Lattice Boltzmann Method Study of Flow Past a Circular Cylinder in 2D

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***Abstract*– In this work, the flow past a circular cylinder in 2D was studied using the lattice Boltzmann method. The lattice Boltzmann models that were used are the single relaxation time (SRT) known as LBGK and BGK-Smagorinsky model for Re = 20; the 2D (D2Q9) nine-velocity square lattice was used. For the BGK-Smagorinsky model; the C_s values of 0.065, 0.1 and 0.13 were used. The results of the simulation gave the values of drag coefficient C_D that are in agreement with other results in the literature. This study has shown that the Smagorinsky model can be used to study channel flow of low Reynolds number past a circular cylinder in 2D with great success.**

***Keywords*– Lattice Boltzmann Method, Circular Cylinder, LBGK Model, BGK-Smagorinsky Model and D2Q9 Model**

I. INTRODUCTION

The lattice Boltzmann method (LBM) has gained recognition and has become one of the trusted computational fluid dynamics (CFD) methods. It has certain advantages over other CFD methods such as its simplicity, scalability on parallel computers, extensibility and the ease with which it can handle complex geometries. The introduction and history can be found in [1]-[4] and references therein.

They are types of lattice Boltzmann models like entropic lattice Boltzmann [5], single relaxation time (SRT) also known as lattice Bhatnagar, Gross and Krook model (LBGK) [6], multiple relaxation time (MRT) [7], BGK and MRT with Smagorinsky model [8] etc.

One of the most studied scenarios in CFD is the flow past a circular cylinder either in 2D or 3D. Circular cylinders are of great interest because of its practical and fundamental importance. It is useful in many industrial applications such as heat exchangers, bridge supports, power lines etc. The flow behavior is important in the design of these structures [9].

Shinichiro [10] studied the vortex-induced motion of 2D circular cylinder using the LBGK method for Reynolds number (Re =500).

Dawoud *et al.* [11] did the comparative study of flow past a circular cylinder in 2D and 3D using the LBGK model for Re =20, 40, 100, 300. The LBM-LES method was used to study the turbulent flow for Re = 2000.

Masanori and Kunio [12] studied the flow around a circular cylinder in 2D using finite difference method for Re = 0.1-10^6.

Schafer and Turek [13] did benchmark computations of flow around a cylinder in 2D (circular) and 3D (square and circular) with collaboration with other researchers used conventional CFD methods for Re =20 and 100.

Filipe *et al.*,[14] studied flow past a circular cylinder in 2D and 3D using RANS (Reynolds-Averaged Navier-Stokes), DES (Detached-Eddy Simulation) and XLES (Extra Large-Eddy Simulation) for Re = 3900.

Suresh and Sanjay [15] studied the oblique vortex shedding from flow around a circular cylinder in 2D using finite element method for Re = 60, 100 and 150.

Mittal and Ray [16] used a new scheme called the higher order compact based in immersed interface to study the flow past a circular cylinder in 2D for Re≤100.

In this study, the LBGK method and BGK with Smagorinsky model are used to simulate the flow past a circular cylinder in 2D. It is a well known fact that for turbulent flows of high Reynolds numbers the Smagorinsky model has been used with great success. The main aim of this study is to use the Smagorinsky model to study the steady flow of low Reynolds number (Re =20) past a circular cylinder in 2D in order to know how successful the model will be. The 2D nine-velocity square lattice (D2Q9) is used.

II. METHOD

A) LBGK Model

For the BGK model the lattice Boltzmann equation is seen as the continuous Boltzmann BGK model which is in discrete form [17] given as:

$$f_\alpha(\vec{x}+\vec{e}_\alpha\delta_t, t+\delta_t) = f_\alpha(\vec{x},t) - \frac{1}{\tau}(f_\alpha - f_\alpha^{eq}) \quad (1)$$

where $f_\alpha \equiv f_\alpha(\vec{x},t)$ gives the distribution function at position $\vec{x}$; $\vec{e}_\alpha$ is the discrete velocity at time t. Equation (1) is known as the LBGK model. For a particle the velocity set is taken to be as:

$$e_\alpha = \begin{cases} (0.0)c & \alpha = 0 \\ \left(\cos\left[(\alpha-1)\frac{\pi}{2}\right], \sin\left[(\alpha-1)\frac{\pi}{2}\right]\right)c & \alpha = 1.....4 \\ \left(\cos\left[(\alpha-5)\frac{\pi}{2}\right], \sin\left[(\alpha-5)\frac{\pi}{2}\right]\right)c & \alpha = 5.....8 \end{cases} \quad (2)$$

where $c = \left|\frac{\delta x}{\delta t}\right|$. The equilibrium distribution function gotten from Maxwell-Boltzmann distribution is given as:

$$f_a^{eq} = \omega_\alpha \rho \left(1 + 3\vec{e}_a.\vec{u} + \frac{9(\vec{e}_a.\vec{u})^2}{2} - \frac{3u^2}{2}\right) \quad (3)$$

