltage Waveform

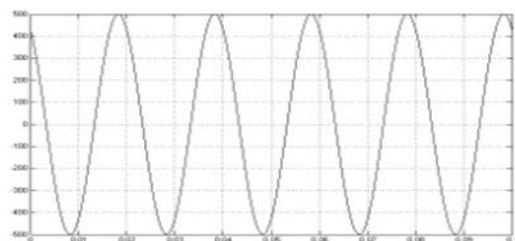


Fig. 15. 500 Volts Input Voltage Waveform

#### V. CONCLUSION

In this research, PWM based three phase AC to AC matrix converter is developed. Simulations are performed on Matlab. It is found out that PWM based matrix converter can be deployed to achieve any desired output and input characteristics. This converter is highly applicable for adjustable speed drives or variable frequency drives because in this proposed model it is shown that irrespective of any supply frequency, variable desired frequency can be attained. Secondly, this proposed model has revolutionized the applications requiring variable voltages. Any desired output

voltage can be achieved without taking into consideration of input voltage and vice versa. Further, harmonic contents are greatly reduced and it has maximized the fundamental content of the desired characteristics.

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