INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY SCIENCES AND ENGINEERING, VOL. 15, NO. 3, JULY 2024

Mathematical Modelling of COVID-19 and Diabetes Comorbidity Under Vaccination

Florence Adongo, Titus Aminer, Benard Okelo, and Benson Onyango

Abstract–**The COVID-19 infection is a double challenge for people infected with comorbidities such as cardiovascular, cerebrovascular and diabetes. Comorbidities have been reported to be risk factors for the complications of COVID-19 infection. In most cases, people with diabetes have much higher rates of serious complications, low rate of recovery and higher mortality rate compare to people without diabetes. According to WHO data from 13 countries evaluated, revealed that 10.2% case of death of patients with and diabetes compared of 2.5% death of patients with COVID-19 only. In this paper, we formulate a mathematical model of COVID-19 and diabetes comorbidity under vaccination. This model will contribute to knowledge in mathematics, which will be used by researchers for future references and analysis.**

Keywords– **COVID-19, Comorbidity, Compartment, Diabetes, Mathematical Model and Vaccination**

I. INTRODUCTION

HUMAN immunity is key for stable and consistent health status. Diabetes is found to be a contributing factor to unstable immunity which is further noted as a predisposing factor for COVID-19 infection. The COVID-19 infection is a double challenge for people infected with comorbidities such as cardiovascular, cerebrovascular and diabetes. Comorbidities have been reported to be risk factors for the complications of COVID-19 infection.

Diabetics are the most affected by COVID-19 disease, whether they are infected or not and they have large number of deaths due to the virus [10], [14]. Lockdown was used to limit the spread of COVID-19, this has impaired access to health care and basic elements of proper disease management of diabetes like eating heathy diet and routine motoring of glucose [12]. According to WHO data from 13 countries evaluated, revealed that 10.2% case of death with patients with diabetes compared of 2.5% death of patients with COVID-19 only [12]. According to WHO findings, 18.3 % of COVID-19 deaths in Africa are among people with diabetes [13].

The first outbreak, which originated in December 2019 in Wuhan, the capital of Hubei province, and rapidly spread to the rest of Hubei and all other provinces in China due to the great mobility of personnel during the Chinese Lunar New Year. The number of COVID-19 cases in other countries gradually increased [8]. The World Health Organization (WHO) declared the virus epidemic in China a public health emergency of international concern. According to WHO, the signs of infection include fever, cough, shortness of breath and breathing difficulty. Other signs include loss of taste or smell as well as muscle aches [6]. In more severe cases, it can lead to pneumonia, multiple organ failure and even death.

People with diabetes face the problem of worse outcomes when infected with COVID-19. In China where most cases occurred, people with diabetes had much higher rates of serious complications and death than people without diabetes. Viral infections can also increase inflammation or internal swelling in people with diabetes. People with diabetes do face an increased risk of diabetic ketoacidosis (DKA) commonly experienced by people with type 1 diabetes. DKA makes it a challenge to manage fluid intake and electrolyte levels which is important in managing sepsis.

Sepsis and septic shock are some of the more serious complications that some people with COVID-19 have experienced American Diabetes Association [1]. In Kenya, at least 500,000 people with diabetes are at a higher risk of dying from COVID-19 if they contract it [5]. A study was done by [4] where patients suspected with COVID-19 were tested and laboratory results confirmed. Forty-one (41) patients admitted in the hospital were analyzed and the outcome was as follows: most of infected patients were men, 30 (73 \%) patients out of 41 patients tested; those who had underlying diseases were less than half, 13 patients (32 \%) distributed as- diabetes 8 patients (20 \%), hypertension 6 patients (15 \%) and cardiovascular disease 6 patients (15 \%); and 13 patients were admitted to ICU and 6 patients died.

Due to the effect, a number of mathematical models have been developed and analysis done on COVID-19, diabetes and for the comorbidity.

A mathematical model of transmission of COVID-19 with underlying condition of diabetes was formulated and analyzed Samuel and Joseph [10]. Numerical results showed that there was a greater number of deceased when they were infected with an underlying condition diabetes. Optimal condition suggested that lockdown and vaccination can reduce the rate of infection if introduced into a population. Mathematical Modeling Co-existence of diabetes and COVID-19 with deterministic and stochastic approach was analyzed by [2].

