Two-Stage Stochastic DCOPF Approach Considering Minimum Load Curtailment

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***Abstract*— In this paper a two-stage stochastic DC Optimal Power Flow (DCOPF) methodology is presented which considers the curtailed load value. Mathematical formulation and optimization problem in both stochastic and deterministic ones also are presented in this paper. A simple mathematical representation of the two-stage stochastic programming is developed. This paper utilizes the Incidence Matrix Approach (IMA) to reformulation the traditional DCOPF. Uncertain parameter which is addressed in this paper is load profile, which is depend on multifarious objects, such as weather, temperature and etc; and short-term load forecasts, also are sensitive to temporal behavior and weather. This paper considers multiple discrete probabilities for these kinds of load demands and corresponding problem representation is addressed. Detailed results from an illustrative example and a case study are presented and discussed. The simulation results show that the presented method is both satisfactory and consistent with expectation. Finally, some relevant conclusions are drawn.**

***Keywords*—** Stochastic Programming, DCOPF, Incidence Matrix and Load Curtailment

List of symbols

x	First stage decision variable
y	Second stage decision variable
ω	Scenario or possible outcome
$Q(x,\omega)$	Second stage objective function
Ω	Set of possible scenarios
Pr	Probability function
$p(\omega)$	Discrete probability of scenario ω
Ω^D	Set of indices of the demands
E_ω	Expectation value of scenario ω
i	Index for bus
j	Index for line
ug	Index for generation unit
ud	Index for load demand
NB	Total number of buses
NG	Total number of generators
NVG	Total number of virtual generators
NL	Total number of lines
NU	Total number of units
ND	Total number of loads
$P(i,ug)$	Power produced by unit ug at bus i
$P^{\max}(i,ug)$	Maximum production level of unit ug at bus i
$P^{\min}(i,ug)$	Minimum production level of unit ug at bus i
$D(i,ud)$	Power demanded by consumer ud at bus i
$C(i)$	Offered price at bus i
$PC(i)$	Curtailment value at bus i
$PG(i)$	Total generation at bus i
$PVG(i,\omega)$	Total curtailment at bus i in scenario ω
$PD(i,\omega)$	Total demand at bus i in scenario ω
$A(i,j)$	Incidence matrix (node and branch)
$A^T(i,j)$	Transpose of Incidence matrix
$X(j,j)$	Diagonal reactance matrix
$\delta(i)$	Voltage angle of bus i
$\lambda(i)$	Dual variable of the balance constraint at bus i
$PL(j)$	Transmission line j capacity
$PL^{\max}(j)$	Maximum transmission line j capacity

Manuscript received July 6, 2011. This work was supported in part by the Islamic Azad University, Science and Research Branch, Fars, Iran.

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I. INTRODUCTION

Short-term planning in power system has a vital role in economic operation of available resources in each country. Unit commitment is one of the main optimization problem which implemented by Independent System Operator (ISO) in recent competitive power markets. The ISO, as a responsible entity, is the responsible entity which operates the power market. Unlike the conventional power system which generating utilities decide to minimize their operation cost, in restructured power system, main objective of generation companies (GENCO's) is to maximize their profit in the market. Aside from the maximizing profit from GENCO's point of view, they haven't any responsibility of maintaining demand load. As it mentioned above, ISO is responsible of satisfying the demand. Indeed, bringing load and supply into balance economically, the central concern of the power delivery chain, requires that power output must match both the real and reactive components of load. Supply and load must be balanced for each node and each hour.

The total power supply within a node is the delivered power of units operating within that node plus the total delivered imported power from other nodes minus the total exported power from that node. Delivered power is power reduced by transmission losses [1]. If the load demand is not supplied at a certain node, some portion of load has been curtailed. From Demand Side Program (DSM) point of view, optional high-reliability prices should be made available to large industrial and other customers that are willing to pay higher prices to avoid service curtailments. The price premium, or surcharge, should be used to pay other consumers to voluntarily curtail use and to fund financial incentives for consumers to purchase and install more energy-efficient appliances, motors, and other devices. If an electricity curtailment can be avoided by the power supply company's purchases of emergency sources of power, the funds can be used to cover the power supply company's incremental cost for that emergency power [2]. Minimization of curtailed load has been interested in the operation cost from point of view of ISO. To do this, the ISO carries out the load forecasting in the first order of merit. Short-term demand forecasting is the starting point for all other activities. Indeed, the purpose of all other activities is to supply the load. Without a forecast of the load no other activity can proceed.

