

# A Study on Free Vibration Under the Effect of Soil-Fluid-Structure Interaction on Concrete Gravity Dam

Iarly Vanderlei da Silveira<sup>1</sup>, Lineu José Pedroso<sup>2</sup>

<sup>1,2</sup>University of Brasilia

**Abstract**—The knowledge of the behavior of dams under dynamic actions is a primordial factor in the safety of these structures. The analysis of the coupled dam-reservoir-foundation system is a complex problem due to the interaction between the water in the reservoir, the soil in the foundation and the dam concrete. Some research on this field considered only the fluid-structure interaction as an important factor in the dynamic response; however, recent research also shows the significant impact of the foundation on the dynamic behavior of the dams. This work aims to evidence the influence of the foundation and reservoir on the dynamic behavior of concrete gravity dams as a function of their parameters in terms of natural frequencies and mode of vibration. The dam-reservoir-foundation interaction will be investigated through the modal analysis by the finite element method via the ANSYS APDL software. For this study, the Pine Flat dam, USA, will be modeled in the ANSYS, and for the validation of the problem with literature, simulations on the dam was analyzed for four different cases: (1) Dam with rigid foundation and empty reservoir; (2) Dam with rigid foundation and full reservoir; (3) Dam with flexible foundation and empty reservoir and (4) Dam with flexible foundation and full reservoir. It is analyzed the first three modes for the structure with rigid and flexible foundation decoupled, as well as the reservoir and coupled modes for the rigid and flexible structure with the reservoir. It is possible to observe the influence of the reservoir and the foundation on the natural frequencies and modes of the coupled system, an aspect that highlights the importance of the integrated dam-reservoir-foundation analysis in concrete gravity dam projects...

**Keywords**– Interaction of Dam-Reservoir-Foundation, Dynamic Analysis, Foundation Flexibility and ANSYS

## I. INTRODUCTION

The problem of soil-structure interaction (SSI) has become an important factor in the calculation of the structures that includes this interaction as in the case of dams that requires special attention in their construction. The high potential risks involved on a possible collapse of the dam imposes a more rigorous analyzes and inspections before, during and after the construction.

The effects of fluid-structure interaction (FSI) are important for the study of seismic or fluid-induced vibrations in dams, for instance. The movement of the structure inevitably causes a movement of the fluid, which remains in contact with the walls of the structure. As a result, the fluid-structure assembly

constitutes a coupled system for which it is often impossible to consider separately the responses and excitations.

The coupling of the dam-reservoir-foundation system includes distinct domains of nature and complex that considers elements of different mechanical characteristics and requiring an analytical treatment often limited depending on the demand of the problem. Thus, the finite element method (FEM) is adequate in approaching to those problems through the ability to discretize complex geometries and solve cases involving different materials.

The methods used for dynamic analysis of concrete gravity dams began to be developed by Westergaard (1933) who demonstrated analytically, through the Laplace equation, the distribution of pressures along the fluid-structure interface. The method proposed by Westergaard [1] assumes that the hydrodynamic effect on a rigid dam is equivalent to the inertial force resulting from an added mass distribution in the dam body. Chopra [2] observed that the response of the short-period vibration structure, such as the concrete dams subjected to seismic was largely influenced by the fundamental mode of vibration, and in his analyzes he also concluded that the vertical components of the ground acceleration had little influence on the structure response. Chopra e Chakrabarti [3] introduced a general procedure for the analysis of the response of concrete dams, including the dynamic effects of water and the flexible foundation to the horizontal and vertical components of soil movement. Fenves e Chopra [4] developed a semi-analytical-numerical technique to analyze the earthquake response of concrete gravity dams using two special cases (a) full reservoir dam supported by a rigid foundation; and (b) an empty reservoir dam supported by a flexible foundation, concluding that in the first case, the dam-reservoir effect and the bottom of the lake are relevant to the dam response, whereas in the second case the dam response is only related to the foundation and the dam. Domanguez et al. *et al.* [5] studied the effect of the reservoir-foundation interaction wherein they proposed an contour integral technique to investigate the response of the dam-reservoir-sediment-foundation systems subject to soil displacements. Arabshahi e Lofti [6] conducted a study on the mechanisms of natural vibration due to damage on the interface at the bottom of the dam, starting from a plasticity-based formulation using the local stresses of the element on the interface to model the sliding as well as the partial cracks along the foundation. Sevim *et al.* [7] determined the dynamic

characteristics of a prototype arch-reservoir-foundation dam system using the modal analysis method including local vibration tests in the arc dam model identifying its natural frequencies and modes of vibration. Seleemah *et al.* [8] analyzed numerically using the ANSYS software the problem the dynamic response of the dam-reservoir-foundation system to concrete gravity dams showing that the results of stresses and displacements are significantly affected when it has flexibility in the foundation.