The weighting factor ω_α given as:

$$\omega_\alpha = \begin{cases} \frac{4}{9} & k = 0 \\ \frac{1}{9} & k = 1.....4 \\ \frac{1}{36} & k = 5.....8 \end{cases} \quad (4)$$

where ρ is the macroscopic density per node and $\vec{u}(u_x, u_y)$ is the velocity, both are recovered from the summations over the lattice links of the particle distribution

$$\rho(\vec{x}, t) = \sum_\alpha f_\alpha(\vec{x}, t) \quad (5)$$

$$\rho(\vec{x}, t).\vec{u}(\vec{x}, t) = \sum_\alpha \vec{e}_\alpha f_\alpha(\vec{x}, t) \quad (6)$$

B) BGK-Smagorinsky Model

The particle distribution function together with the equilibrium distribution function are used in solving large scales of turbulence flows. Equation (1) [17] can be rewritten as:

$$f_\alpha(\vec{x} + \vec{e}_\alpha \delta_t, t + \delta_t) = f_\alpha(\vec{x}, t) - \frac{1}{\tau_{tot}}(f_\alpha - f_\alpha^{eq}) \quad (7)$$

where τ_{tot} is the total relaxation time which is made up of τ_o (molecular relaxation time) and τ_t (turbulence relaxation time). The total relaxation time can be written as:

$$\tau_{tot} = 3\frac{\delta t}{\delta x^2}(\vartheta_o + \vartheta_t) + \frac{1}{2} \quad (8)$$

where ϑ_t is the eddy viscosity which is governed by the Smagorinsky model

$$\vartheta_t = (C_s \Delta)^2 |\bar{S}| \quad (9)$$

C_s is the Smagorinsky constant; Δ is the filter size. $\bar{S}_{ij}$ is the strain-rate tensor which has the magnitude $|\bar{S}| = \sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$. For the purpose of this study the C_s values used are 0.065 [18], 0.1 [19]-[20] and 0.13 [21] which were used for channel flows by these researchers.

C) Computer Implementation

The implementation of the LBGK and BGK-Smagorinsky models were done via the open source LBM code openLB [22] and the boundary condition of Boudizi [23] was used. The simulation parameters are given in Table I. The 2D circular cylinder configuration is shown in Fig. 1; which is based on the benchmark work of Schafer and Turek [13].

III. RESULT AND DISCUSSION

The simulation for this study was done for Re = 20.The cylinder diameter is D = 0.1m and the height H = 0.41m. Table 2 lists the drag coefficient C_D and comparing the result with others results from literature. Figure 2 describes the detail of flow past a circular cylinder for LBGK model while Figures 3-5 describe the detail of flow past a circular cylinder for BGK-Smagorinsky model and C_s = 0.065, 0.1 and 0.13. At Re = 20, there is steady flow with light separation.

The LBGK model gave the value of drag coefficient C_D that is very close to the benchmark result. For the BGK-Smagorinsky model; when C_s = 0.065 the value of the drag coefficient C_D is closer to the benchmark result than when C_s = 0.1 and 0.13. Figures 6 and 7 showed the graph comparing the results of drag coefficient C_D from LBGK and BGK-Smagorinsky (when C_s = 0.065) models with the benchmark result.

IV. CONCLUSION

In this study, the flow past a circular cylinder in 2D was studied using the LBGK and BGK- Smagorinsky models. For the BGK-Smagorinsky model the C_s values of 0.065, 0.1 and 0.13 were used. The simulation results from these models gave the values of drag coefficient C_D that are in agreement with other results in the literature. This study has established the fact that for low Reynolds number (Re =20) the Smagorinsky model has been successful since it is well known for studying flow of high Reynolds numbers.

Table I: Simulation parameters

Resolution (N)	20
Characteristical pressure (N/m^2)	0
Physical density (kg/m^3)	1
Characteristical length (m)	0.1
Speed (m/s)	0.2
Lattice relaxation time	0.506
Lattice velocity	0.002
Time steps (s)	5×10^{-5}
Physical kinematic viscosity (m^2/s)	0.001

Table II: Drag coefficient C_D values

Schafer and Turek[13]	Dawoud*et al.*, [11]	Present study			
		LBGK	BGK-S(C_s)		
			0.065	0.1	0.13
5.57-5.59	5.28	5.614	5.617	5.621	5.627

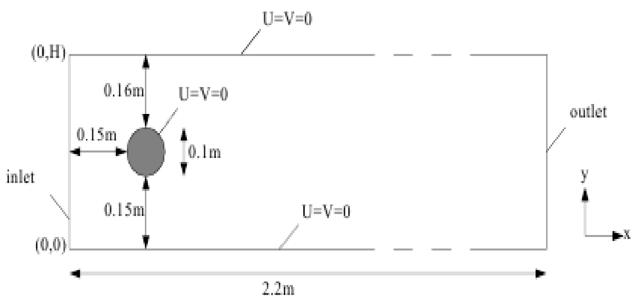

Fig. 1: 2D circular cylinder geometry [13]

