A Graph Theoretic Approach to Quality Evaluation of a Typical Gas Turbine System

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***Abstract*— In the present paper, the quality evaluation of a Gas turbine system has been carried out by considering its different characteristics which govern the qualitative aspects of the gas turbine system. The graph theoretic approach has been adopted for the quality evaluation of the gas turbine system. By using this methodology, the gas turbine system has been modeled and various attributes contributing to the quality of the gas turbine system have been identified. A digraph of the characteristics which contributes to the quality of the gas turbine system significantly has been developed. The interdependency of the attributes as well as their inheritances has been identified at the gas turbine subsystem level and its representation in matrix form has been further used for the calculation of quality index of the gas turbine system through its variable permanent quality function. The sensitivity analysis of the gas turbine systems have been carried out for studying the effects of attributes over the quality of the gas turbine systems as well as their optimal selection over its various operating stages.**

***Keywords*—** Gas Turbine System Quality, Gas Turbine System Quality Index, Diagraph and Matrix Methods

I. INTRODUCTION

A Gas turbine system requires a proper balance between quality, performance and cost. Quality, however, is generally given higher priority than the other two as no matter how better performance or how low cost may be, the gas turbine system value to its customers in the market would be compromised by less than the perfect quality. Different ways of representation of quality have been used by various authors. Either it is correlated with conformance to features and specifications of a product or it is related to customer desired and value for its money. However, the managing the functional requirements in conformance to customer desires with in the limits of probable investment in system and its recognized value is again a challenging task for the system designers. Many a times, it is correlated with the objectives of customer satisfaction but with reasonable profit or returns to the organization in order to keep their presence in the global but competitive market. Some quality attributes for a typical system were identified [1] and later on, some other attributes like maintainability, safety, environmental impact and the life cycle cost were also considered significant [2] for the effective quality evaluation of a system.

In the past few decades, the gas turbine systems and its technological advancements have almost showed its remarkable presence in petrochemicals, aviations, marines, industrial and power generation sectors. Hence, it has become inevitable to improve the quality of such systems for better performance of such systems. The Graph theory has emerged as a useful tool for understanding the system parameters and effective decision making for improving the system quality. For various categories of the gas turbine systems, this theory has also been effectively used. The real time study related to reliability evaluation [3] and commercial availability [4] of steam power plant as well as the system modeling of coal based power plants [5] has been carried by using this methodology. Similarly, the various aspects of quality evaluation of a thermal power plant [6] have also been studied by using graph theoretic approach. Similarly, [7] has used this approach for the quality evaluation of an industry and a mathematical model for the same is been developed.

For the GE gas turbines, QFD technique has been implemented to understand the performance characteristics of the gas turbine systems [8] and various factors related to customer requirements and the functional requirement of such systems have been ranked on a normalized scale. Further, various factors affecting the operational cost as well as the performance characteristics [9] of such plants have been evaluated.

Various aspects of performance improvement have also been given due attention in gas turbine industry. The stated reasons of the performance monitoring [10] of the gas turbines have been discussed along with the ways of obtaining the values of such gas turbine monitoring systems by the customers. The performance parameters of gas turbines have been studied [11, 12] along with the factors affecting their designs. Comparative analyses have also been carried out [13] for different gas turbine solutions with the objective of one of the priority i.e. better efficiency after quality.

Since the gas turbine systems have to be used for the different operational considerations, hence, the priorities of success of such systems are always better quality with effective operational flexibility and maximum combined cycle efficiency. The cost effective analysis with improved operational flexibility over the maintenance and warranty issues of the power plants have also been suggested [14]. As

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the gas the gas turbine market is demand of the present competitive market, various aspects of its success with operational flexibility [15, 17] along with business driven factors [16] have also been studied in detail contributing towards its better quality and performance.

II. QUALITY CHARACTERISTICS OF GAS TURBINE SYSTEM

The quality of a gas turbine system is defined as the measure of the attributes that effects the quality of the subsystems in a typical gas turbine system, which help it to achieve the desired goals through the best matching performance of subsystems, working media, the system integration and operational strategy of the gas turbines by using latest, but best suitable tools and technologies subjected to minimum cost and time. The desired goals of the qualitative analysis includes achieving best functional performance of the gas turbine system with maximum satisfaction to the customer with best compatible compromise between the customer desires and functional needs of the gas turbine system with best inherent impact of all other quality governing attributes of the system. Thus, the quality attributes of the gas turbine system includes all such parameters which are directly or indirectly responsible for achieving the desired quality of the gas turbine system.

The Functional Performance characteristics of a gas turbine system are generally the desired objectives represented in scientific manner with conformance to customer desirable characteristics. Since a number of system variables or the attributes are interdependent, it makes all the quality characteristics also interdependent in one way or the other. A close relationship of such interdependencies among the attributes or the characteristics cannot be established in the form of universal empirical relations; hence, the visual means of representation of such systems like digraph representation plays an important role in understanding and analyzing the quality aspects of gas turbine systems..

A. Gas Turbine System Quality Digraph

A diagraph is used to represent the governing attributes and their interdependencies within the system. A gas turbine system quality digraph represents the qualitative measure of the characteristics or the attributes (D_i's) through its nodes and edges related to the interdependencies of the attributes (D_{ij}'s). A qualitative gas turbine system attributes digraph models the quality attributes of the gas turbine systems and their relative importance. The digraph consists of a set of nodes V= $\{V_i\}$, with i= 1,2,3,……M and a set of directed edges D=$\{D_{ij}\}$. A node V_i represents the ith qualitative attribute and the edges represents the relative importance between such quality attributes of the gas turbine system. The number of nodes in the digraph represents the total number of quality attributes considered for the qualitative evaluation of the gas turbine systems. In the present digraph method , if a node 'i' exhibits relative importance over node 'j' during the qualitative evaluation of the gas turbine system, then a directed edge is represented from node 'i' to node 'j' (i.e. D_{ij}). If a node 'j' exhibits relative importance over node 'i' then a directed edge is drawn from node 'j' to node 'i' (i.e. D_{ji}).

To demonstrate the qualitative attributes digraph of the gas turbine system, an example of qualitative evaluation of simple gas turbine system in an integrated environment has been considered. The qualitative characteristics of interest for the quality analysis of gas turbine system are Functional Performance (FP), Operational Availability (OA), Serviceability (SV), Operational Flexibility (OF), Environmental Impact (EI), Customer Desire Conformance (CD), Aesthetics (AT), Perceived Quality (PQ), Durability (DR), Life Cycle Cost (LC), Operational Safety (OS), Feature and Design Conformance (FD). These characteristics also act as attributes for the overall quality evaluation of the given gas turbine system.

The Functional Performance (FP) is a must- desired outcome of the Customer desire Performance (CD). Better representation of Functional Performance (FP) of the gas Turbine system in terms of Customer needs i.e. the Customer Design conformance (CD) and the Perceived quality (PQ) ensures not only the acceptability of the system in the market, but also its better Operational availability (OA). Aesthetics (AT) play an important role for the acceptability of the system in terms of Perceived quality (PQ). One the close relationships are identified for the Functional Performance (FP) and the Customer needs i.e. Customer Design conformance (CD) and the Perceived quality (PQ) along with Aesthetics (AT), efforts are generally made to get best compromised solutions in terms of functional outcomes i.e. the Operational Availability (OA), Serviceability (SV), Operational Flexibility (OF), Environmental Impact (EI), Life Cycle Cost (LC), Operational Safety (OS), Feature and Design Conformance (FD).

Many a times, desirables and the deliverables in terms of the effect of attributes over the quality parameters are conflicting. So, a balance is generally made between the two by identifying the close interdependency of these attributes as well as the interdependency among the sub-attributes which affect these quality parameters or the attributes directly or indirectly. An effort has also been done for quantifying such level of interdependencies among the attributes and the sub-attributes in the present work.