The University of Brasília, within the study team of the Fluid-Structure Dynamic Group (FSDG) of the postgraduate course in Structures and Civil Construction, has developed many studies on dams, dynamic fluid-structure interaction and related studies, such as example: Oliveira [9], Pedrosa [10], Ribeiro [11], Souza Júnior [12], Silva [13], Souza [14], Melo [15], Ribeiro [16] and Mendes [17].

The present article is based on a text by Chopra and Chakrabarti [3] that presented a dynamic analysis study evaluating the influence of the soil and the reservoir on the response of the concrete gravity dam in terms of the modes of vibration and the natural frequencies of the dam-reservoir-foundation system.

**II. THERORECTICAL FORMULATION**

For the coupled modeling of a concrete gravity dam involving its interaction with the fluid and the foundation used the theory of finite elements, for discretization of the fluid, dam and foundation equations.

For the dam-reservoir coupling model, assuming small displacements hypothesis for the structure and fluid with a compressible material and negligible viscosity, a two-dimensional wave equation (1) is assumed:

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{1}$$

Being *c* the wave velocity, *p* is the hydrodynamic acoustic pressure and *t* is time. In order to obtain the solution of Equation (1) it is fundamental the understating of the boundary conditions [18] of the problem, presented in Fig. 1:

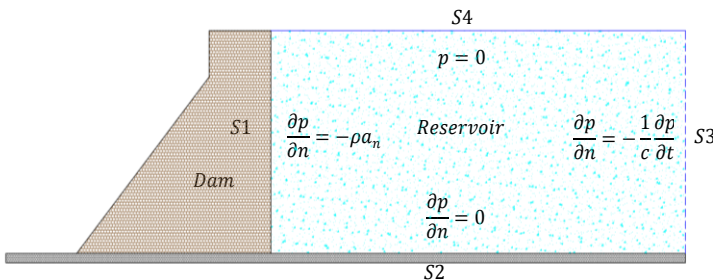


Fig. 1: Boundary conditions for the fluid domain

On the interface S1 occurs an interaction between the fluid and the structure which is the result of the inertial force imposed, in the normal direction  $\vec{n}$ , by the movement of the dam, being the pressure variation  $\left(\frac{\partial p}{\partial t}\right)$  applied on the face of

the reservoir, it is given by the fluid-structure condition (equation 2):

$$\frac{\partial p}{\partial n} = -\rho a_n \tag{2}$$

Being  $\rho$  the fluid density and  $a_n$  the structure acceleration vector in the normal direction to the common limit between fluid and structure.

The bottom region of the reservoir, S2, is considered rigid in this step. Assuming a horizontal motion of water, it is possible to neglect the pressure gradient (equation 3):

$$\frac{\partial p}{\partial n} = 0 \tag{3}$$

The S3 region upstream of the dam is influenced by the vibration of the dam, where waves are created by hydrodynamic pressure and propagate towards the end of the reservoir, if the length of the reservoir were considered infinite, these waves would disappear. Assuming from this a reservoir of infinite length we arrive at the boundary condition of Sommerfield (condition of no return of the wave – equation 4):

$$\frac{\partial p}{\partial n} = -\frac{1}{c} \frac{\partial p}{\partial t} \tag{4}$$

The region S4 represents a free surface and, neglecting the surface wave effect, its condition is defined as (equation 5):

$$p = 0 \tag{5}$$

The mathematical problem described by equation (1) with the respective boundary conditions, discretized by the finite element method, leads to the equation of motion of the reservoir (fluid) given by the matrix expression (6):

$$M_f \ddot{P}_e + C_f \dot{P}_e + K_f P_e + \rho_w Q^T (\ddot{u}_e + \ddot{u}_g) = 0 \tag{6}$$

Where  $M_f$ ,  $C_f$ , and  $K_f$  are, respectively, the fluid mass, damping and stiffness matrix;  $P_e$ ,  $\ddot{u}_e$ , and  $\ddot{u}_g$  are, respectively, the nodal pressure, nodal relative acceleration, and nodal acceleration vector of the soil, respectively. The term  $\rho_w Q^T$  refers to the coupling term between the fluid and the structure [19].