Typically, a utility has a department that is responsible for day-ahead forecasting. This group maintains all the statistical factors that influence load. A day-ahead forecast requires quantifying all these factors and entering them into a model. Stochastic manner of the short-term forecasted load, introduces some critical problems. Because of management of short-term planning of power system is based on the deterministic load, it makes conflicts to do the Unit Commitment (UC) program. To overcome this problem, stochastic DCOPF approach is presented in this paper. This approach considers a normal distribution probability function of expected load.

As it mentioned above, the ISO is confronting with uncertainty of the demand at each node in each hour of planning. Stochastic programming is one of the strong tools for overcoming the uncertainty at each node of power system.

Bi-level programming background is provided, for instance, in [3]. References using bi-level programming in different contexts include [4], which provides a model to find an optimal expansion plan that mitigates the impact of deliberated network attacks, and [5], which addresses the generation capacity expansion planning problem. Reference [6] introduces the stochastic programming approach to electric energy procurement in competitive power market.

This paper provides a procedure for optimal short-term operation of the power system to be derived and implemented by the ISO. The approach is static in the sense that it considers a single target hour in the operation analysis.

This procedure relies on a two-stage formulation [3]. The first stage problem represents a short-term operational objective function. The second stage represents the objective of the ISO, i.e., minimizing curtailment cost while "maximizing social welfare." The numerical proxy considered here, is maximizing the average social welfare over all considered scenarios [7]. The scenarios considered include cases with probable load densities.

The remainder of this paper is organized as follows. Theoretical consideration of stochastic programming and corresponding mathematical formulation is addressed in next section. Modeling of the DCOPF based on incidence matrix in both deterministic and stochastic cases are presented in section. III Simulation case and results are introduced in section IV. Conclusion of this paper is conducted in last section.

II. THEORETICAL CONSIDERATIONS

Stochastic programming has become an important problem area. With current standard off-the-shelf software including modeling systems such as AMPL and GAMS, powerful large-scale general-purpose solvers such as CPLEX and specialized stochastic programming solvers such as OSL-SE and DECIS, end-users can develop realistic stochastic programming models and solve them on standard desktop hardware. The two-stage stochastic linear programming problem can be stated as:

$$\begin{aligned} &\underset{x}{Min}\quad c^T x + E_\omega Q(x,\omega) \\ &\qquad Ax = b \\ &\qquad x \geq 0 \end{aligned} \tag{1}$$

Where:

$$\begin{aligned} &Q(x,\omega) = \underset{y}{Min}\ d_\omega^T y \\ &\qquad T_\omega x + W_\omega y = h_\omega \\ &\qquad y \geq 0 \end{aligned} \tag{2}$$

Here E_ω is the expectation, and ω denotes a scenario or possible outcome with respect to the probability space (Ω, Pr).

The variables x are called first-stage variables, as they have to be decided upon before the outcome of the stochastic variable ω is observed. The variables y are second-stage variables: They can be calculated after the outcome of ω is known [8].

In general, discrete distributions P only are considered, so it can be written as:

$$E_\omega Q(x,\omega) = \sum_{\omega\in\Omega} p(\omega) Q(x,\omega) \tag{3}$$

Using this it can be formulated as a large LP that forms the deterministic equivalent problem:

$$\begin{aligned} &Min \\ &\quad c^T x + \sum_\omega p(\omega) d_\omega^T y_\omega \\ &\qquad Ax = b \\ &\qquad T_\omega x + W_\omega y_\omega = h_\omega \quad \forall\omega \\ &\qquad x \geq 0\ ,\ y_\omega \geq 0 \end{aligned} \tag{4}$$

The chain of events in this model is as follows: first the decision maker implements the first stage decisions x.