III. MATRIX REPRESENTATION OF GAS TURBINE SYSTEM'S QUALITY ATTRIBUTES DIGRAPH

Matrix representation of the Gas turbine system's quality attributes digraph presents a one-to-one representation. The digraph representation is very suitable for visual analysis, but is not suitable for computer processing. Moreover, if the system is large, its corresponding graph is complex and this complicates its visual understanding. Hence, a matrix called Quality attributes matrix is defined. This matrix considers all the attributes (D_i's) and their relative importance (i.e. D_{ij}) with respect to each other for the Gas turbine system. The Quality attributes matrix of the Gas Turbine system quality attribute

$$A = \begin{array}{cc} attributes & \begin{array}{cccccccccccc} FP & OA & SV & OF & EI & CD & AT & PQ & DR & LC & OS & FD \end{array} \\ \begin{array}{c} FP \\ OA \\ SV \\ OF \\ EI \\ CD \\ AT \\ PQ \\ DR \\ LC \\ OS \\ FD \end{array} & \begin{bmatrix} D_1 & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9} & a_{1,10} & a_{1,11} & a_{1,12} \\ a_{2,1} & D_2 & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} & a_{2,11} & a_{2,12} \\ a_{3,1} & a_{3,2} & D_3 & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} & a_{3,11} & a_{3,12} \\ a_{4,1} & a_{4,2} & a_{4,3} & D_4 & a_{4,5} & a_{4,6} & a_{4,7} & a_{4,8} & a_{4,9} & a_{4,10} & a_{4,11} & a_{4,12} \\ a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & D_5 & a_{5,6} & a_{5,7} & a_{5,8} & a_{5,9} & a_{5,10} & a_{5,11} & a_{5,12} \\ a_{6,1} & a_{6,2} & a_{6,3} & a_{6,4} & a_{6,5} & D_6 & a_{6,7} & a_{6,8} & a_{6,9} & a_{6,10} & a_{6,11} & a_{6,12} \\ a_{7,1} & a_{7,2} & a_{7,3} & a_{7,4} & a_{7,5} & a_{7,6} & D_7 & a_{7,8} & a_{7,9} & a_{7,10} & a_{7,11} & a_{7,12} \\ a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & D_8 & a_{8,9} & a_{8,10} & a_{8,11} & a_{8,12} \\ a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & D_9 & a_{9,10} & a_{9,11} & a_{9,12} \\ a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & D_{10} & a_{10,11} & a_{10,12} \\ a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & a_{11,5} & a_{11,6} & a_{11,7} & a_{11,8} & a_{11,9} & a_{11,10} & D_{11} & a_{11,12} \\ a_{12,1} & a_{12,2} & a_{12,3} & a_{12,4} & a_{12,5} & a_{12,6} & a_{12,7} & a_{12,8} & a_{12,9} & a_{12,10} & a_{12,11} & D_{12} \end{bmatrix} \end{array} \quad (1)$$

digraph is represented as: Where 'D_i' is the value of the *i*th attribute represented by node 'V_i' and 'D_{ij}' is the relative importance of the ith attribute over the *j*th attribute. This relative importance has been represented by the edge 'i-j' in the digraph. The permanent of this matrix 'A' i.e. the Per(A), is defined as the qualitative function of the gas turbine system. The 'Per(A)' is a standard matrix function and is generally used in combinatorial mathematics. It is calculated in the same manner as the determinant, but all negative terms obtained after expansion for the calculation of the determinant of the matrix are replaced with positive equivalent terms. This computation result in a monomial where every term has a significance related to the overall evaluation of the gas turbine system quality function and no loss of significant term is observed during the computation.

The gas turbine system quality function (GTSQF) is represented as:

$$GTSQF = Per(A) \quad (2)$$

The expanded form of the above equation in terms of various groupings and subgroupings has been given in Appendix.

The equation (2) is the complete expression in the form of terms arranged in 13 groupings for the quality evaluation of the gas turbine system, as it considers the presence of all the attributes and all the possible relative importance between the attributes. The terms are the sets of distinct diagonal elements (D_i's) and the loops of off-diagonal elements of different sizes (i.e. $D_{ij}.D_{ji}$, $D_{ij}.D_{jk}.D_{ki}$). In the permanent Per(A), various groupings have their own physical significance.

- The first term (grouping) represents a set of twelve independent subsystem characteristics as D_1, D_2, D_3,….D_{12}.
- As there are no self loops with in the system itself, second groupings are absent.
- Each term of the third grouping represents a set of two elements attribute loops (i.e. $D_{ij}.D_{ji}$) and is the resultant dependence of attribute *i* and *j* and the evaluation measure of N-2 connected terms.
- Each term of the fourth grouping represents a set of three element attribute loops ($D_{ij}.D_{jk}.D_{ki}$ or its pair $D_{ik}.D_{kj}.D_{ji}$) and the evaluation measure of N-3 unconnected elements or attributes with in the system.
- The fifth grouping contains two subgroups. The terms of first subgrouping consists of four element attribute loops (i.e. $D_{ij}.D_{jk}.D_{kl}.D_{li}$) and the subsystem evaluation index component ($D_m.D_n$…… D_u). The terms of the second grouping are the product of two element attributes loops ($D_{ij}.D_{ji}$) ($D_{kl}.D_{lk}$)) and the subsystem evaluation index component ($D_m.D_n$…… D_u).
- The terms of the sixth grouping are arranged in two subgroupings. The terms of the first subgroupings are of five element attribute loop (i.e. $D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mi}$) or its pair ($D_{im}.D_{ml}.D_{lk}.D_{kj}.D_{ji}$) the subsystem evaluation index component ($D_n.D_p$…… D_u). The second subgrouping consists of a product of two attributes loops (i.e. $D_{ij}.D_{ji}$) and a three attribute loop (i.e. $D_{kl}.D_{lm}.D_{mk}$) or its pair (i.e. $D_{km}.D_{ml}.D_{lk}$) and the subsystem evaluation index component ($D_n.D_p$…… D_u).
- The terms of seventh groupings are arranged in four subgroupings. The first subgrouping of the seventh grouping is a set of 3- two element attribute loops (i.e. $D_{ij}.D_{ji}$, $D_{kl}.D_{lk}$, $D_{mn}.D_{nm}$) and the subsystem evaluation index component ($D_p.D_q$…… D_u). The terms of second subgrouping of seventh grouping are of two element attribute loop (i.e. $D_{ij}.D_{ji}$) and four element attribute loop (i.e. $D_{kl}.D_{lm}.D_{mn}.D_{nk}$) with the subsystem evaluation index component ($D_p.D_q$…… D_u). The terms of the third subgrouping of the seventh grouping are of 2- three element attribute loops (i.e. $D_{ij}.D_{ji}.D_{ki}$ and $D_{lm}.D_{mn}.D_{nl}$) with the subsystem evaluation index component ($D_p.D_q$…… D_u). The terms of fourth subgrouping of seventh grouping are of six elemental attribute loop (i.e. $D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mn}.D_{ni}$) and one subsystem evaluation index component ($D_p.D_q$…… D_u).
- The terms of eighth grouping are arranged in four subgroupings. The first subgrouping of the eighth grouping is a set of three element attribute loop (i.e. $D_{mn}.D_{np}.D_{pm}$), two element structural diads as ($D_{ij}.D_{ji}$) and ($D_{kl}.D_{lk}$) and the subsystem evaluation index component ($D_q.D_r$… D_u). The second subgrouping is a set of a two element diad (Dij.Dji), a five element attribute loop (i.e. $D_{kl}.D_{lm}.D_{mn}.D_{np}.D_{pk}$) and the subsystem evaluation index component ($D_q.D_r$… D_u). The third subgrouping consists of a three element attribute loop (i.e. $D_{ij}.D_{jk}.D_{ki}$), a four element attribute loop (i.e. $D_{lm}.D_{mn}.D_{np}.D_{pl}$) and the subsystem evaluation index component ($D_q.D_r$… D_u). Similarly, the fourth subgrouping

of the eighth grouping is a seven elemental attribute loop (i.e. $D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mn}.D_{np}.D_{pi}$) and the subsystem evaluation index component ($D_q.D_r\ldots D_u$).