The discretization equation of the structure and foundation can be formulated through an approximation by using a plane stress finite element.

This element represents the structural dynamic equation presented in Equation (7):

$$M_s \ddot{u}_e + C_s \dot{u}_e + K_s u_e = -M_s \ddot{u}_g + Q P_e \tag{7}$$

Wherein  $M_s$ ,  $C_s$  and  $K_s$  are, respectively, the structural mass, damping and stiffness matrix;  $u_e$  is the nodal

displacement vector relative to the soil; the term  $QP_e$  represents the nodal force associated to the hydrodynamic pressure produced by the reservoir;  $\ddot{u}_e$  and  $\ddot{u}_g$  are the relative nodal acceleration and nodal acceleration of the soil, respectively. The term Q represents the coupling matrix.

The fluid-structure-foundation coupling equation can be written using Equations (6) and (7):

$$\begin{bmatrix} M_s & 0 \\ M_{fs} & M_f \end{bmatrix} \begin{bmatrix} \ddot{u}_e \\ \ddot{P}_e \end{bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix} \begin{bmatrix} \dot{u}_e \\ \dot{P}_e \end{bmatrix} + \begin{bmatrix} K_s & K_{fs} \\ 0 & K_f \end{bmatrix} \begin{bmatrix} u_e \\ P_e \end{bmatrix} = \begin{bmatrix} -M_s \ddot{u}_g \\ QP_e \end{bmatrix} \quad (8)$$

Being  $K_{fs} = -Q$  and  $M_{fs} = \rho_w Q^T$ . Equation (7) represents a second order differential equation, with an anti-symmetric matrix, and can be solved by the direct integration method. The consideration of the foundation effect is allowed by the numeric modelling in finite elements and this equation represents the most complex dynamic case for dynamic problems. For the case of free vibration, the damping matrix ([C]) and the external forces, on the right-hand side of the equation, are null.

III. DATA AND MODEL DEFINITION

The Pine Flat dam located near the King’s Fresno river in California – US, was chosen for study because of the results already found in the literature [3] for the validation of the model. The dimensions and geometry of the dam are shown in Fig. 2 and Table I.

A The dam has a height of 121.92 m, with a crest length of 560.83 and base length of 96.80 m.

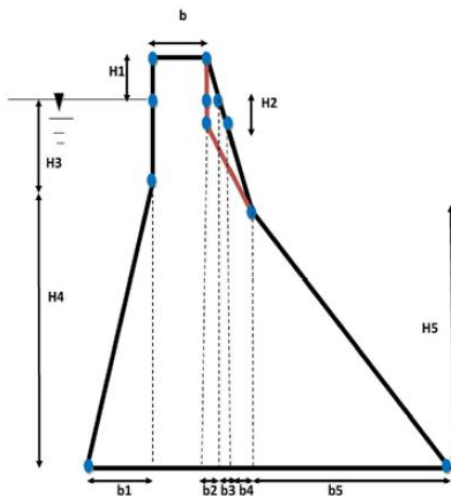


Fig. 2: The geometry variables of Pine Flat Dam [21]

TABLE I: THE GEOMETRY VARIABLES OF PINE FLAT DAM

Parameter	b	b1	b2	b3	b4	b5	H1	H2	H3	H4	H5
Value (m)	9.754	5.105	0	0	9.623	71.323	5.791	4.267	14.021	102.108	91.44

The concrete of the Pine Flat dam has the following physical properties: specific mass  $\rho_e = 2430 \text{ kg/m}^3$ ; Young’s modulus  $E_e = 22400 \text{ MPa}$ ; Poisson ratio  $\nu = 0,20$ . The physical Properties of the water are: sound speed  $c = 1000 \text{ m/s}$  and specific mass  $\rho_w = 1000 \text{ kg/m}^3$ . The foundation rock is assumed to be massless ( $\rho_s = 0 \text{ kg/m}^3$ ) and with a Young’s modulus  $E_s = 68923 \text{ MPa}$ .

Four cases were initially analyzed by varying the conditions of the foundation (rigid or flexible) and the volume of the reservoir (full or empty), in order to verify the behavior and validate the model. After the validation, the decoupled and coupled modes of the system were verified for the 4 cases.

The finite element modeling assumed the foundation and dam material as linearly elastic, isotropic and homogenous. It was used the plane finite element (PLANE 182) to model the structure and foundation. This element can be used as a plane element (plane stress or plane strain states) or as an asymmetrical element. It is defined by four nodes with two degrees of freedom in each node: translations respective to the nodal directions x and y.

