Analysis Variation Thickness with Stainless Steel

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***Abstract*— In the present work, study of thin cylindrical shells made of a stainless steel is presented. Material properties are graded in the thickness direction of the shell. The objective is to study the natural frequencies a cylindrical shell. The study is carried out using third order shear deformation shell theory. The analysis is carried out using Hamilton's principle. The governing equations of motion of cylindrical shells are derived based on shear deformation theory. Results are presented on the frequency characteristics and the effects of boundary conditions.**

***Keywords*–** Cylindrical, Shell and Stainless Steel

I. INTRODUCTION

Cylindrical shells have found many applications in the industry. They are often used as load bearing structures for aircrafts, ships and buildings. Understanding of vibration behavior of cylindrical shells is an important aspect for the successful applications of cylindrical shells. Researches on free vibrations of cylindrical shells have been carried out extensively [1-5]. Recently, the present authors presented studies on the influence of boundary conditions on the frequencies of a multi–layered cylindrical shell [6]. In all the above works, different thin shell theories based on Love–hypothesis were used. Vibration of cylindrical shells with ring support is considered by Loy and Lam [7]. Study of cylindrical shell structures is important. In this paper a study on cylindrical shells is presented. The study is carried out based on third order shear deformation shell theory. The analysis is carried out using Hamilton's principle. Results are presented on the frequency characteristics a cylindrical shell with stainless steel.

II. ANALYSIS

The strain-displacement relationships for a thin shell [8].

$$\in_{11}=\frac{1}{A_1(1+\frac{\alpha_3}{R_1})}\left[\frac{\partial U_1}{\partial \alpha_1}+\frac{U_2}{A_2}\frac{\partial A_1}{\partial \alpha_2}+U_3\frac{A_1}{R_1}\right] \tag{1}$$

$$\in_{22}\ \frac{1}{A_2(1+\frac{\alpha_3}{R_2})}\left[\frac{\partial U_2}{\partial \alpha_2}+\frac{U_1}{A_1}\frac{\partial A_2}{\partial \alpha_1}+U_3\frac{A_2}{R_2}\right] \tag{2}$$

$$\in_{33}=\frac{\partial U_3}{\partial \alpha_3} \tag{3}$$

$$\in_{12}=\frac{A_1(1+\frac{\alpha_3}{R_1})}{A_2(1+\frac{\alpha_3}{R_2})}\frac{\partial}{\partial \alpha_2}\left(\frac{U_1}{A_1(1+\frac{\alpha_3}{R_1})}\right)+\frac{A_2(1+\frac{\alpha_3}{R_2})}{A_1(1+\frac{\alpha_3}{R_1})}\frac{\partial}{\partial \alpha_1}\left(\frac{U_2}{A_2(1+\frac{\alpha_3}{R_2})}\right) \tag{4}$$

$$\in_{13}=A_1(1+\frac{\alpha_3}{R_1})\frac{\partial}{\partial \alpha_3}\left(\frac{U_1}{A_1(1+\frac{\alpha_3}{R_1})}\right)+\frac{1}{A_1(1+\frac{\alpha_3}{R_1})}\frac{\partial U_3}{\partial \alpha_1} \tag{5}$$

$$\in_{23}=A_2(1+\frac{\alpha_3}{R_2})\frac{\partial}{\partial \alpha_3}\left(\frac{U_2}{A_2(1+\frac{\alpha_3}{R_2})}\right)+\frac{1}{A_2(1+\frac{\alpha_3}{R_2})}\frac{\partial U_3}{\partial \alpha_2} \tag{6}$$

In this equations A_1 and A_2 are the fundamental form parameters or Lame parameters, U_1, U_2 and U_3 are the displacement at any point ($\alpha_1, \alpha_2, \alpha_3$), R_1 and R_2 are the radius of curvature related to α_1, α_2 and α_3 respectively.

The third- order theory of Reddy used in the present study is based on the following displacement field:

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2)+\alpha_3.\phi_1(\alpha_1,\alpha_2)+\alpha_3^2.\psi_1(\alpha_1,\alpha_2)+\alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2)+\alpha_3.\phi_2(\alpha_1,\alpha_2)+\alpha_3^2.\psi_2(\alpha_1,\alpha_2)+\alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{7}$$

Substituting Eq. (7) into nonlinear strain-displacement relation (1) - (6), it can be obtained for the third-order theory of Reddy.

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} \tag{9}$$

Where $(\varepsilon^0, \gamma^0)$ are the membranes strains and $(k, k', \gamma^2, \gamma^3)$ are the bending strains, known as the curvatures.

III. FORMOLATION

For a cylindrical shell with R is the radius, L the length and h the thickness of the shell. The reference surface is chosen to be the middle surface of the cylindrical shell where an orthogonal coordinate system x, θ, z is fixed. The displacements of the shell with reference this coordinate system are denoted by U_1, U_2 and U_3 in the x, θ and z

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directions, respectively. For a thin cylindrical shell, the stress - strain relationship are defined as

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{44} & 0 & 0\\ 0 & 0 & 0 & Q_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\in_{11}\\ \in_{22}\\ \in_{23}\\ \in_{13}\\ \in_{12}\end{Bmatrix} \qquad (10)$$

