

# Second Law Analysis of Nanofluid Flow in Coiled Tube under Constant Heat Flux

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**Abstract**— This paper analytically examines the effects of adding nanoparticle on the entropy generation of water- $\text{Al}_2\text{O}_3$  nanofluid flows through a coiled tube under uniform wall heat flux condition in turbulent regime. It is found that adding nanoparticles improves the thermal performance of water- $\text{Al}_2\text{O}_3$  flow with Re numbers less than 40,000. The pumping power to heat transfer rate ratio by adding nanoparticles in constant  $\delta$  and different  $\delta$  investigated. Moreover optimum conditions (based on the entropy generation sense) for turbulent nanofluid flows are obtained.

**Keywords**— Coiled Tube, Nanofluid, Optimization and Second Low Analysis

## I. INTRODUCTION

When metallic or non-metallic particles of higher thermal conductivity, whose dimensions are less than 100 nm, are dispersed in conventional heat transfer liquids, the effective thermal conductivity of the resulting medium increases substantially. Masuda et al. [1] showed that the thermal conductivity and the viscosity of liquids are altered dramatically by dispersing ultra-fine particles of  $\alpha$ -aluminum oxide ( $\text{Al}_2\text{O}_3$ ), silicon dioxide ( $\text{SiO}_2$ ) and titanium dioxide ( $\text{TiO}_2$ ). Subsequently, this finding was conclusively established from experiments of other researchers; notably, Choi [2], Wang et al. [3] and Eastman et al. [4]. For the same Nusselt number of fluid flow in a given flow passage, if the thermal conductivity increases then the convective heat transfer also increases in the same proportion. Nanofluids have valuable applications in the area of heating buildings through the hydronic coils, cooling automotive engines through the radiators and in heat exchangers in all types of industries. In all these applications the fluid flow is generally in the turbulent regime, because higher heat transfer is achieved through the turbulent flow.

Li and Xuan [5] and Xuan and Li [6] investigated experimentally the convective heat transfer and flow characteristics for Cu-water nanofluid flowing through a straight tube with a constant heat flux under laminar and turbulent flow conditions. Cu nanoparticles with diameters below 100 nm were used in their study. The results of the experiment showed that the suspended nanoparticle remarkably enhanced the heat transfer performance of the conventional base fluid and their friction factor coincided well with that of the water. Furthermore, they also proposed the new convective heat transfer correlations for prediction of the

heat transfer coefficients of the nanofluid for both laminar and turbulent flow conditions. Das et al. [7] have investigated the increase of thermal conductivity with temperature for nanofluids with water as the base fluid and nanoparticles of  $\text{Al}_2\text{O}_3$  or CuO as the suspension material using the temperature oscillation technique.

Maiga et al. [8] have presented numerical results for laminar and turbulent convective heat transfer of nanofluids through a uniformly heated tube using the Fluent code. They investigated both water- $\text{Al}_2\text{O}_3$  and ethylene glycol- $\text{Al}_2\text{O}_3$  nanofluid flows and they found that the inclusion of nanoparticles to a base fluid increases the wall-heat-transfer and shear stress in both laminar and turbulent regimes.

Pak and Cho [9] conducted experiments to determine the heat transfer coefficient in pipe flow and viscosity for water- $\text{Al}_2\text{O}_3$  and water- $\text{TiO}_2$  nanofluids. Their findings were that Nusselt number correlations tended to increase with increasing particle concentration and Reynolds number. However, the nanofluids tested had lower Nusselt numbers than water at equal velocity conditions. Viscosity for the tested nanofluids was substantially higher than that for water. Temperatures were not reported for the Nusselt number and heat transfer coefficient experiments. Williams et al. [10] experimentally determined Nusselt numbers for water- $\text{Al}_2\text{O}_3$  and water- $\text{ZrO}_2$  nanofluids under turbulent flow conditions, and had similar findings.

Many researchers have studied the entropy generation of thermal systems to find their optimum design condition. EGM (entropy generation minimization) is the method of modeling and optimization of the devices accounting for both heat transfer and fluid flow irreversibilities. For example Ko and Ting [11] have applied this concept to find the most appropriate flow conditions of a fully developed, laminar forced convection flow through a helical coil tube for which entropy generation is minimized.