Moreover, the interface of the soil-structure can be discretized using the command NUMMRGE for all nodes and elements in the surfaces of the contact (planes and interaction) or by CONTA 172 and TARGE 169, elements that make a link between them and which were used in this work. The contact elements cover the solid elements that describe the limit of a deformable body and are potentially in contact to the target surface. This surface is discretized by a group of elements of “target” segments and coupled to its contact associated surface through a group of shared real constants, allowing the imposition of any translational or rotational displacement, temperature, stress and magnetic potential in the target segment element, besides forces and moments over the target element.

For the two-dimensional study of the reservoir used the finite element Fluid 29, a four-node element with 3 degrees of freedom, with translations in the x and y direction and the pressure, used in fluid modeling and in the interfaces of fluid-structure interaction problems. The discretized equation takes into account the coupling of the acoustic pressure and the structural movement at the interface.

The model was discretized with the previously mentioned elements having 345 elements PLANE 182 for the structure, 440 elements Fluid 29 for the reservoir and 581 elements PLANE 182 for the foundation (Fig. 3).

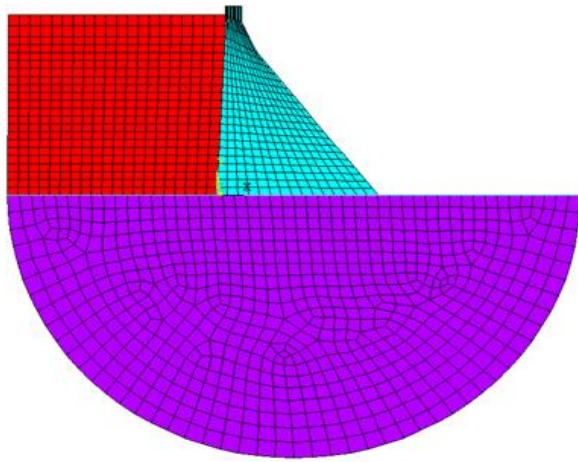


Fig. 3: Modelling in finite elements for the coupled system

IV. RESULTS

The results of the dynamic response analysis in this work are given in terms of natural frequency and modal shape, considering four cases varying the conditions of the dam-reservoir-foundation interaction namely:

- Case 1: Dam with empty reservoir and rigid foundation;
- Case 2: Dam with empty reservoir and flexible foundation;
- Case 3: Dam with full reservoir and rigid foundation;
- Case 4: Dam with full reservoir and flexible foundation.

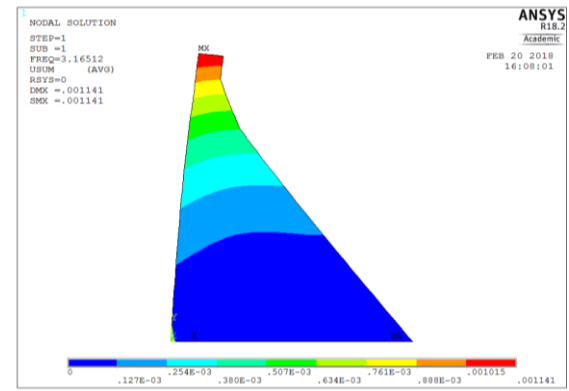
For the validation of the numerical model in finite elements, the first natural frequency of the dam modes for Cases 1 to 4 was determined and compared with the results found in the literature [3].

For the calculation of the error: difference between the values obtained by Chopra ( $f_{ref}$ ) and the present work ( $f_{pt}$ ) was used expression (9):

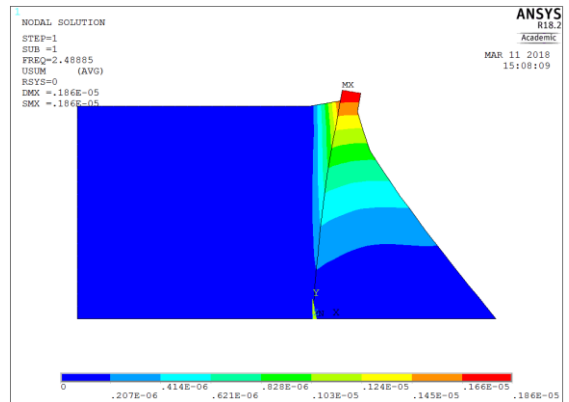
$$error(\%) = \frac{|f_{ref} - f_{pt}|}{f_{ref}} \quad (9)$$

It can be perceived from the results presented in Table II that there is a good agreement between the results found in this paper and those presented by Chopra [3]. It can also be noticed that the errors were small, aspect that qualifies the quality of the elaborated model, and ensures its validity. The first modal shape for each case is presented in Fig. 4.