Then the system will be subjected to the random process described by (Ω, Pr), which results in an outcome $\omega \in \Omega$ [8].

III. DETERMINISTIC AND STOCHASTIC DCOPF

Earlier studies of LMP calculation with DCOPF ignore the line losses. Thus, the energy price and the congestion price follow a perfect linear model with a zero loss price. However, challenges arise if nonlinear losses need to be considered in LMP calculations [9]. Stochastic behavior of demanded load implies that the results of short-term operation maybe face with load curtailment or excess the demand and increase the mismatch. Since the lossless DCOPF can be modeled as the minimization of the total production cost subject to energy balance and transmission constraints, the stochastic manner of demand impacted the results. Therefore, if the total production is less than total demand, system can exercise the shortcoming and load curtailment at some nodes appear.

A. Deterministic Incidence Based DCOPF

In general DCOPF modeling, the voltage magnitudes are assumed to be unity and reactive power is ignored. Also, it is assumed that there is no demand elasticity. This model may be written as LP:

$$Min \quad \sum_{i=1}^{NG}[PG(i)*C(i)]+\sum_{i=1}^{NVG}[PVG(i)*PC(i)] \tag{5}$$

Subject to:

$$PG(i)=\sum_{ug=1}^{NU}P(i,ug) \tag{6}$$

$$PD(i)=\sum_{ud=1}^{ND}D(i,ud) \tag{7}$$

$$\begin{bmatrix} PG(i)+PVG(i)-PD(i)= \\ \sum_{j=1}^{NL}A(i,j)*PL(j) \end{bmatrix} \perp \quad \lambda(i) \tag{8}$$

$$\sum_{i=1}^{NB}A^T(i,j)*\delta(i)=\sum_{j=1}^{NL}X(j,j)*PL(j) \tag{9}$$

$$-PL^{\max}(j)\le PL(j)\le PL^{\max}(j) \tag{10}$$

$$P^{\min}(i,ug)\le P(i,ug)\le P^{\max}(i,ug) \tag{11}$$

$$PVG(i)\le PD(i) \tag{12}$$

The objective function of deterministic DCOPF is minimization of total generation and curtailment cost, which is represented as (5). It also should be noticed that, in this case, generation companies at each bus own all generating units which located at identical bus. For the sake of simplicity linear generation cost function is considered in this paper.

Aggregated generation and demand at each bus are represented in (6) and (7), respectively. Generation and demand balance addressed in (8) by implementing the incidence matrix, this equation corresponds with injection power through power transmission lines connected to bus i and local curtailed load (modeled as virtual generating unit). Locational marginal price is the dual variable of the balance constraint at bus i and indicated as λ(i). Power transmitted through transmission lines is indicated as (9) using correspondence diagonal reactance matrix, X. Transmission line limits and power generation boundary

Constraints (10) and (11) enforce the transmission capacity limits of each line and each generation unit, respectively. Since total load curtailment in each bus is less than total demand at corresponding node, it is modeled as (12). It also should be note that the ISO's objective function is maximizing social welfare and to do this, it considers the value of the curtailment of consumer load. From ISO's point of view, the value of curtailment is equal with Value of Loss of Load (VOLL)

B. Stochastic Incidence Based DCOPF

Based on the stochastic nature of demanded load, it can be modeled as the stochastic DCOPF. In this section stochastic modeling of DCOPF based on incidence matrix approach is presented as a two-stage linear stochastic programming:

$$Min \quad \sum_{i=1}^{NG}[PG(i)*C(i)]+\sum_{i=1}^{NVG}\sum_{\omega\in\Omega^D}[PVG(i,\omega)*PC(i)] \tag{13}$$