- Similarly, other terms of the expression are defined up to the thirteenths grouping. Each term of the grouping as well as the subgroupings have their own independent identities which are useful for the designers and the development analysts for one-to-one qualitative analysis of the Gas turbine systems.

Thus the permanent function characterizes a system for selected number of attributes as it contains all possible components of attributes and their relative importance.

IV. GAS TURBINE SYSTEM QUALITY INDEX

The gas turbine system quality index is a measure of the attributes that effects the quality of the subsystems in a typical gas turbine system to achieve the desired goals through the best matching performance of subsystems, working media, the system integration and operational strategy of the gas turbines by using technologically reliable and robust tools subjected to the constraints of minimum cost and time. The gas turbine system quality function as defined above in equation (2) is used for the evaluation of the gas turbine system quality index as it contains the presence of all the attributes and their relative importance. The numerical value of the gas turbine system quality function is called the gas turbine system quality index. As the gas turbine system quality function mentioned above contains only the positive terms, higher the value of inheritance level (i.e. D_i's) and/or the interdependency (i.e. aij's), higher will be the value of the gas turbine system quality function. In order to calculate, this index, the detailed information about 'D_i' and 'D_{ij}' is required.

The value of D_i is obtained from the knowledge database available in the form of experimental test results. This information may be qualitative or quantitative in nature. In case, the quantitative value is not available for the D_i, then a ranked value judgment on a scale i.e. 0 to 1 is adopted. Further, the information available for different D_i's may have difference nature and carry different kind of units. In such cases, it is desirable to normalize the quantitative values of the D_i's on the same scale as the qualitative values i.e. 0 to 1. Since the nature of the D_i's plays an important role during the process of normalization, it is advisable to categorize the benefit type and cost type attributes prior to normalization. An attribute can be considered as benefit type of attribute, if its positive variation results in increase of the permanent function and vice-versa. Similarly, an attribute can be considered as the cost type attribute, if its positive variation results in decrease in the value of the permanent function and vice-versa. For example, if an attribute is of benefit type i.e. increase or decrease in the attribute value contributes in the same sense as that of the objective or index of the problem then the assigned values (D_i's) within the limits of 0-1 are normalized using the relation:

$D_i = (10/D_{iu})*D_{ii}$ for $D_{ii}=0$ and

$D_i= \{10/(D_{iu}-D_{il})\}*(D_{ii}-D_{il})$ for $D_{ii}>0$ (3)

Where D_{il}= lowest range value of the attribute

D_{iu} : highest range value of the attribute

D_{ii}: value of the attribute (diagonal value in the matrix representation D(MxM), M is the order of the Matrix

However, if the attribute is of cost type, then the normalization of the attribute value is generally done over range of 0-1 by using the following relation:

$D_i = 10(1-D_{ii}/D_{iu})$ for $D_{ii}=0$ and

$D_i= \{10/(D_{iu}-D_{il})\}*(D_{iu}-D_{ii})$ for $D_{ii}>0$ (4)

where notations have their usual meanings. Table 1 suggests the equivalent value over a scale of 0-1 for the qualitative measure of an attribute.

TABLE 1: INHERITANCE NORMALIZED VALUES OF ATTRIBUTES

Qualitative measure of attributes	Assigned value of the attributes (Di)
Exceptionally low	0.05
Extremely low	0.1
Very low	0.2
Low	0.3
Below normal	0.4
Normal	0.5
Above normal	0.6
High	0.7
Very high	0.8
Extremely high	0.9
Exceptionally high	0.95

Similarly, the relative importance between the two characteristics or attributes is also assigned a value on a scale of 0-1 and is arranged into classes as mentioned in Table 2.

TABLE 2: RELATIVE IMPORTANCE OF ATTRIBUTES

Dependency effect of attribute 'j' on attribute 'i'	Assigned value of the attributes (D_{ij})
Exceptionally low	0.05
Extremely low	0.1
Very low	0.2
Low	0.3
Below normal	0.4
Normal	0.5
Above normal	0.6
High	0.7
Very high	0.8
Extremely high	0.9
Exceptionally high	0.95

Due to complexity of the system as a whole, it becomes infeasible to calculate the relative inter- dependency of one attribute over the other. However, for simplicity, a relationship has been suggested in the literature for such cases which assigns the relative importance of *i*th attribute over *j*th attribute and vice-versa as under:

$$\left.\begin{aligned} D_{ij} &= 1 - D_{ji} \\ D_{ji} &= 1 - D_{ij} \end{aligned}\right\} \qquad (5)$$

The gas turbine system quality index for each type of available gas turbine system is evaluated by using equation (2) by substituting the values of D_i's and D_{ij}'s. The gas turbine systems may be arranged in the ascending order or descending order of the evaluated gas turbine system quality index. The gas turbine system which has highest quality index is considered to be the best choice as per the qualitative standards. Since, the graph theoretic approach as adopted can incorporate more number of attributes during the modeling and matrix representation of the gas turbine systems; it becomes easier to upgrade such systems with modifications in the attribute constraints represented in matrix form.

V. IDENTIFICATION AND COMPARISON OF GAS TURBINE SYSTEMS

The gas turbine system quality function is useful for identification and comparison of gas turbine systems for a given set of quality ascertaining attributes. The number of terms in each grouping of the gas turbine system quality function for all kind of gas turbine systems for a given set of such attributes will be same. However, their values will be different. Two gas turbine systems may be similar from the qualitative aspects, if their system quality attribute digraphs are isomorphic. Two such digraphs are isomorphic, if they have identical permanent function matrix set representation. This means not only the number of terms in the groupings as well as subgrouping as same but also their values are also same. Based on this, system quality identification set of the gas turbine systems is represented as:

$$[(J_1^T /J_2^T /J_3^T /J_4^T /J_{51}^T /J_{52}^T /J_{61}^T /J_{62}^T /J_{71}^T /J_{72}^T /J_{73}^T /J_{74}^T /J_{81}^T /J_{82}^T /J_{83}^T /J_{84}^T \ldots J_{13,18}^T /J_{13,19}^T /J_{13,20}^T /J_{13,21}^T) (V_1^T /V_2^T /V_3^T /V_4^T /V_{51}^T /V_{52}^T /V_{61}^T /V_{62}^T /V_{71}^T /V_{72}^T /V_{73}^T /V_{74}^T /V_{81}^T /V_{82}^T /V_{83}^T /V_{84}^T \ldots V_{13,18}^T /V_{13,19}^T /V_{13,20}^T /V_{13,21}^T)] \quad (6)$$

where J_i^T represents the total number of terms in the '*i*th grouping, J_{ij}^T represents the total number of terms of the *J*th subgrouping in the *i*th grouping. Similarly, V_i^T represents the numerical value the '*i*th grouping and V_{ij}^T represents the numerical value of the *j*th subgrouping in the ith grouping. Numerical values of the D_i's and D_{ij}'s are substituted in the subgrouping or the grouping to obtain the permanent system quality index.