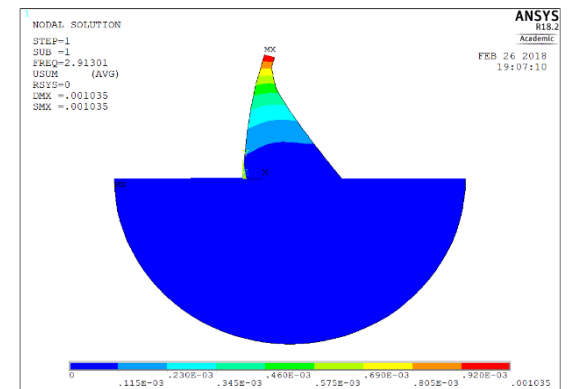
Case	Foundation	Reservoir	Chopra	The present Works	Error (%)
1	Rigid	Empty	3.1546	3.1651	0.33
2	rigid	Full	2.5189	2.4888	1.22
3	Flexible	Empty	2.9325	2.9130	0.66
4	Flexible	Full	2.3310	2.2974	1.44



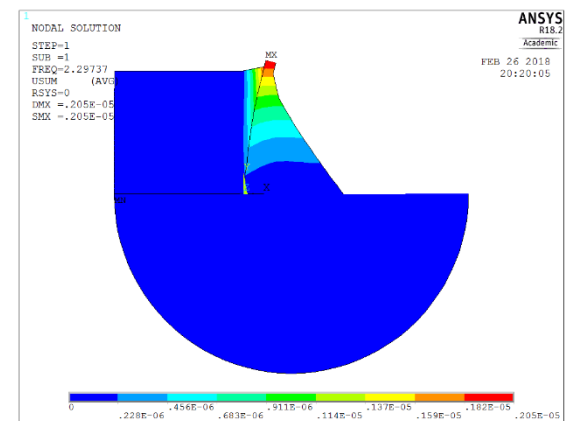
(a)



(b)



(c)



(d)

Fig. 4: The first shape of the dam for different Cases: (a) Case 1; (b) Case 2; (c) Case 3; (d) Case 4

Then, the first three modes of the decoupled and coupled system (dam-reservoir) were analyzed with and without the flexible foundation to verify the influence of each one on the structure response as shown in Fig. 5 to Fig. 10.

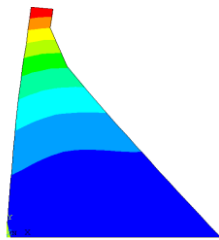
The analytical solution [22] of the reservoir represented by a closed-open cavity in both the x and y directions is given by equation (10), wherein  $i, j = 1, 2, 3, \dots$ ,  $c$  is the velocity of sound in the fluid,  $L$  the length of the reservoir and  $H$  the height, being in this case  $L \gg H$ .

$$f_{ij} = \frac{1}{2\pi} \left\{ (\pi c)^2 \left[ \frac{n_x^2}{4L^2} + \frac{n_y^2}{4H^2} \right] \right\}^{0.5} \quad (10)$$

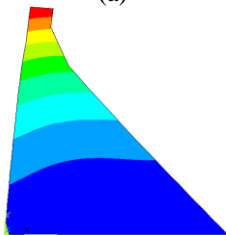
Para  $n_x = (2i - 1)$  e  $n_y = (2j - 1)$ .

- DECOUPLED MODES

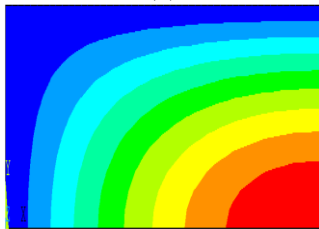
- 1° mode



$f_1 = 3.165$   
(a)



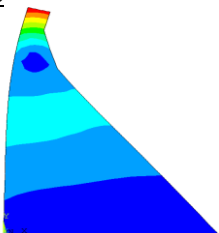
$f_1 = 2.913$   
(b)