Subject to:

$$PG(i)=\sum_{ug=1}^{NU}P(i,ug) \tag{14}$$

$$PD(i,\omega)=\sum_{ud=1}^{ND}D(i,ud,\omega) \tag{15}$$

$$\begin{bmatrix} PG(i)+PVG(i,\omega)-PD(i,\omega)= \\ \sum_{j=1}^{NL}A(i,j)*PL(j) \end{bmatrix} \perp \lambda(i) \tag{16}$$

$$\sum_{i=1}^{NB}A^T(i,j)*\delta(i)=\sum_{j=1}^{NL}X(j,j)*PL(j) \tag{17}$$

$$-PL^{\max}(j)\le PL(j)\le PL^{\max}(j) \tag{18}$$

$$P^{\min}(i,ug)\le P(i,ug)\le P^{\max}(i,ug) \tag{19}$$

$$PVG(i,\omega)\le PD(i,\omega) \tag{20}$$

The objective function of stochastic SCOPF is minimization of total operation and curtailment cost, too. The main stochastic parameter is demanded load. In the problem formulation each bus of power system has a load profile based on nature of consumer needs. In this stage, the load profile could be extracted from previous record, and usually, a normal distribution probability function is fitted to the real data, which the discrete probability function of load profile could be derivate from records.

Similarly with the deterministic model, (14), (17)-(19) are same as (6), (9)-(11), respectively in the stochastic model. Because of stochastic nature of the load, (15) implies this issue.

Generation and demand balance is also addressed in (16) by implementing the incidence matrix, this equation corresponds with injection power through power transmission lines connected to bus i , as a first decision variable, and local curtailed load, as a second stage variable which should be minimized. It also should be noticed that total curtailment at each node is must be less than total demanded load at that identical node.

In the next section, the proposed methodology is examined in a 5-bus PJM test system, which is a popular benchmark for evaluating the LMP calculation.

IV. SIMULATION STUDIES

In order to validate the proposed both deterministic and stochastic incidence matrix based DCOPF, a PJM five bus, six lines test system, which is a standard test case, is considered here. The benchmark parameters are listed in tables I and II. Demanded load at buses B, C and D, are similarly 300MW, in base case and is considered as “Mid” in the stochastic ones. The system is slightly modified from the PJM 5-bus system [10] and will be used for the rest of this paper. The generation cost at Sundance (unit 4.1) is modified from the original \$30/MWh to \$35/MWh to differentiate its cost from the Solitude (unit 3.1) for better illustration.

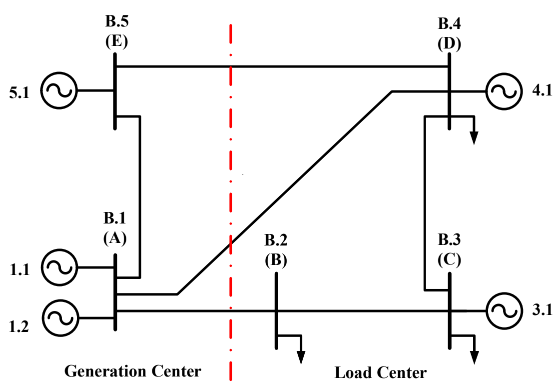

Fig. 1. Base case of the PJM 5-Bus example

The system can be roughly divided into two areas, a generation center consisting of Buses A and E with three low-cost generation units and a load center consisting of Buses B, C, and D with 900 MWh load and two high-cost generation units. The transmission line impedances are given in Table I, where the reactance is obtained from [10] and the resistance is assumed to be 10% of the reactance. Here only thermal flow limit of Line DE (Line 6) is considered for illustrative purpose.

Table I. Line impedance and flow limits

Line Number	1	2	3	4	5	6
Connection	AB	AD	AE	BC	CD	DE
R%	0.281	0.304	0.064	0.108	0.297	0.297
X%	2.81	3.04	0.64	1.08	2.97	2.97
Limit(MW)	999	999	999	999	999	240

Table II. Generation unit's data

Unit	Location	Indication	Pmax	Pmin	Offer
Alta	A	1.1	110	0	14
Park City	A	1.2	100	0	15
Solitude	C	3.1	520	0	30
Sundance	D	4.1	200	0	35
Brighton	E	5.1	600	0	10

A. DCOPF Calculation in Deterministic Case

As it shown in [9], LMP of each bus is the dual variable of load balance equation. In the deterministic case, total demand is 900MW and installed capacity is 1530MW. Each generation company, GENCO, offer its identical price to maintaining consumer load. Incidence and reactance matrixes are addressed in appendix tables A1 and A2 respectively. Summary of load dispatch is presented in table III.