The aim of present paper the entropy generation and pumping power of nanofluid flow is computed and the optimum condition for flow parameters is specified. The entropy generation and pumping power of water- $\text{Al}_2\text{O}_3$  nanofluid flow through a coiled tube under constant wall heat flux condition is analytically investigated. nanofluid flow is studied in turbulent regime and nanoparticles volume concentration up to 4% is considered.

II. GEOMETRY OF COILED TUBE

A coiled tube has been shown in Fig. 1. In this figure,  $d/2$  is inner radius of the tube and  $D/2$  is curvature radius of the coil, and  $b$  is the coil pitch. The curvature ratio,  $\delta$ , is defined as the coil-to-tube radius ratio,  $d/D$ . The other three important dimensionless parameters namely, Reynolds number ( $Re$ ), Nusselt number ( $Nu$ ), and Dean number ( $Dn$ ) are defined as follow.

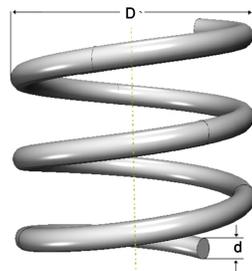


Fig. 1. Geometry of the coiled tube

$$Re = \frac{\rho U d}{\mu}, Nu = \frac{h d}{k}, Dn = Re \left( \frac{d}{D} \right)^{0.5} \tag{1}$$

Where,  $U$  and  $h$  are average velocity and convective heat transfer coefficient respectively.

III. ENTROPY GENERATION ANALYSIS

Taking the coil passage of length  $dx$  as the thermodynamic system, the first and second laws can be expressed as

$$\dot{m} dh = q' dx \tag{2}$$

$$\dot{S}'_{gen} = \dot{m} \frac{ds}{dx} - \frac{q'}{T + \Delta T} \tag{3}$$

where  $\dot{m}$ ,  $q'$  and  $\dot{S}'_{gen}$  are the mass flow rate in the coiled tubes, the heat transfer rate and the entropy generation rate per unit coil length, respectively. By using the thermodynamic relation

$$T ds = dh - v dp \tag{4}$$

$\dot{S}'_{gen}$  can be written as

$$\dot{S}'_{gen} = \frac{q' \Delta T}{T^2 \left( 1 + \frac{\Delta T}{T} \right)} + \frac{\dot{m}}{T \rho} \left( - \frac{dp}{dx} \right) \tag{5}$$

The pressure drop is [12]

$$dp = - \frac{f \rho U^2}{2d} dx \tag{6}$$

Based on the relationship between friction factor  $f$  and pressure drop, and the heat transfer coefficient  $\bar{h}$  and Nusselt number,  $\dot{S}'_{gen}$  can be expressed by

$$\dot{S}'_{gen} = \frac{q' \Delta T}{T^2 \pi Nu k + T q'} + \frac{32 \dot{m}^3 f}{T \rho^2 d^5 \pi^2} \tag{7}$$

The only difference between the final form and the derivation of Bejan [13] is that the  $\Delta T/T$  term in Eq. (5) has been retained in the present derivation for accuracy, although the effect of the term may not be significant since its value is relatively minor when  $\Delta T$  is much smaller than  $T$ . The non-dimensional entropy generation number  $N_S$  [13, 14] is defined as  $\dot{S}'_{gen} / (q'/T)$  and can be determined from Eq. (7) as

$$N_S = (N_S)_T + (N_S)_P \tag{8}$$

Where

$$(N_S)_T = \frac{1}{Nu \eta_1 + 1} \tag{9}$$

$$(N_S)_P = \frac{f Re^5}{\eta_2} \tag{10}$$

$\eta_1$  and  $\eta_2$  are two dimensionless duty parameters, defined as

$$\eta_1 = \frac{\pi k T}{q'} \tag{11}$$

$$\eta_2 = \frac{32 \dot{m}^2 \rho^2 q'}{\mu^5 \pi^3} \tag{12}$$

In turbulent flow the Dean number does not correlate the flow measurements data well, whereas Reynolds number can be used as a model parameter to predict the friction factor accurately. The friction factor in the turbulent regime is given by Mori and Nakayama [15]:

$$f = \frac{64}{Re} \left( Re \delta^2 \right)^{\frac{1}{20}} \tag{13}$$

Xin and Ebadin [16] measured the average Nusselt number using constant heat flux boundary condition and reported the following correlation for Nusselt number in the turbulent regime:

$$Nu = 0.00619 Re^{0.92} Pr^{0.4} (1 + 3.455 \delta) \tag{14}$$

IV. PUMPING POWER TO HEAT TRANSFER RATIO

The ratio of pumping power to heat transfer rate is

$$P_r = \frac{\pi d^2}{4} \frac{\Delta P U}{\left( \frac{q'}{\pi d h} \right)} \tag{15}$$