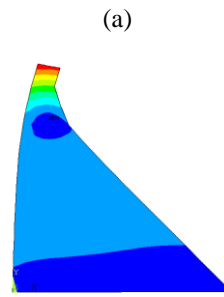
$f_1 = 3.067 / f_{analytical} = 3.064$   
(c)

Fig. 5: Uncoupled 1° modes (a) Rigid Structure, (b) Flexible Structure, (c) Reservoir

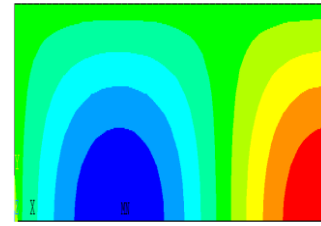
- 2° mode



$f_2 = 6.708$



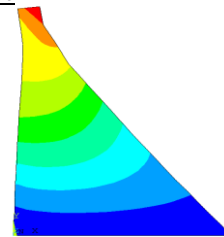
$f_2 = 6.090$   
(b)



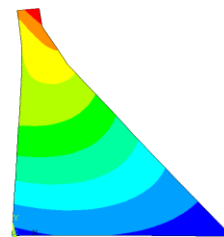
$f_2 = 3.673 / f_{analytical} = 3.671$   
(c)

Fig. 6: Uncoupled 2° mode: (a) Rigid Structure, (b) Flexible Structure, (c) Reservoir

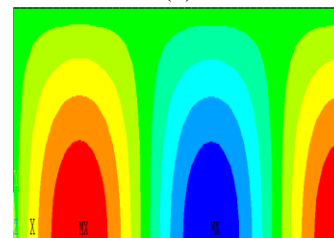
- 3° mode



$f_3 = 8.813$   
(a)



$f_3 = 7.289$   
(b)



$f_3 = 4.658 / f_{analytical} = 4.654$   
(c)

Fig. 7: Uncoupled 3° mode (a) Rigid Structure, (b) Flexible Structure, (c) Reservoir