The sensitivity analysis were performed on the parameters involved in R{0} and the observation made showed that reduction on the transmission co-efficient from diabetes susceptible class to diabetes class is the most critical factor to control the co-comorbidity. From the numerical findings, observations showed that chances of getting COVID-19 by diabetes patients is high if they come into contact with COVID-19 than non-diabetes patients.

Ikram [7] studied an original mathematical model which described the dynamics of transmission of COVID-19 virus in the population with diabetes and came up with optimal controls such as the awareness program to diabetic people, strict glycemic control with a multidisciplinary medical follow-up in hospital and early diagnosis for diabetic people to reduce the number of people with complications. Most mathematical models that have been researched concentrated on the transmission of the diseases and optimal controls [2], [3], [4], [7], [9] among others. There are huge gaps in terms of access to early diagnostics, treatment and care. It was recommended people with comorbidity to be given priority for vaccination over those without [12].

In this paper, we are introducing vaccination class as vaccines play a bigger role on reducing the rate of transmission and recovery. After infection, how do we control recovery and death rate that is higher in persons with comorbidity than on those with COVID-19 alone?

In this paper, mathematical model formation is discussed for the two diseases.

II. MATHEMATICAL MODEL

Model description and formulation

We formulate a model in which the total human population at any time t, denoted by $N_{H,}$ is subdivided into six classes: Susceptible individuals (S_H(t)); Individual with diabetes (D_H(t)); Carrier individuals (infected and infectious but asymptomatic) with diabetes (L_{CD}(t)); Infected individuals (symptomatic) but with diabetes (I_{CD}(t)); Vaccinated individuals but with diabetes (V_D(t)); and Recovered individuals but with diabetes (R_D(t)). Hence, $N_H(t) = S_H(t) + D_H(t) + L_{CD}(t) + V_D(t) + I_{CD}(t) + R_D(t)$.

ASSUMPTIONS

1) Vaccinated individuals are free from COVID-19 but have diabetes.
2) Individuals recovered from COVID-19 but not diabetes.
3) Natural death occurs on each sub population at same rate.
4) Illness can cause death
5) Contact between infected and susceptive can lead to disease transmission.

Compartmental descriptions

Compartment S_H (Susceptible individuals)

Assumed a constant recruitment ρ_H into susceptible class which increase susceptible population. Susceptible are reduced by infection into diabetic class D_H by rate of λ_d and vaccination class V_H at the rate of θ and natural death μ.

$$\frac{d\mathrm{SH}}{\mathrm{dt}} = \rho\mathrm{H} - (\lambda + \mu + \theta)\mathrm{SH}$$

Compartment D_H (Individual with diabetes)

It is increased by individuals moving from susceptible at the rate of ʎ$_{d.}$ These are people infected by diabetes. This class is reduced by people infected with COVID-19 both to asymptomatic and symptomatic class and rates of λ_1 and λ_2 respectively, those who are vaccinated to vaccination class at rate Θ_d, natural death μ and death caused by diabetes disease at the rate of δ_d.

$$\frac{d\mathrm{DH}}{\mathrm{dt}} = \lambda\mathrm{SH} - (\delta\mathrm{d} + \lambda1 + \lambda2 + \theta\mathrm{d})\mathrm{DH}$$

Compartment V_D (vaccinated individuals)

The individuals are vaccinated in this class from susceptible population S_H, Diabetic population D_H and recovery class. Individuals with diabetes can be vaccinated against COVID-19 and move to vaccinated and lastly those who recover from COVID-19 can be vaccinated to avoid re-infection and to increase stability of the immunity system. All these increase the number of individuals in this class. From $S_{H,}$ D_H and R_D to vaccinated class V_D at the rate of θ, θ_d and κ respectively. Mortality occurs due to diabetes at the rate of δ and natural death μ. These deaths reduce the number of individuals in this class:

$$\frac{d\mathrm{VD}}{\mathrm{dt}} = \theta\mathrm{SH} + \theta\mathrm{DH} + \kappa\mathrm{RD} - (\delta + \mu)\mathrm{SH}$$

Compartment L_{CD} (Carriers individuals (infected and infectious but asymptomatic) with diabetes)