Using Eq. (6), the pumping power to heat transfer ratio  $Pr$  is

$$Pr = \frac{1}{8} f \frac{Ec}{St} \tag{16}$$

Where the Eckert number and Stanton number is defined as

$$Ec = \frac{U^2}{C_p \left( \frac{q'}{\pi d h} \right)} \tag{17}$$

$$St = \frac{h}{\rho U C_p} \tag{18}$$

**V. THERMOPHYSICAL PROPERTIES OF NANOFLUID**

Assuming small temperature variations the thermophysical properties (density, specific heat, viscosity and thermal conductivity) of the nanofluid may be calculated as a function of nanoparticle volume concentration ( $\phi$ ), base fluid and nanoparticles properties. Using the general formula for the mixtures, the following equation can be obtained to evaluate the density of nanofluid:

$$\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_p \tag{19}$$

Where indices ‘‘p’’, ‘‘bf’’ and ‘‘nf’’ refer to particle, base fluid, and nanofluid respectively. As mentioned in Buongiorno [17], assuming that the nanoparticles and the base fluid are in thermal equilibrium, the nanofluid specific heat is derived from:

$$Cp_{nf} = (1 - \phi)Cp_{bf} + \phi Cp_p \tag{20}$$

These equations, which are based on the physical principle of the mixture rules, have been found appropriate for use with nanofluids through experimental validation by Pak and Cho [9] and Xuan and Roetzel [18].

Viscosity and thermal conductivity of water- $Al_2O_3$  nanofluid are evaluated by the model developed by Maiga et al. [19] based on experimental works of Masuda et al. [20], Lee et al. [21] and Choi et al. [22]. For water- $Al_2O_3$  it was proposed:

$$\mu_{nf} = (123\phi^2 + 7.3\phi + 1)\mu_{bf} \tag{21}$$

$$k_{nf} = (4.97\phi^2 + 2.72\phi + 1)k_{bf} \tag{22}$$

In these equations it is assumed that the temperature variation is smaller than  $10^{\circ}$ . The true effect of augmentation technique (such as adding nanoparticles) on the thermodynamic performance can be evaluated by comparing the irreversibility of the heat exchanger apparatus before and after the implementation of the augmentation technique. To this end the augmentation entropy generation number is defined:

$$N_{S,a} = \frac{N_S}{N_{S,0}} \tag{23}$$

Where  $N_{S,0}$  represent the degree of irreversibility when the fluid is distilled water ( $\phi=0$ ). According to Eq. (25) adding nanoparticle is thermodynamically advantageous when  $N_{S,a}$  values are less than 1.

**VI. RESULT AND DISCUSSION**

Fig. 2 displays the entropy generation number for turbulent flow of water- $Al_2O_3$  nanofluid versus  $Re$ , in different volume concentrations. It is found that by increasing  $Re$  number the entropy generation number first decreases and then increases, so there is a  $Re$  number for each volume concentration of nanoparticle in which the entropy generation is minimum. The  $Re$  number in which the entropy generation is minimum, decreases by increasing nanoparticle volume concentration from about 230,000 for pure water to 154,000 for  $\phi=4\%$ . Also it is clear from Fig. 8 that adding nanoparticles to the turbulent flow of nanofluid has different effects on the entropy generation before and after minimum entropy generation region.