VI. METHODOLOGY

The graph theoretic approach evaluates the permanent qualitative index of a gas turbine system in terms of single numerical index, which takes into account the all qualitative measures and their interdependencies while analyzing and evaluating the system. The various steps of the proposed approach, which would be helpful in evaluation of the permanent qualitative index of the gas turbine systems, are enlisted in sequential manner as below:

1. Identify the various characteristics or the broad attributes of the gas turbine system which are responsible for defining the system quality as a whole. On the basis of application domain and the operational constraints, different types of gas turbine systems may have different characteristics or the attributes specifying its quality.
2. Classify the various characteristics or the attributes into clusters such that each cluster of attributes represents or exhibits a common set of characteristics of the system pertaining to its quality. These clusters may be treated as groupings and the constituent attributes of these clusters may be termed as factors or subsystems responsible for performance or response of the grouping.
3. Logically develop a diagraph between the factors as well as the broad attributes or the characteristics depending upon their interdependencies.
4. Develop a variable permanent function matrix at the system level on the basis of digraph developed in step 3.
5. Identify the subfactors affecting each factor.
6. For each factor, develop the digraph among the subfactors based on the interactions among them.
7. Develop a variable permanent matrix at the subsystem level for each factor on the basis of subfactors level digraphs developed in step 6.
8. Using the logical values of the quality measures as well as their interdependencies, obtain the permanent functions at the system as well as subsystem level. The off- diagonal elements of the matrix representation may be obtained from the graphs, knowledge database interpretation or from the excerpts of the expert's opinion.
9. Evaluate the permanent of the variable permanent function at the macro system level i.e. Gas turbine system using the permanent functions developed at system level. This permanent has been obtained by analyzing, retrieving and processing the qualitative data of the gas turbine systems without loosing any information as per the combinatorial practices of graph theory.
10. Various gas turbine systems can be compared on the basis of permanent system quality index thus obtained. Necessary improvement strategies may be implemented ahead for enhancing the quality of the gas turbine system.

VII. EXAMPLES

To demonstrate the proposed qualitative index evaluation of the gas turbine systems, two examples as below are considered:

A. Example 1

In this example, the graph theoretic based quality evaluation has been carried out for the qualitative attributes of the gas turbine system. The broad performance characteristics of the gas turbine system have been used to model and analyze the qualitative aspects of the system. A comparison has been made for the qualitative similarity and dissimilarity of the gas turbine systems has also been carried out. The various steps of the methodology adopted are described as below:

1. First, the various characteristics of the gas turbine system have been identified which decides the quality of the gas turbine system and various contributing factors that give the required characteristic of the gas turbine system. The

various characteristics as identified are same as mentioned in gas turbine system quality digraph representation. These are Functional Performance (FP), Operational Availability (OA), Serviceability (SV), Operational Flexibility (OF), Environmental Impact (EI), Customer Desire Conformance (CD), Aesthetics (AT), Perceived Quality (PQ), Durability (DR), Life Cycle Cost (LC), Operational Safety (OS), Feature and Design Conformance (FD).

level of interdependency between the attributes is qualitatively expressed over the normalized scale using the contributing sense of the attribute(s) or the Characteristic(s) towards the desired objective function for quality analysis. On the basis of the operational strategies and expert's opinions, the level of quality measures of the gas turbine system quality attributes as well as their dependencies have been specified in Table 3.

TABLE 3: QUALITY MEASURE AND DEPENDENCE OF GAS TURBINE SYSTEM ATTRIBUTES

Sr. No.	Quality attribute 'I' related to Gas turbine System performance	Quality measure of attribute 'I'	Attributes 'j' which effect the attribute 'I' as per dependency level mentioned below										
			Exceptionally Low	Extremely Low	Very Low	Low	Below Normal	Normal	Above normal	High	Very High	Extremely high	Exceptionally high
1	Functional performance (FP)	0.8		8	5, 7	10	6	2, 3	4, 9	11		12	
2	Customer Desire Conformance (CD)	0.6		7	8	4	3	10, 11	6	1, 9, 12		5	
3	Operational availability (OA)	0.7	7, 8	5	2	11	6, 10	9		1, 12	4		
4	Durability (DR)	0.7	5	7, 8		2, 11	10	6		9	12	1, 3	
5	Feature and Design Conformance (FD)	0.6					7, 8	4, 10	3, 6, 11, 12	9		2	1
6	Serviceability (SV)	0.6	8	5, 7	3, 4		11, 12	2	9		1	10	
7	Aesthetics (AT)	0.5	3, 4, 6, 8, 9, 11	1, 2, 5, 10							12		
8	Perceived Quality (PQ)	0.7	3, 4, 6, 7, 11	1, 2, 5, 9, 10					12				
9	Operating Flexibility (OF)	0.6	7, 11	5, 10	6, 8, 12		3	1		2, 4			
10	Operating Safety (OS)	0.6	7, 8	5, 9, 11, 12	4		3	1	2, 6				
11	Environmental Impact (EI)	0.5	7, 8, 10	3, 4, 5, 9		2, 12				6			1

2. During the characteristics based quality evaluation, each and every characteristics of the system acts as an attribute. Each characteristic itself represents a cluster of sub-attributes or the subfactors which contribute to the impact of that cluster or characteristic in the qualitative analysis of the gas turbine system. However, the criterion for approximating the inheritance of these characteristics towards qualitative evaluation of the said system is given in Table 1. Similarly, the qualitative dependence of the quality measures is also mentioned in Table 2.
3. In order to develop a logical digraph for all significant characteristics as gas turbine system quality attributes, a typical gas turbine system is studied from Macroscopic level to Microscopic level i.e. Gas turbine system level to subcomponent level and the interdependency of the attributes or such characteristics are analyzed. The probable

The graph theoretic representation of these quality characteristics or the attributes of the gas turbine system is shown in Figure 1.

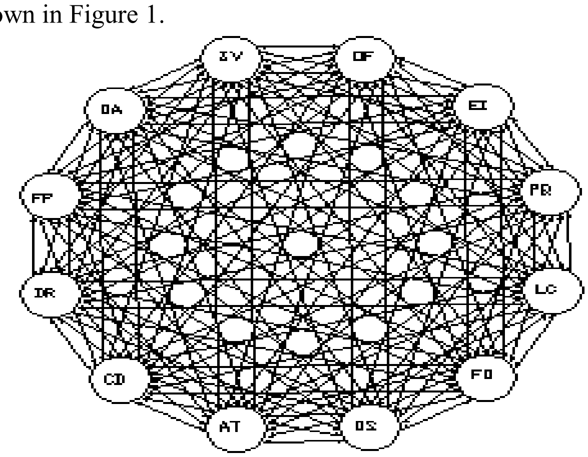

Fig.1: Digraph for Gas Turbine System Attributes

4. The digraph at the system level developed at step 3 is represented in matrix form, where the order of the matrix is the number of attributes responsible for quality characteristics of the system. The diagonal elements of this matrix are the inheritance values of the system quality characteristic attributes and the off- diagonal elements are the interdependency values of these attributes. Using Table 3 and equation (1), the equivalent matrix representation of the system quality characteristic diagraph is given as:

$$A=\begin{array}{cc} \mathit{attributes} & \begin{array}{cccccccccccc} FP & OA & SV & OF & EI & CD & AT & PQ & DR & LC & OS & FD \end{array} \\ \begin{array}{c} FP \\ OA \\ SV \\ OF \\ EI \\ CD \\ AT \\ PQ \\ DR \\ LC \\ OS \\ FD \end{array} & \begin{bmatrix} 0.8 & 0.5 & 0.5 & 0.6 & 0.2 & 0.4 & 0.2 & 0.1 & 0.6 & 0.3 & 0.7 & 0.9 \\ 0.7 & 0.6 & 0.4 & 0.3 & 0.9 & 0.6 & 0.1 & 0.2 & 0.7 & 0.5 & 0.5 & 0.7 \\ 0.7 & 0.2 & 0.7 & 0.8 & 0.1 & 0.4 & 0.05 & 0.05 & 0.5 & 0.4 & 0.3 & 0.7 \\ 0.9 & 0.3 & 0.9 & 0.7 & 0.05 & 0.5 & 0.1 & 0.1 & 0.7 & 0.4 & 0.3 & 0.8 \\ 0.95 & 0.9 & 0.6 & 0.5 & 0.6 & 0.6 & 0.4 & 0.4 & 0.7 & 0.5 & 0.6 & 0.6 \\ 0.8 & 0.5 & 0.2 & 0.2 & 0.1 & 0.6 & 0.1 & 0.05 & 0.6 & 0.9 & 0.4 & 0.4 \\ 0.1 & 0.1 & 0.05 & 0.05 & 0.1 & 0.05 & 0.5 & 0.05 & 0.05 & 0.1 & 0.05 & 0.8 \\ 0.1 & 0.1 & 0.05 & 0.05 & 0.1 & 0.05 & 0.05 & 0.7 & 0.1 & 0.1 & 0.05 & 0.6 \\ 0.5 & 0.7 & 0.4 & 0.7 & 0.1 & 0.2 & 0.05 & 0.1 & 0.6 & 0.1 & 0.05 & 0.1 \\ 0.5 & 0.6 & 0.4 & 0.2 & 0.1 & 0.6 & 0.05 & 0.05 & 0.1 & 0.6 & 0.1 & 0.1 \\ 0.95 & 0.3 & 0.1 & 0.1 & 0.1 & 0.7 & 0.05 & 0.05 & 0.1 & 0.05 & 0.5 & 0.3 \\ 0.95 & 0.8 & 0.6 & 0.5 & 0.1 & 0.7 & 0.7 & 0.95 & 0.5 & 0.3 & 0.2 & 0.7 \end{bmatrix} \end{array} \qquad (7)$$

5. The gas turbine system quality characteristics or the attributes are critically examined from the system level to its subsystem level (i.e. the component level) in the form of subfactors. These subfactors also contribute towards the achievement of maximum system quality through their characteristic normalized values placed in the system quality function matrix. The Table 4 enlists a number of such subfactors contributing towards impact of system characteristics:
6. These subfactors which affect the characteristic attributes have similar kind of inheritance as well as interdependency among each other as the quality characteristics of the gas turbine system have with respect to each other. The level of inheritance and interdependency of such subfactors is also evaluated using the same criteria as adopted for system attributes or quality characteristics.

In order to develop a logical digraph for a particular characteristic attribute, the rational interdependency of the subfactors affecting the system characteristic attribute is analyzed and a qualitative representation of such interdependencies is carried out using the same normalization techniques as used for the system characteristic attributes.

For example, in order to develop the digraph related to functional performance system characteristic attribute, all the significant subfactors such as equipment design philosophy (ED), operation philosophy (OP), Maintenance philosophy (MP), equipment upgrades and modernization (EM), personal performance (PP), Matching compatibility performance (MC) are analyzed for their inheritance measure and the interdependency measure. On the basis of design theories and operational strategies, the level of the quality measures of the attributes as well as their

TABLE 4: SUBFACTORS OF THE GAS TURBINE SYSTEM ATTRIBUTES

Sr. No.	Gas turbine System attributes	Subfactors affecting the gas turbine system attributes or the characteristics
1	Functional performance	(i). Equipment design (ii).Operation philosophy (iii). Maintenance philosophy (iv). Equipment upgrades and modernization (v). Personal performance- (skills, training, knowledge, motivation, organization, communication, cooperation) (vi). Matching compatibility performance
2	Operational availability	(i). reliability (ii). Maintainability (iii). Logistic support capability- (logistic support, chain, organization) (iv). Fuel quality (v). Comprehensive preventive maintenance dynamics (vi). Operating mode (vii). Control system technology (viii). Auxiliary system technology (ix). Personal performance (x). Thermodynamic loadings (xi). mechanical loadings
3	Serviceability	(i). Easability of installation and commissioning (ii). External failures average time (iii). Condition monitoring & checks frequency (iv). Start up frequency (v). Shut down frequency (vi) Reparability of turbine parts
4	Operational flexibility	(i). Fuel flexibility (ii). Lubrication flexibility (iii). Cooling flexibility, (iv). Regulatory mechanism response quality (v). Drive- mechanism flexibility (vi). Protection system reliability (vii). Inter-stage operation reliability (viii). Personal performance (ix). Control system design philosophy
5	Environmental impact	(i). No_x emission level (ii). Cooling technology effectiveness (iii). Combustor effectiveness (iv). Smoke level (v). Peak temperature of gas turbines (vi). Diluent's injection fraction effectiveness (vii). Air contamination level (viii). Fuel characteristics (ix). Air noise level
6	Durability	(i). Endurance strength (ii). Loading effect resistance (iii). Upgradeability etc.
7	Customer desire conformance	(i). Combined cycle efficiency (ii). No_x emission level (iii). Specific cost (iv). Simple cycle efficiency (iv). Operating flexibility (v). Reliability and availability
8	Aesthetics	(i). Shape and size standardization feasibility (ii). Operational colour coding feasibility (iii). System environment compatibility (iv). Logistic standards implementation feasibility (v). Ergonomic design, (vi). Finish and surface texture level
9	Perceived quality	(i). Reputation of product (ii). Impact of advertising (iii). Philosophy, (iv). Business strategy (v). Marketing strategy,
10	Life cycle cost	(i). Design cost (ii). Production cost (iii). Capital cost (iv). Operation and maintenance cost (v). Fuel cost
11	Operational safety	(i). Identification of faults (ii). Accessibility to subcomponents, (iii). Availability of troubleshooting database (iv). Ergonomic design, (v). Controllability of operations (vi). Protection system robustness, (vii). Subsystem integration feasibility (viii). functional invariability
12	Design specification conformance	(i). Cooling technology adaptation (ii). Firing temperature peak-ability (iii). Operating pressure ratio capability (iv). Material technology adaptation (v). Mass flow rate of working media

TABLE 5: INHERITANCES AND DEPENDENCE OF QUALITY ATTRIBUTES RELATED TO FUNCTIONAL PERFORMANCE

Sr. No.	Quality attribute 'I' related to Functional Performance	Quality measure of attribute 'I'	Attributes 'j' which effect the attribute 'I' as per dependency level mentioned below										
			Exceptionally Low	Extremely Low	Very Low	Low	Below Normal	Normal	Above normal	High	Very High	Extremely high	Exceptionally high
1	Equipment design philosophy (ED)	0.8				2	6	4	3		5		
2	Operation philosophy (OP)	0.5				5	4	3	1, 6				
3	Maintenance philosophy (MP)	0.6		1		4	2	5		6			
4	Equipment upgrades and Modernization (EM)	0.6		6	2		3	5	1				
5	Personal performance (PP)	0.7		1, 6	2, 4	3							
6	Matching compatibility performance (MC)	0.9		3	4, 5						2	1	

The graph theoretic representation of the specified system characteristic attribute i.e. functional performance in terms of its subfactors, their inheritance and interdependencies have been represented as:

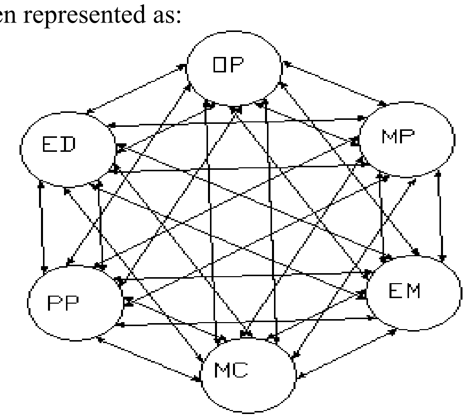

Fig. 2: Digraph for Functional Performance Attributes of Gas Turbine System

7. Using the same technique as adopted for the system quality function matrix, the digraph at the subsystem level for all the system characteristic attributes are also represented in matrix form, where the order of the matrix is the number of sub-attributes responsible for quality characteristics of the subsystem. The equivalent subsystem characteristic permanent matrix for the Functional performance characteristic diagraph is represented as:

$$A = \begin{array}{c} attributes \\ ED \\ OP \\ MP \\ EM \\ PP \\ MC \end{array} \begin{array}{c} \begin{array}{cccccc} ED & OP & MP & EM & PP & MC \end{array} \\ \begin{bmatrix} 0.8 & 0.3 & 0.6 & 0.5 & 0.8 & 0.4 \\ 0.6 & 0.5 & 0.5 & 0.4 & 0.3 & 0.6 \\ 0.1 & 0.4 & 0.6 & 0.3 & 0.5 & 0.7 \\ 0.6 & 0.2 & 0.4 & 0.6 & 0.5 & 0.1 \\ 0.1 & 0.2 & 0.3 & 0.2 & 0.7 & 0.1 \\ 0.9 & 0.8 & 0.1 & 0.2 & 0.2 & 0.9 \end{bmatrix} \end{array} \qquad (8)$$

Similar attempt may be made for the representation of other subsystem matrices for the system quality attributes of the gas turbine systems.