- COUPLED MODES

• 1° mode

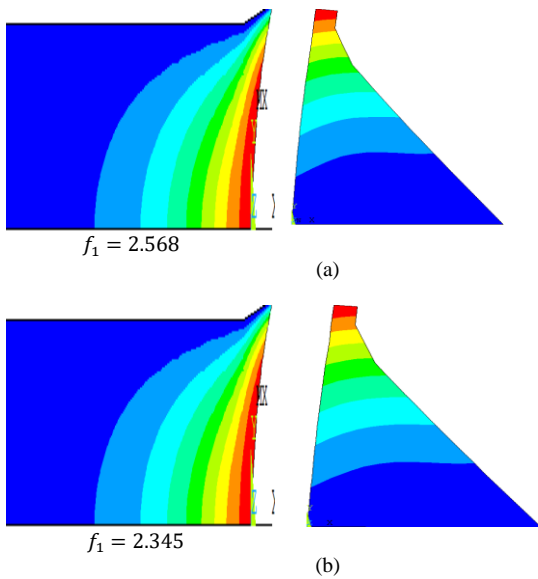


Fig. 8: Coupled 1° mode (a) Rigid Structure, (b) Flexible Structure

• 2° mode

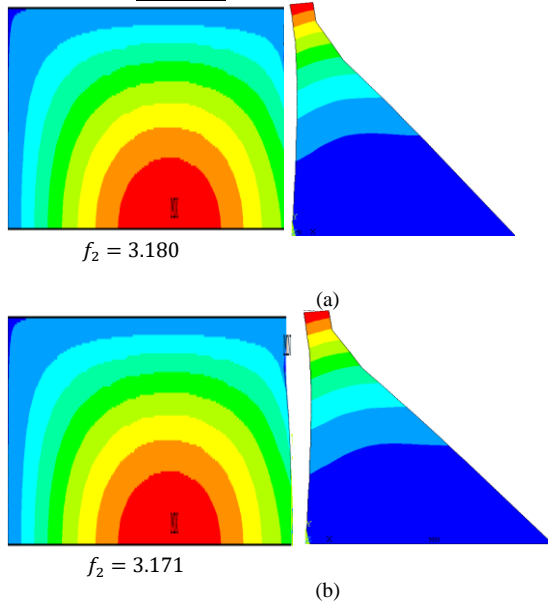


Fig. 9: Coupled 2° mode (a) Rigid Structure, (b) Flexible Structure

• 3° mode

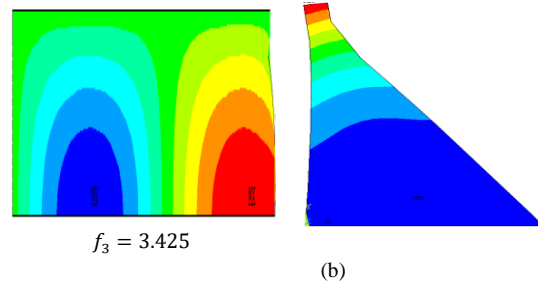
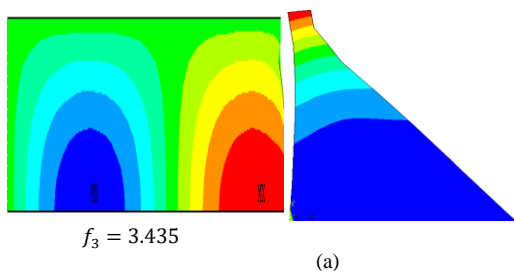


Fig. 10: Coupled 2° mode (a) Rigid Structure, (b) Flexible Structure

The vibration modes of the coupled fluid-structure problem presented a difficulty in the analysis and interpretation, since modal shapes with complex characteristics are found [14].

In order to identify them, it is necessary to first observe the modal shapes of the structure or fluid alone. The modal shapes for the coupled problem can reproduce: Structure mode, Fluid mode and a combination of both - "Mixed" mode. The first is characterized by structure mode control (additional mass mode) that the cavity adapts (follows) to the shape of the structure, whereas in fluid dominated modes, the structure adjusts (aligns) the configuration of the fluid pressure field. The mixed mode presents characteristic of the two domains, reproducing modal forms similar to structure and cavity.

By analyzing the coupled cases it was observed that the first coupled mode is controlled by the Structure mode, the second and the third by the Fluid Modes. The influence on the behavior of the problem is also perceived by the presence of the flexible foundation, further reducing the coupled frequencies.

The coupled structure for both cases with rigid or flexible foundation provides natural frequencies very close to the decoupled case of the reservoir, an aspect that influences the problem, in addition to presenting visually of its deformed pressure modes, similar to the deformed decoupled ones, in particular for the second and the third coupled mode respectively, respectively, to the first and second decoupled mode of the reservoir.

V. CONCLUSIONS

From the preliminary analyzes carried out in this work, the results allow to highlight some conclusions:

- The dam-reservoir-foundation interaction must have an important role in the project of dams, due to the influence, in the structure, of the coupling between these materials; the lowest frequency was obtained when it was considered a flexible foundation and full reservoir; the highest frequency was obtained for the case where it was considered a rigid foundation and empty reservoir; the consideration of the foundation flexibility induces a decrease in the natural frequencies, thus showing the importance of considering the effects that the soil imposes on the structure, as well as the influence of the hydrodynamic pressure exerted on the structure (additional mass), which in turn reduces the natural frequencies of the coupled system. In both cases the reduction of frequencies in coupled systems can bring the set

to a frequency range where earthquakes could significantly influence the forced response.

• The behavior of the coupled and uncoupled system analyzed can contribute to the comprehension of the problem involving the structure, reservoir and foundation interaction.