Fig. 3 represents the effects of adding nanoparticles on augmentation entropy generation number at different  $Re$  numbers. As it can be seen at low  $Re$  numbers ( $Re < 40000$ ) the augmentation entropy generation number is less than unity which means that using nanoparticles at these  $Re$  numbers is efficient and increases the thermodynamic performance, whereas at high  $Re$  numbers ( $Re > 140000$ ) the use of nanoparticles results in raising the entropy generation sharply. For each volume concentration an optimum  $Re$  number exists that has the smallest entropy generation relative to the pure water which are represented by triangular symbols in Fig. 3. It is found that these optimum  $Re$  numbers are between 40,000 and 41,500 (respectively for  $\phi=4\%$  and  $\phi=1\%$ ). Moreover it is found that the maximum decrease in entropy generation is for  $\phi=4\%$  at  $Re=40000$  which is about 22% while the highest increase of entropy generation is about 101% and occurs for  $\phi=4\%$  at  $Re=350000$ .

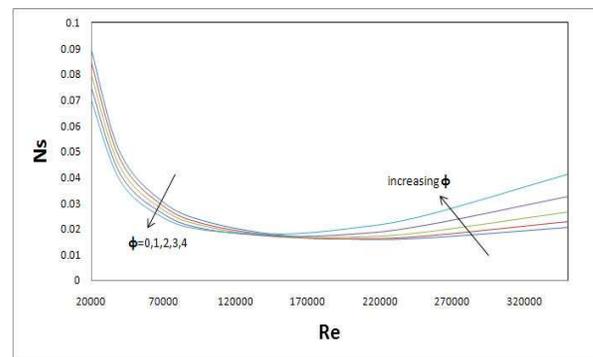


Fig. 2. Entropy generation number in turbulent flow

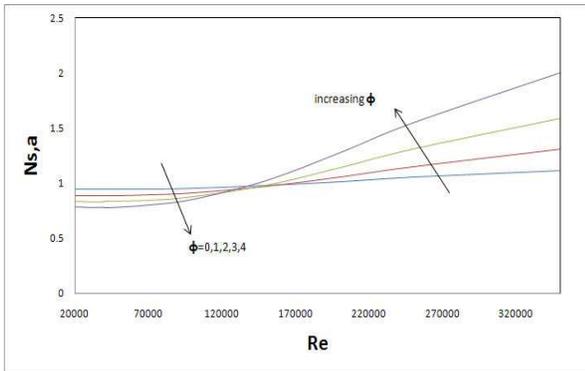


Fig. 3. Augmentation entropy generation number in turbulent flow

In Fig. 4, augmentation entropy generation number is plotted versus volume concentration of nanoparticles for different Re numbers. It is shown that for very small Re numbers ( $Re < 150000$ ) the augmentation entropy generation number declines linearly. By increasing the Re number at  $Re = 45000$  the effects of fluid flow irreversibility becomes more important and the advantageous of decreasing heat transfer irreversibility is wasted by increasing the fluid flow irreversibility. Indeed at this Re number adding nanoparticles has no significant effects on the entropy generation and augmentation entropy generation number is approximately unity at a range of volume concentrations from 0% to 4%. By continuing to increase the Re number, the fluid flow irreversibility becomes dominant and adding nanoparticles results in an extreme ascent of entropy generation number.

Fig. 5 illustrates the entropy generation number versus  $\delta$  number in turbulent flow where nanoparticle volume concentration ranges from 0% to 4%. It can be seen that Ns decrease with the increase of  $\delta$  for pure water and declines by increasing the volume concentration of nanoparticles.

Variation of the pumping power to heat transfer rate ratio  $P_r$  with Re numbers shown in Fig. 6 for water- $Al_2O_3$  for constant  $\delta$  in turbulent flow. It is shown that the pumping power to heat transfer rate ratio  $P_r$  increases by adding nanoparticles in constant  $\delta$  similar laminar flow.

Fig. 7 illustrates the pumping power to heat transfer rate ratio  $P_r$  number versus  $\delta$  where nanoparticle volume

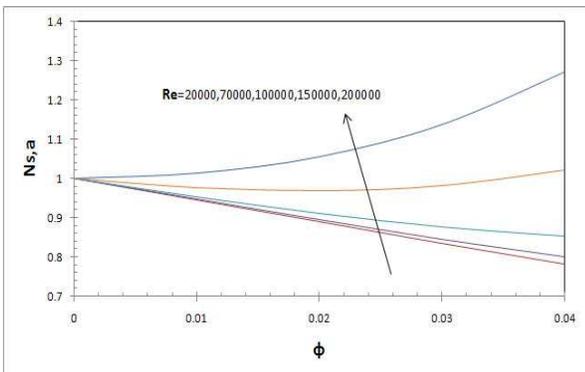


Fig. 4. Augmentation Entropy generation number in turbulent flow versus volume concentration