8. Since the variable permanent function matrix for the functional performance attribute is of the order of 6, the permanent for this function will contain 6! terms and is represented as :

$$\begin{aligned} Per(A) = & \prod_{1}^{6} V_i + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{ji}).D_k.D_l.D_m.D_n \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{jk}.a_{ki}).D_l.D_m.D_n \\ & + \{\sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{ji}).(a_{kl}.a_{lk}).D_m.D_n \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{jk}.a_{kl}.a_{li}).D_m.D_n \} \\ & + \{ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{kl}.a_{lm}.a_{mk}).(a_{ij}.a_{ji}).D_n \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{jk}.a_{kl}.a_{lm}.a_{mi}).D_n \} \\ & + \{\sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{ji}).(a_{kl}.a_{lk}).(a_{mn}.a_{nm}) \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{ji}).(a_{kl}.a_{lm}.a_{mn}.a_{nk}) \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{jk}.a_{ki}).(a_{lm}.a_{mn}.a_{nl}) \\ & + \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n (a_{ij}.a_{jk}.a_{kl}.a_{lm}.a_{mn}.a_{ni})\} \qquad (9) \end{aligned}$$

The permanent of the above subsystem permanent function is a numerical value and is responsible for judgment of inheritance factors of the main system quality attribute i.e. the functional performance. Similarly, the permanent related to other subsystem matrices are also calculated. In present case, the permanent value as calculated for the functional performance of the gas turbine subsystem matrix is 5.145329 for a given set of attribute normalized set.

TABLE 6: QUALITY MEASURES AND DEPENDENCE RELATED TO CUSTOMER DESIRE –FUNCTIONAL REQUIREMENT RELATIONSHIPS

Sr. No.	Gas turbine system quality Attribute description	Quality measure of attribute 'I'	Attributes 'j' which effect the attribute 'I' as per dependency level mentioned below: Exceptionally Low	Extremely Low	Very Low	Low	Below Normal	Normal	Above normal	High	Very High	Extremely high	Exceptionally high
Case- I : Quality attribute 'I' related to customer desire conformance													
1	High Combined cycle efficiency (HC)	0.9			5	7	4, 6	3					2
2	Low Nox emissions (NX)	0.8	1			5	4, 6, 7	3					
3	Low Specific Costs (SC)	0.6			5	4, 6, 7		1, 2					
4	High Availability (HA)	0.5				5, 7	6		1, 2	3			
5	Operating Flexibility (OF)	0.5					6, 7			2, 4	1, 3		
6	High Reliability (HR)	0.7				7			1, 2, 4, 5	3			
7	High Simple cycle efficiency (SE)	0.6							2, 5	1, 3, 4, 6			
Case -II: Quality attribute 'I' related to Functional requirements													
1	Cooling Technology (CT)	0.9	2						3	4, 5			
2	Firing Temperature (FT)	0.8						4	3, 5			1	
3	Pressure Ratio (PR)	0.7					2	1	5	4			
4	Materials Technology (MT)	0.6				1, 3		2, 5					
5	Mass flow of the working media (MF)	0.4				1	2	3, 4					
Case – III: Quality attribute 'I' related to customer desire- Functional requirement relationships													
1	High Combined cycle efficiency (HC)	0.9		10, 12	5	7, 8, 11	4, 6	3				9	2
2	Low Nox emissions (NX)	0.8	1	11, 12		5, 10	4, 6, 7	3				8, 9	
3	Low Specific Costs (SC)	0.6			5	4, 6, 7, 8, 9, 10, 11		1, 2				12	
4	High Availability (HA)	0.5		10, 12		5, 7, 8, 9, 11	6		1, 2	3			
5	Operating Flexibility (OF)	0.5		9, 10, 12		8, 11	6, 7			2, 4	1, 3		
6	High Reliability (HR)	0.7		10, 12		7, 8, 9, 11			1, 2, 4, 5	3			
7	High Simple cycle efficiency (SE)	0.6		9, 11, 12		8			2, 5	1, 3, 4, 6		10	
8	Cooling Technology (CT)	0.9	9	2					10	1, 3, 4, 5, 6, 7, 11,12			
9	Firing Temperature(FT)	0.8		1, 2				11	10, 12	3, 4, 6		5, 7, 8	
10	Pressure Ratio (PR)	0.7		7			9	8	12	2, 3, 11		1, 4, 5, 6	
11	Materials Technology (MT)	0.6				8, 10		9, 12		1, 3, 4, 5, 6		2, 7	
12	Mass flow of the working media (MF)	0.4		3		8	9	10, 11				1, 2, 4, 5, 6, 7	

9. After normalizing and evaluating the weightage of inheritance of system characteristic attributes on the basis of their permanents obtained from the respective governing subsystem permanent functions, the permanent of the system characteristic quality function matrix is obtained. Since, a complex level of interdependency exists within the system-system as well as subsystem-subsystem attributes, a through understanding of the entire system is desirable including the expert's advice. At present, the normalized values of the inheritances have been taken as diagonal elements of the gas turbine system quality function matrix developed earlier at step 5. The value of the permanent thus obtained is called gas turbine system quality Index and is to be used further for the comparison of various gas turbine systems quality.
10. The identification set for the gas turbine system quality functions is represented as below:

$[(J_1^T /J_2^T /J_3^T /J_4^T /J_{51}^T /J_{52}^T /J_{61}^T /J_{62}^T /J_{71}^T /J_{72}^T /J_{73}^T /J_{74}^T /J_{81}^T /J_{82}^T /J_{83}^T /J_{84}^T \ldots J_{13,18}^T /J_{13,19}^T /J_{13,20}^T /J_{13,21}^T)\ (V_1^T /V_2^T /V_3^T /V_4^T /V_{51}^T /V_{52}^T /V_{61}^T /V_{62}^T /V_{71}^T /V_{72}^T /V_{73}^T /V_{74}^T /V_{81}^T /V_{82}^T /V_{83}^T /V_{84}^T \ldots V_{13,18}^T /V_{13,19}^T /V_{13,20}^T /V_{13,21}^T)]$: [(1/ 0/ 66/ 440/ 1485/ 2970/ 15940/ 19008/ 13860/ 83160/ 36960/ 110880/ 166320/ 399168/ 332640/ 570240/ ….terms in higher order subgroupings)(0.0037340357/ 0/ 0.1089119/ 0.3355173/ 1.009191/ 1.275905/ 4.623179/ 4.369334/ 3.554557/ 12.83652/ 4.29211/ 13.11733/ 16.65007/ 30.5394/ 19.08051/ 32.87354/ … sum of higher order subgrouping terms)] (10)

where J_i^T represents the total number of terms in the '*i*th grouping, J_{ij}^T represents the total number of terms of the *j*th subgrouping in the *i*th grouping. Similarly, V_i^T represents the numerical value the '*i*th grouping and V_{ij}^T represents the numerical value of the *j*th subgrouping in the *i*th grouping.