This class consist with individuals with both COVID-19 and diabetes but no symptoms of COVID-19 (asymptomatic) individuals are increased in this class from diabetes class (D_H) and reinfection from recovery class (R_D) and reduced by natural death μ, COVID-19 induced death δ, death due to two diseases δ_{cd}, rate of recovery γ that is affected by the presence of diabetes with modifying factor χ_1 and rate of moving from asymptomatic to symptomatic τ. The equation for this class becomes:

$$\frac{d\mathrm{LCD}}{\mathrm{dt}} = \lambda1\mathrm{DH} + \psi1\mathrm{RD} - (\delta\mathrm{dc} + \gamma1\chi1 + \tau + \delta + \mu)\mathrm{LCD}$$

Compartment I_{CD} (infected (symptomatic) with diabetes)

This class consists of individuals with both COVID-19 and diabetes with symptoms of COVID-19 (symptomatic) individuals are increased in this class from diabetes class (D_H), reinfection from recovery class (R_D) at the rate of ψ_2 and rate of moving from asymptomatic to symptomatic τ. Reduction is done by natural death μ, COVID-19 induced death δ, death due to two diseases δ_{cd}, rate of recovery γ_2 that is affected by the presence of diabetes with modifying factor χ_2 . The equation for this class becomes:

$$\frac{d\mathrm{ICD}}{\mathrm{dt}} = \lambda2\mathrm{DH} + \psi2\mathrm{RD} + \tau\mathrm{LCD} - (\delta\mathrm{dc} + \gamma2\chi2 + \delta + \mu)\mathrm{ICD}$$

Compartment R_D (Recovered individuals but with diabetes)

Individuals from this class have recovered from COVID-19 but have diabetes. Individuals moving into this class from classes L_{CD} and I_{CD} at rate of $\gamma_1\chi_1$ and $\gamma_2\chi_2$ respectively. They are reduced as some got re-infected at rate ψ_1 and ψ_2 back to classes L_{CD} and I_{CD} respectively. After recovery those who were not vaccinated get vaccination and move to vaccination class at rate of κ. The equation for this class becomes:

$$\frac{dRD}{dt} = \gamma 2\chi 2ICD + \gamma 1\chi 1LCD - (\delta d + \psi 2 + \kappa + \psi 1 + \mu)RD$$

The force of infection associated with COVID-19 is:

$$\lambda = \beta(LCD + \alpha ICD)/N$$

Where β is the effective contact rate for COVID-19 infection and α is a modification factor $\alpha \geq 1$ account for the fact that individuals display symptoms are more infectious than individuals in latent class asymptomatic due to high viral load.

Variables and descriptions

S_H- Susceptible population
D_H- Diabetic individuals
V_D- Vaccinated individuals but diabetic
L_{CD}- Carriers individuals (asymptomatic) with diabetic
I_{CD}- Infected individuals (symptomatic) with diabetic
R_D- Recovered individuals but diabetic

Parameters and description

α- Modifying factor for rate of infection of infected class
β-Effective contact rate for COVID-19
δ-Death rate due to COVID-19
κ-Rate of recruitment from recovery to vaccinated class with diabetic
λ-Rate of recruitment to diabetic class
λ_1- Rate of recruitment to carrier class with diabetic
λ_2- Rate of recruitment to infected class with diabetic
ψ_1-Rate of re-infection to carrier class with diabetic
ψ_2-Rate of re-infection to infected class with diabetic
γ_1- Rate of re-recovery for carrier individuals with diabetic
γ_1- Rate of re-recovery for infected individuals with diabetic
μ-Natural death rate
τ-Rate of transfer of carrier individuals to infected class
χ_1-Modifying factor for recovery of carriers
χ_2-Modifying factor for recovery of infected
θ-Rate of recruitment from susceptible to vaccinated class
θ_d-Rate of recruitment from diabetic to vaccinated class
ρ_H- Rate of recruitment to susceptible class

From the above definition of variables, parameters and compartments, the resulting flow diagram for COVID-19 and Diabetes comorbidity under vaccination is given below:

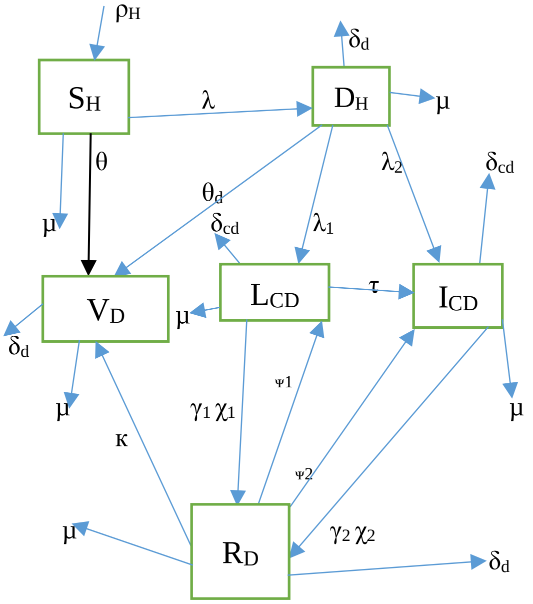

Fig. 1: The flow diagram for COVID-19 and Diabetes comorbidity under vaccination

The mathematical model is defined by a system of differential equations based on the flow diagram above.

$$\frac{dSH}{dt} = \rho H - (\lambda + \mu + \theta)SH$$

$$\frac{dDH}{dt} = \lambda SH - (\delta d + \lambda 1 + \lambda 2 + \theta d)DH$$

$$\frac{dVD}{dt} = \theta SH + \theta dDH + \kappa RD - (\delta + \mu)VD$$

$$\frac{dLCD}{dt} = \lambda 2DH + \psi 2RD + \tau LCD - (\delta dc + \gamma 2\chi 2 + \delta + \mu)ICD$$

$$\frac{dICD}{dt} = \lambda 2DH + \psi 2RD + \tau LCD - (\delta dc + \gamma 2\chi 2 + \delta + \mu)ICD$$

$$\frac{dRD}{dt} = \gamma 2\chi 2ICD + \gamma 1\chi 1LCD - (\delta d + \psi 2 + \kappa + \psi 1 + \mu)RD$$

REFERENCES

[1] American Diabetes Association., How COVID-19 Impacts People With Diabetes, https:www.diabetes.org/Covid-19-covid-19/ 2020.
[2] S. Anusha and S. Athithan., Mathematical Modelling Co-existing of diabetes and COVID-19: Deterministic and Stochastic approach, department of Mathematics, Faculty of engineering and technology, SRM institute of Science and Technology Kattankulathur tamil nadu 6032203, India, 2020.
[3] A. Boutayeb, E. Twizell, K. Achouayb and A. Chetouani, A mathematical model for the burden of

diabetes and its complications, Biomedical Engineering Online, 2004.

[4] H. Chaolin, W. Yeming, L. Xingwang and Gul., Mathematical Model for Coronavirus disease 2019 (COVID-19) Containing Isolstion class, Hindawi; BIomed REsearch international https:doi.org/10.1155/2020/3452402.

[5] CITIZEN NEWS. Diabetes, obesity declared risk factors as Kenya recorded 375 new cases %https://citizen.co.ke/news/Covid-19, 2020.

[6] European centers for Disease Control and Prevention., clinical characteristics of CIVID-19, Retrieved 4 April 2020.

[7] I. Ikram, I. Nadia. and E. Saloua, A new model of COVID-19 among diabetes population: a mathematical analysis and optimal control approach for intervention strategies, Communications in Mathematical biology and Neuroscience https: //doi.org/10.28919/cmbn/8264 vol-123, (2023).

[8] National Health Commission daily reports. Online access, National Health Commission (NHC)of People's Republic of China}, http://www.nhc.gov.cn/yjb/pzhgli/new*list.sht

[9] O. Samuel and A. Joseph, A Mathematical Model of Transmission Dynamics of SARS-CoV-2(COVID-19) with an Underlying Condition of diabetes, International Journal of Mathematics and Mathematical Sciences, https:doi.org/10.1155/2022/7984818 (2022).

[10] J. Valderas, B. Starfield and C. Salisbury, Defining comorbidity: Implications for Understanding Health and Health services, {doi:10.1370/afm.983, Vol.7, 2009.

[11] World Health Organization, diabetes-word health organization, https://www.who/ 2020.

[12] World Health Organization (WHO), COVID-19 more deadly in Africans with diabetes, https://www.afro.who.int/news/covid-19-more deadly-africans-diabetes 2021.

[13] World Health Organization (WHO), Fact sheet diabetes, https://www.afro.who.int/news-room/fact-sheet/detail/diabetes 2021.

[14] C. Yang and J. Wang, COVID-19 and underlying health conditions: a modeling investigation, Mathematical biosciences and Engineering, Vol. 18(4), (2021), 3790-3812.