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#### REFERENCES

- [1] Westergard, H. M. (1933). *Water Pressure on Dams during Earthquakes*, TRANSACTIONS ASCE Vol.98.
- [2] Chopra, A. K. *Earthquake resistant design of concrete gravity dams*. In: Journal of The Structural Division, ASCE, v. 104, n. ST6, p. 953-971, jun. 1978.
- [3] Chopra, A. K. and P. Chakrabarti (1981). *Earthquake analysis of concrete gravity dams including dam-water-foundation rock interaction*. Earthquake engineering & structural dynamics 9(4): 363-383
- [4] Fenves, G. and A. K. Chopra (1984). *Earthquake analysis and response of concrete gravity dams*, Report No. UCB/EERC-84/10, Earthquake Engineering Research Centre, University of California, Berkeley, 213 pp.
- [5] Domiguez, J., R. Gallego and Bernardo. R (1997). "Effects of porous sediments on seismic response of concrete gravity dams." Journal of engineering mechanics 123(4): 302-311.
- [6] Arabshahi, H. and V. Lotfi (2008). *Earthquake response of concrete gravity dams including dam foundation interface nonlinearities*. Engineering Structures 30(11): 3065-3073.
- [7] Sevim, B., Bayraktar, A., Altunisik, A.C., Adanur, S., Akkose, M. (2010). *Dynamic Characteristics of a Prototype Arch Dam*. In: Experimental Mechanics. DOI 10.1007/s11340-010-9392-9.
- [8] Seleemagh. A., Zeidan, B., Rashed, A., EL Prince, M., C. *Effect of ground motion excitations on seismic response of concrete gravity dams*. International Symposium on Dams In A Global Environmental Challenges, ICOLD 2017, Prague, Czech Republic, 3-7 July, 2017.
- [9] Oliveira, F. F. (2002). *Análise de Tensões e Estabilidade Global de Barragens de Gravidade de Concreto*. Dissertação de Mestrado, Publicação E.DM-01A/02, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 160p.
- [10] Pedroso, L. J. (2003). *Interação Fluido-Estrutura*. In: Notas de Curso e Apostila didática, UnB-FT/ENC, Vs.3 – Brasília, DF
- [11] Ribeiro, P. M. V. (2006). *Uma Metodologia Analítica para a Avaliação do Campo de Tensões em Barragens Gravidade de Concreto Durante Terremotos*. Dissertação de Mestrado em Estruturas e Construção Civil, Publicação E.DM -003º/06. Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, Brasília, DF, 140p.
- [12] Souza Junior, L. C. (2006). *Uma Aplicação dos Métodos dos Elementos Finitos e Diferenças Finitas à Interação Fluido-Estrutura*. Dissertação de Mestrado em Estruturas e Construção Civil, Publicação E.DM-008º/06, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 197p.
- [13] Silva, S. F. (2007). *Interação dinâmica barragem-reservatório: modelos analíticos e numéricos*. Tese de Doutorado em Estruturas e Construção Civil, Publicação E. TD-05 A/07, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, Brasília, DF, 220p.
- [14] Souza, S. M. (2007). *Contribuição para uma Metodologia de Análise Acoplada Fluido-Estrutura para Cavidades Acústicas de Paredes Flexíveis*. Dissertação de Mestrado em Estruturas e Construção Civil, Publicação E.DM-004A/07, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 197p.
- [15] Melo, C. A. E. (2009). *Estudo do Acoplamento Comporta-Reservatório sob ações Sísmicas*. Dissertação de Mestrado em Estruturas e Construção Civil, E.DM 012A/09, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 318 p.
- [16] Ribeiro, P. M. V. (2010). *Soluções Analíticas para Cavidades Acústicas Bidimensionais com Aplicação ao Estudo de Problemas de Interação Dinâmica Barragem-Reservatório*. Tese de Doutorado em Estruturas e Construção Civil, Publicação E. TD-004A/10, Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, Brasília, DF, 306p.
- [17] Mendes, N.B.(2013). *Estudo comparativo analítico-numérico de aspectos da interação fluido-estrutura em cascas com aplicações a barragens em arco*. Dissertação de Mestrado em Estruturas e Construção Civil. Publicação E.DM-12A/13. Departamento de Engenharia Civil e Ambiental, Universidade de Brasília, DF, 318p
- [18] Chopra, A K. and P. Chakrabarti (1981). "Earthquake analysis of concrete gravity dams including dam-water-foundation rock interaction" Earthquake engineering & structural dynamics 9 (4): 363-383.
- [19] Sevedpoor, S.M., J. Salajegheh and E. Salajegheh, 2009. *Shape optimal design of arch dams including dam-water-foundation rock interaction using a grading strategy and approximation concepts*, Applied Mathematical Modelling, 34: 1149-1163
- [20] Bakenaz A. Zeidan (2014). *Seismic Analysis of Dam-Reservoir-Foundation Interaction for concrete gravity dams*, International Symposium on Dams in a Global Environmental Challenges, ICOLD 2014, Bali, Indonésia.
- [21] Khosravi, S.H., 2011. *Optimum of Concrete gravity dams with dynamic constraints and considering dam- reservoir-foundation rock interaction*. Master of Science thesis, Islamic azad University of Kerman.
- [22] PEDROSO, L. J. 2005. *Equação da Onda – 1D: Método da Matriz de transferência*. Publicação Didática. UnB-FT/ENC.



**Iarly Vanderlei da Silveira** Mr. Iarly Vanderlei was born in San Francisco - PB, Brazil. He has a degree in Civil Engineering at the Federal University of Campina Grande and he has a master's degree in Structures and Construction at the University of Brasília, and is currently a doctoral student at the same university and research line. He has published several research articles in international peer reviewed journals on concrete gravity dam structures, in particular with soil-structure and fluid-structure interaction.