B. Example 2

In this example, interdependency of the functional requirements of the gas turbine systems and their inheritances have been analyzed and used to evaluate the gas turbine subsystem quality perception index based on the inheritance and interdependency of functional requirements and the customer desires of the gas turbine systems.

It is worth to mention here that the customer is never interacted directly with the functional requirements or the design specifications, but all his desires are represented with best compromised solution in terms of the functional requirements. It is only the functional requirement, on the basis of which entire structure of system quality is represented for such systems.

(i). [8] provides detailed information on the preference of the customer desires as well as the functional requirements along with their level of interdependence. The preference ratings for both the customer desires and the functional requirements have been evaluated on the scale as used to represent the level of interdependency of the system level attributes. On the basis of operational philosophy, the interrelationships among the Customer desires and the Functional requirements for the best compromised solution, has been calculated as in Table 6.

(ii). Using the procedural steps of the proposed methodology as used in Example 1, the three different digraphs have been modeled to represent the interdependency of the customer desire and the functional requirements evaluated on the same logical scale as used for the system attributes quality function. These digraphs have been shown in Figure (3), (4) and (5).

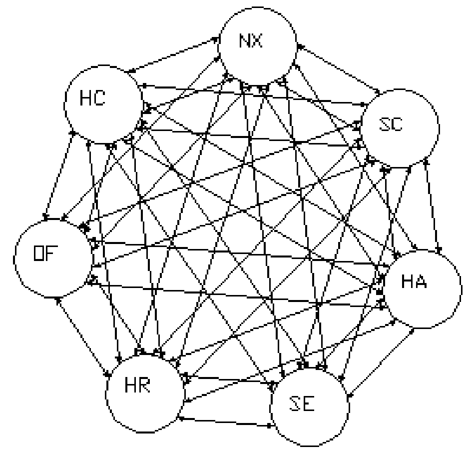

Fig. 3: Digraph Representation for Customer Desire (CD) Conformance

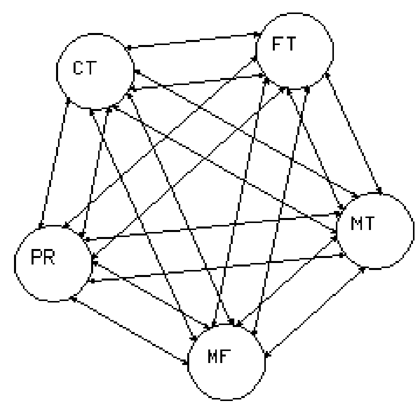

Fig. 3: Digraph Representation for Functional Requirement (FR) Conformance

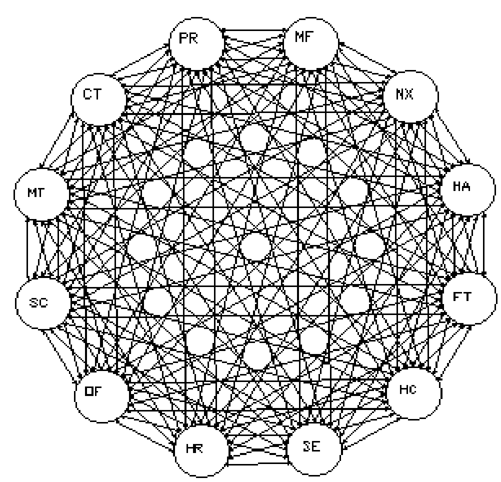

Fig. 5: Digraph for Combined Desire-requirement Attributes Relationships

(iii). On the basis of the qualitative measures and the interdependencies of the attributes related to Customer desire (CD) and the Functional requirement (FR) of the gas turbine system, the qualitative effect of the subfactors affecting the CD and FR attributes are identified and their individual effects of quality measures and interdependency among the subfactors are identified and evaluated by using the same normalized scale as used in the Example 1. using the quality measures of the interaction among the subfactors and the digraph developed, the system quality matrix for each case i.e. the Customer desire attributes, Functional requirement attributes and the Customer desire- Functional requirement relationships are developed which may be used further for the calculation of permanent functions of the system matrices to analyze the effects of present subgrouping with in the system equations. For the three cases as mentioned above, the system matrices are represented as:

- **Case-I**: Effect of attributes related to Customer desire on the quality characteristics of the gas turbine system

$$A^1 = \begin{array}{cc} \text{attributes} & \begin{array}{ccccccc} HC & NX & SC & HA & OF & HR & SE \end{array} \\ \begin{array}{c} HC \\ NX \\ SC \\ HA \\ OF \\ HR \\ SE \end{array} & \begin{bmatrix} 0.9 & 0.95 & 0.5 & 0.4 & 0.2 & 0.4 & 0.3 \\ 0.05 & 0.8 & 0.5 & 0.4 & 0.3 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0.6 & 0.3 & 0.2 & 0.3 & 0.3 \\ 0.6 & 0.6 & 0.7 & 0.5 & 0.3 & 0.4 & 0.7 \\ 0.8 & 0.7 & 0.8 & 0.7 & 0.5 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.7 & 0.6 & 0.6 & 0.7 & 0.3 \\ 0.7 & 0.6 & 0.7 & 0.7 & 0.6 & 0.7 & 0.6 \end{bmatrix} \end{array} \qquad (11)$$

- **Case-II**: Effect of attributes related to Functional requirements on the quality characteristics of the gas turbine system

$$A^2 = \begin{array}{cc} \text{attributes} & \begin{array}{ccccc} CC & FT & PR & MT & MF \end{array} \\ \begin{array}{c} CC \\ FT \\ PR \\ MT \\ MF \end{array} & \begin{bmatrix} 0.9 & 0.05 & 0.6 & 0.7 & 0.7 \\ 0.9 & 0.8 & 0.6 & 0.5 & 0.6 \\ 0.5 & 0.4 & 0.7 & 0.7 & 0.6 \\ 0.3 & 0.5 & 0.3 & 0.6 & 0.5 \\ 0.3 & 0.4 & 0.5 & 0.5 & 0.4 \end{bmatrix} \end{array} \qquad (12)$$

- **Case-III:** Effect of attributes related to combined Customer desire & Functional requirements on the quality characteristics of the gas turbine system

$$A^3 = \begin{array}{cc} \text{attributes} & \begin{array}{cccccccccccc} HC & NX & SC & HA & OF & HR & SE & CC & FT & PR & MT & MF \end{array} \\ \begin{array}{c} HC \\ NX \\ SC \\ HA \\ OF \\ HR \\ SE \\ CC \\ FT \\ PR \\ MT \\ MF \end{array} & \begin{bmatrix} 0.9 & 0.95 & 0.5 & 0.4 & 0.2 & 0.4 & 0.3 & 0.3 & 0.9 & 0.1 & 0.3 & 0.1 \\ 0.05 & 0.8 & 0.5 & 0.4 & 0.3 & 0.4 & 0.4 & 0.9 & 0.9 & 0.3 & 0.1 & 0.1 \\ 0.5 & 0.5 & 0.6 & 0.3 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.9 \\ 0.6 & 0.6 & 0.7 & 0.5 & 0.3 & 0.4 & 0.7 & 0.3 & 0.3 & 0.1 & 0.3 & 0.1 \\ 0.8 & 0.7 & 0.8 & 0.7 & 0.5 & 0.4 & 0.4 & 0.3 & 0.1 & 0.1 & 0.3 & 0.1 \\ 0.6 & 0.6 & 0.7 & 0.6 & 0.6 & 0.7 & 0.3 & 0.3 & 0.3 & 0.1 & 0.3 & 0.1 \\ 0.7 & 0.6 & 0.7 & 0.7 & 0.6 & 0.7 & 0.6 & 0.3 & 0.1 & 0.9 & 0.05 & 0.1 \\ 0.7 & 0.1 & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 & 0.9 & 0.05 & 0.6 & 0.7 & 0.7 \\ 0.1 & 0.1 & 0.7 & 0.7 & 0.9 & 0.7 & 0.9 & 0.9 & 0.8 & 0.6 & 0.5 & 0.6 \\ 0.9 & 0.7 & 0.7 & 0.9 & 0.9 & 0.9 & 0.1 & 0.5 & 0.4 & 0.7 & 0.7 & 0.6 \\ 0.9 & 0.7 & 0.7 & 0.7 & 0.7 & 0.7 & 0.9 & 0.3 & 0.5 & 0.3 & 0.6 & 0.5 \\ 0.9 & 0.9 & 0.1 & 0.9 & 0.9 & 0.9 & 0.9 & 0.3 & 0.4 & 0.5 & 0.5 & 0.4 \end{bmatrix} \end{array} \qquad (13)$$