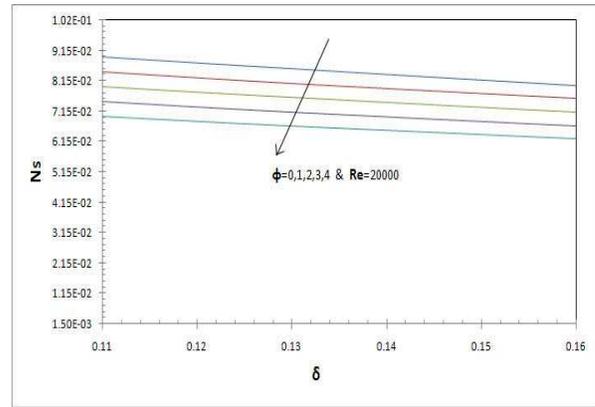


Fig. 5. Augmentation entropy generation number in turbulent flow versus  $\delta$

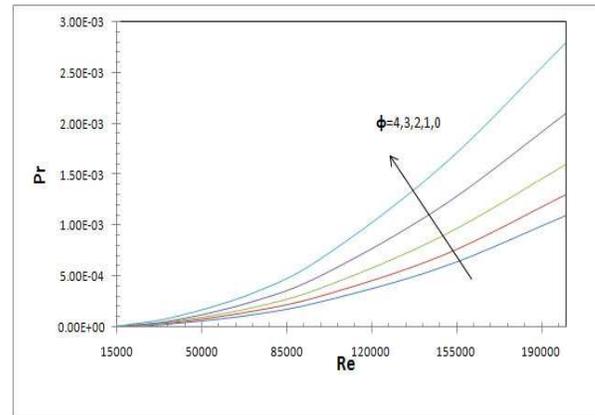


Fig. 6. Pumping power to heat transfer rate ratio  $P_r$  versus Re in turbulent flow

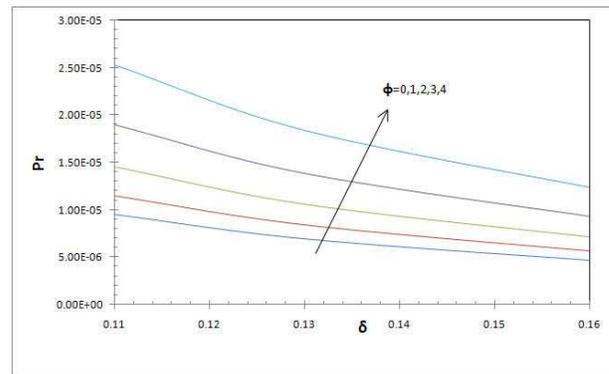


Fig. 7. Pumping power to heat transfer rate ratio  $P_r$  versus  $\delta$  in turbulent flow

concentration ranges from 0% to 4%. It can be seen that  $P_r$  decrease with the increase of  $\delta$  for pure water and increase by increasing the volume concentration of nanoparticles.

## VII. CONCLUSION

In this paper entropy generation and pumping power of the nanofluids flow through a coiled tube under constant wall heat flux is investigated. Based on the results obtained in this paper it can be concluded that in the sense of thermodynamic performance, adding nanoparticles to the base fluid is efficient

only when the heat transfer irreversibility is dominant. In turbulent flow of water- $\text{Al}_2\text{O}_3$  nanofluid, it is concluded that adding nanoparticles is useful when Reynolds number is smaller than 40,000 and is baneful for Reynolds number greater than 140,000. Moreover it is found that for a specified nanoparticle volume concentration the optimum Reynolds number is about 41,500. The greatest decline of the irreversibility which is occurred at  $\text{Re} = 40000$  for  $\phi = 4\%$  is about 22%. The pumping power to heat transfer rate ratio  $P_r$  increases by adding nanoparticles in constant  $\delta$ . The pumping power to heat transfer rate ratio decrease with the increase of  $\delta$  for pure water and increase by increasing the volume concentration of nanoparticles.

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