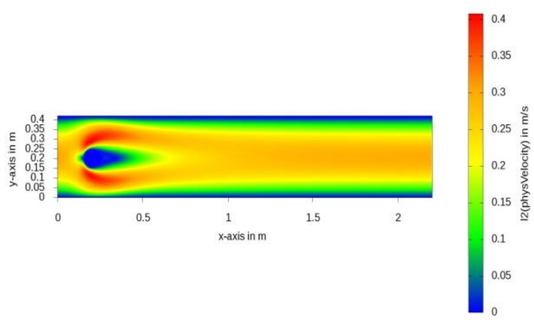

Fig. 2: Flow past a circular cylinder using LBGK

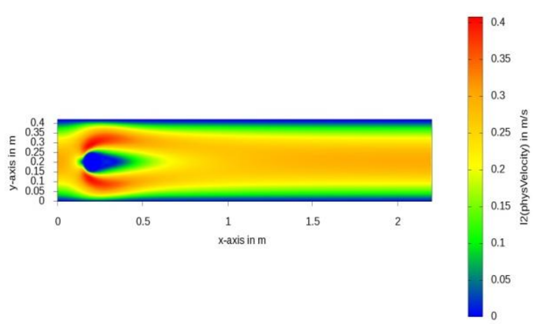

Fig. 3: Flow past a circular cylinder using BGK-Smagorinsky model when C_s = 0.065

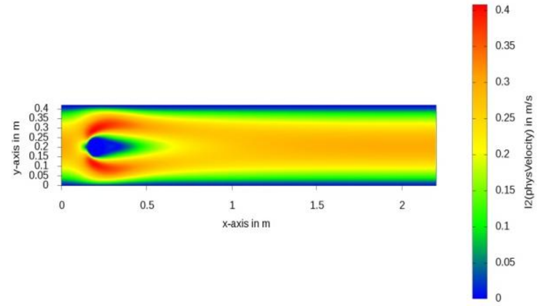

Fig. 4: Flow past a circular cylinder using BGK-Smagorinsky model when C_s = 0.1

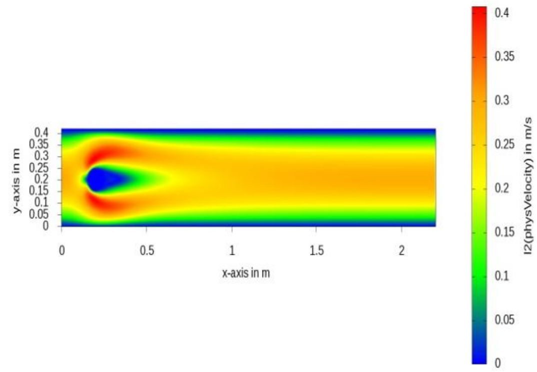

Fig. 5: Flow past a circular cylinder using BGK-Smagorinsky model when C_s = 0.13

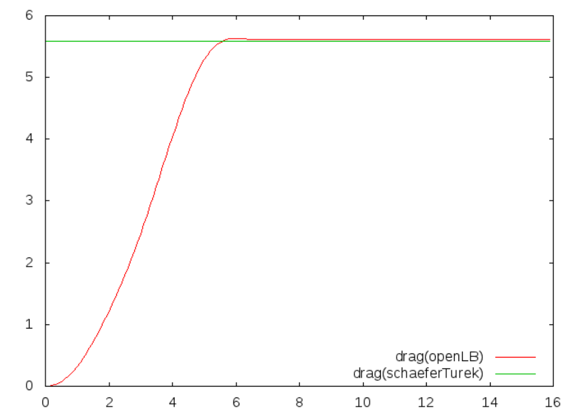

Fig. 6: Comparison between result obtained from LBGK model and benchmark result

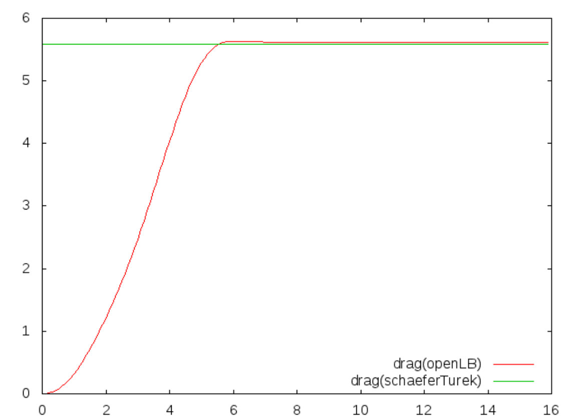

Fig. 7: Comparison between result obtained from BGK-Smagorinsky model when C_s = 0.065 and benchmark result

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