(iv). The variable permanent function of the subsystem quality matrices are expanded into the form of terms containing various subgroupings within the groupings with no self contained loops. The values of the permanent of each of the matrices will be a numerical index and is termed as perception index of the respective subsystem contributing towards system quality. For example, for the present case, the values of the permanents calculated for case-I and case-II are 47.33942 and 5.250910 respectively and may be used as reference parameter for comparison of isomorphic cases.

(v). Using equation (6) , the concise representation of the identification set for each of the cases mentioned above for the system matrices reflects the significance of various groupings and the subgrouping because of affect of quality measures and interdependency of other attributes on the system quality. These identification sets are:

Case-I:

$[(J_1^T/J_2^T/J_3^T/J_4^T/J_{51}^T/J_{52}^T/J_{61}^T/J_{62}^T/J_{71}^T/J_{72}^T/J_{73}^T/J_{74}^T/J_{81}^T/J_{82}^T/J_{83}^T/J_{84}^T)\ (V_1^T/V_2^T/V_3^T/V_4^T/V_{51}^T/V_{52}^T/V_{61}^T/V_{62}^T/V_{71}^T/V_{72}^T/V_{73}^T/V_{74}^T/V_{81}^T/V_{82}^T/V_{83}^T/V_{84}^T)]$: [(1/ 0/ 21/ 70/ 105/ 210/ 420/ 504/ 105/ 630/ 280/ 840/ 210/ 504/ 420/ 720/)(0.04536/ 0/ 0.5544579/ 1.502965/ 1.523246/ 3.440350/ 4.887028/ 6.273170/ 0.7901559/ 5.477227/ 2.611651/ 7.830887/ 1.252369/ 3.247791/ 2.919071/ 5.003694)] (14)

Case-II:

$[(J_1^T/J_2^T/J_3^T/J_4^T/J_{51}^T/J_{52}^T/J_{61}^T/J_{62}^T)\ (V_1^T/V_2^T/V_3^T/V_4^T/V_{51}^T/V_{52}^T/V_{61}^T/V_{62}^T)]$:[(1/ 0/ 10/ 20/ 15/ 30/ 20/ 24) (0.12096/ 0/ 0.7044/ 1.17078/ 0.527925/ 1.336945/ 0.5849651/ 0.8049351)] (15)

Case- III:

$[(J_1^T/J_2^T/J_3^T/J_4^T/J_{51}^T/J_{52}^T/J_{61}^T/J_{62}^T/J_{71}^T/J_{72}^T/J_{73}^T/J_{74}^T/J_{81}^T/J_{82}^T/J_{83}^T/J_{84}^T$…………… $J_{13,18}^T/J_{13,19}^T/J_{13,20}^T/J_{13,21}^T)\ (V_1^T/V_2^T/V_3^T/V_4^T/V_{51}^T/V_{52}^T/V_{61}^T/V_{62}^T/V_{71}^T/V_{72}^T/V_{73}^T/V_{74}^T/V_{81}^T/V_{82}^T/V_{83}^T/V_{84}^T$…… $V_{13,18}^T/V_{13,19}^T/V_{13,20}^T/V_{13,21}^T)]$:[(1/ 0/ 66/ 440/ 1485/ 2970/ 15940/ 19008/ 13860/ 83160/ 36960/ 110880/ 166320/ 399168/ 332640/ 570240/) (0.005486746/ 0/ 0.1646656/ 1.089271/ 1.653982/ 5.639472/ 17.43588/ 27.44607/ 6.75295/ 69.69619/ 40.13968/ 121.4445/ 79.79746/ 252.4408/ 274.5477/ 472.1742/ higher order terms)] (16)

Since, the representation of these sets are dynamic in nature, effect of any subgrouping on the attribute based system quality Index can be studied and interpreted for further analysis.

VIII. CONCLUSION

In the present work, a methodology for evaluation of quality of a Gas Turbine system is proposed. This is based on digraph and matrix methods by considering the significant characteristics of the gas turbine. The twelve characteristics which parameterized the quality of the gas turbine system are identified. The graph theoretical methodology consists of gas turbine system quality digraph, gas turbine system quality matrix and the gas turbine system quality function. The gas turbine system quality function is a realistic mathematical quality model. The matrix functions are representative function of the quality and are useful for the characterization, comparison, analysis and evaluation of the quality of such systems. The quality characterization and isomorphism are used for the comparison of the gas turbine systems based on quality. The gas turbine system quality index, which is the permanent value of the gas turbine system quality matrix, is identified. The step by step procedure for the gas turbine system quality evaluation is explained considering different cases. The methodology is explained with two typical cases of examples. This procedure is not only useful for designers in the development of reliable and robust gas turbine systems but also to diagnose the failures of such systems. This methodology has been appreciated by the experts of domain as a self assessment tool, since this tool provides sufficient information at various levels for analyzing such cases.

APPENDIX

The permanent function given by equation (2) is represented as below:

$$\textbf{Per (A)} = \prod_{1}^{12} V_i$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).D_k.D_l.D_m.D_n.D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{ki}).D_l.D_m.D_n.D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \{ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{kl}.D_{li}).D_m.D_n.D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).(D_{kl}.D_{lk}).D_m.D_n.D_p.D_q.D_r.D_s.D_t.D_u \}$$

$$+ \{ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{ki}).(D_{lm}.D_{ml}).D_n.D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mi}).D_n.D_p.D_q.D_r.D_s.D_t.D_u \}$$

$$+ \{ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).(D_{kl}.D_{lk}).(D_{mn}.D_{nm}).D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).(D_{kl}.D_{lm}.D_{mn}.D_{nk}).D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{ki}).(D_{lm}.D_{mn}.D_{nl}).D_p.D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mn}.D_{ni}).D_p.D_q.D_r.D_s.D_t.D_u \}$$

$$+ \{ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).(D_{kl}.D_{lk}).(D_{mn}.D_{np}.D_{pm}).D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{ji}).(D_{kl}.D_{lm}.D_{mn}.D_{np}.D_{pk}).D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{ki}).(D_{lm}.D_{mn}.D_{np}.D_{pl}).D_q.D_r.D_s.D_t.D_u$$

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mn}.D_{np}.D_{pi}).D_q.D_r.D_s.D_t.D_u \} \ldots\ldots\ldots\ldots$$

+ up to 21st subgrouping of 13th grouping i.e.

$$+ \sum_i\sum_j\sum_k\sum_l\sum_m\sum_n\sum_p\sum_q\sum_r\sum_s\sum_t\sum_u (D_{ij}.D_{jk}.D_{kl}.D_{lm}.D_{mn}.D_{np}.D_{pq}.D_{qr}.D_{rs}.D_{st}.D_{tu}.D_{ui}) \}$$

all random counts i, j, k, l, m, n, p, q, r, s, t, u are integers

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