Table III. Generation dispatch results and LMP for base case

Bus	Indication	Generation	LMP
1	1.1	110.00	15.826
	1.2	100.00	
2	-	-	23.680
3	3.1	0	26.699
4	4.1	116.079	35.000
5	5.1	573.921	10.000

Total operation cost is 12841.892 \$/h and load curtailment cost is considered to be 100\$/MWh in the entire system. In the base case, LMP's are equal with the results which obtained in [9].

B. DCOPF Calculation in Stochastic Case

In this section, stochastic modeling of the proposed methodology is simulated. For the sake of comparison with the base case, stochastic behaviors of load is only considered.

Initially, three different scenarios are considered to describe demand situations. Table IV and V characterize these demand scenarios. The second column table IV provides the demand distribution at each bus with three scenarios: “Lo”, “Mid” and “Hi”, while imply low demand, middle and high ones, respectively. Weights of the correspondence scenarios are addressed in table V. total scenarios are 27 cases which is solved by DECIS solver of GAMS [11].

Table IV. Distribution of stochastic demand

Bus	Lo	Mid	Hi
2	200	300	400
3	250	300	350
4	225	300	375

Table V. Probability of stochastic demand

Bus	Lo	Mid	Hi
2	0.25	0.50	0.25
3	0.25	0.50	0.25
4	0.25	0.50	0.25

The DCOPF results in this case are tabulated in table VI. Total operation cost in this case is 19561.227$/h which is almost 6720$/h expense than base case. Corresponding LMP for this case is presented in table VI.

Table VI. Generation dispatch results and LMP for stochastic case

Bus	Indication	Generation	LMP
1	1.1	110.00	16.977
	1.2	100.00	
2	-	-	26.384
3	3.1	118.561	30.000
4	4.1	200.000	39.943
5	5.1	596.439	10.000

In comparison with the deterministic case, total dispatch of generation units is 1125 MW in the stochastic case, while the identical generation corresponding with deterministic case is 900 MW. This difference is because of the ISO's concerns in short-term operation.

For the sake of illustration the curtailment load, demand growth of 40% for entire buses is considered. In this case, maximum total load is 1575 MW, which is generated in scenario 27th. In this case, all generating unit, except unit 5.1, are dispatched. Because of the network constraints and unit maximum generation, it is impossible to maintaining entire load. In such case, unit 5.1 is scheduled at 582.075 MW. Since line DE (L6) is congested, and unit 4.1, which is expensive ones, is reach maximum limit, load curtailment is occurred. Maximum curtailed load is 62.925 MW. It also should be noticed that maximum load in this node is 525 MW (1.4*375MW).

V. CONCLUSION

The proposed two-level stochastic incidence matrix-based DCOPF is useful approach to implementing for large scale power system analysis regardless of time horizon analysis. In short-term analysis, such as Day A-head market clearing, mid-term analysis such as maintenance scheduling or fuel allocation and in the long-term analysis such as expansion planning studies; this approach would be applicable.

This can reduce the computational effort since it does not require the algorithm to run till convergence. Therefore, it fits a simulation or planning purpose well if the accuracy is reasonably acceptable in uncertain case studies. The simulation results on the benchmark PJM 5-bus system show the feasibility and applicability of the proposed method in short-term, analysis. Simulation results also show that the presented method is both satisfactory and consistent with expectation.

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A. *Appendix*

Table A1: Incidence matrix of PJM 5-bus test system

A(i,j)	1	2	3	4	5	6
1	**1**	**1**	**1**	0	0	0
2	**-1**	0	0	**1**	0	0
3	0	0	0	**-1**	**1**	0
4	0	**-1**	0	0	**-1**	**1**
5	0	0	-1	0	0	**-1**

Table A2: Reactance matrix of PJM 5-bus test system

X(j,j)	1	2	3	4	5	6
1	**0.0281**	0	0	0	0	0
2	0	**0.0304**	0	0	0	0
3	0	0	**0.0064**	0	0	0
4	0	0	0	**0.0108**	0	0
5	0	0	0	0	**0.0297**	0
6	0	0	0	0	0	**0.0297**