For a isotropic cylindrical shell the reduced stiffness Q_{ij} (i, j=1, 2 and 6) are defined as:

$$Q_{11}=Q_{22}=\frac{E}{1-\nu^2}\ ,Q_{12}=\frac{\nu\,E}{1-\nu^2} \qquad (11)$$

$$Q_{44}=Q_{55}=Q_{66}=\frac{E}{2(1+\nu)} \qquad (12)$$

where E is the Young's modulus and ν is Poisson's ratio. Defining

$$\{A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},G_{ij},H_{ij}\}=\int_{-h/2}^{h/2}Q_{ij}\{1,\alpha_3,\alpha_3^2,\alpha_3^3,\alpha_3^4,\alpha_3^5,\alpha_3^6\}\,d\alpha_3 \qquad (13)$$

where Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij},F_{ij},G_{ij},H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness. For a thin cylindrical shell the force and moment results are defined as

$$\begin{Bmatrix}N_{11}\\ N_{22}\\ N_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}d\alpha_3\ ,\quad \begin{Bmatrix}M_{11}\\ M_{22}\\ M_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}\alpha_3^3\,d\alpha_3 \qquad (14)$$

$$\begin{Bmatrix}P_{11}\\ P_{22}\\ P_{12}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}\alpha_3^3\,d\alpha_3\ ,\begin{Bmatrix}P_{13}\\ \\ P_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \\ \sigma_{23}\end{Bmatrix}\alpha_3^3\,d\alpha_3 \qquad (15)$$

$$\begin{Bmatrix}Q_{13}\\ Q_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \sigma_{23}\end{Bmatrix}d\alpha_3\ ,\quad \begin{Bmatrix}R_{13}\\ R_{23}\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_{13}\\ \sigma_{23}\end{Bmatrix}\alpha_3^{\,2}\,d\alpha_3 \qquad (16)$$

IV. THE EQUETIONS OF MOTION FOR GENERIC SHELL

The equations of motion for vibration of a generic shell can be derived by using Hamilton's principle which is described by

$$\delta\int_{t_1}^{t_2}(\Pi-K)\,dt=0\ ,\ \Pi=U-V \qquad (17)$$

Where K,Π,U and V are the total kinetic, potential, strain and loading energies, t_1 and t_2 are arbitrary time. The kinetic, strain and loading energies of a cylindrical shell can be written as:

$$K=\frac{1}{2}\int_{\alpha_1}\int_{\alpha_2}\int_{\alpha_3}\rho(\dot U_1^2+\dot U_2^2+\dot U_3^2)\,dV \qquad (18)$$

$$U=\int_{\alpha_1}\int_{\alpha_2}\int_{\alpha_3}(\sigma_{11}\in_{11}+\sigma_{22}\in_{22}+\sigma_{12}\in_{12}+\sigma_{13}\in_{13}+\sigma_{23}\in_{23})\,dV \qquad (19)$$

$$V=\int_{\alpha_1}\int_{\alpha_2}(q_1\delta U_1+q_2\delta U_2+q_3\delta U_3)A_1A_2\,d\alpha_1 d\alpha_2 \qquad (20)$$

The infinitesimal volume is given by

$$dV=A_1A_2\,d\alpha_1 d\alpha_2 d\alpha_3 \qquad (21)$$

The displacement fields for a cylindrical shell and the displacement fields which satisfy these boundary conditions can be written as:

$$\begin{aligned}
u_1&=\overline{A}\,\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
u_2&=\overline{B}\,\phi(x)\,\sin(n\theta)\cos(\omega t)\\
u_3&=\overline{C}\,\phi(x)\cos(n\theta)\,\cos(\omega t)\\
\phi_1&=\overline{D}\,\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
\phi_2&=\overline{E}\,\phi(x)\,\sin(n\theta)\cos(\omega t)
\end{aligned} \qquad (22)$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, ϕ_1 and ϕ_2 are the displacement fields for higher order deformation theories for a cylindrical shell, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions. The axial function $\phi(x)$ is chosen as the beam function as

$$\phi(x)=\gamma_1\cosh\left(\frac{\lambda_m x}{L}\right)+\gamma_2\cos\left(\frac{\lambda_m x}{L}\right)-\zeta_m\left(\gamma_3\sinh\left(\frac{\lambda_m x}{L}\right)+\gamma_4\sin\left(\frac{\lambda_m x}{L}\right)\right) \qquad (23)$$

The geometric boundary conditions for free and clamped boundary conditions can be expressed mathematically in terms of $\phi(x)$ as:

Substituting Eq. (22) into Eq. (18) - (22) for third order theory we can be expressed:

$$\det\,(C_{ij}-M_{ij}\,\omega^2)=0 \qquad (24)$$

Expanding this determinant, a polynomial in even powers of ω is obtained:

$$\beta_\circ\omega^{10} + \beta_1\omega^8 + \beta_2\omega^6 + \beta_3\omega^4 + \beta_4\omega^2 + \beta_5 = \circ \qquad (25)$$

where $\beta_i\,(i = 0,1,2,3,4,5)$ are some constants. Eq. (25) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell based third-order theory. The smallest of the five roots is the natural angular frequency studied in the present study.

V. RESULTS AND DISCUSSION

To validate the present analysis, results for cylindrical shells are compared with Loy and Lam [9]. The comparisons show that the present results agreed well with those in the literature.

TABLE I: COMPARISON OF NATURAL FREQUENCY (Hz) FOR A CYLINDRICAL SHELL

$L=20.3cm$, $R=5.08cm$, $h=0.25cm$, $E=2.07788*10^{11}Nm^{-2}$, $\nu=0.317756$

$\rho=8166kgm^{-3}$

n	m	Loy[9]	Present
2	1	2043.8	2043.6
	2	5635.4	5635.2
	3	8932.5	8932.1
	4	11407.5	11407.2
	5	13253.2	13252.8
	6	14790.0	14789.8

From the comparisons presented in Table I, it can be seen that the present results agree well with those in the literature. In this paper, studies are presented for a cylindrical shell. The effects of the configuration are studying the frequencies of cylindrical shells. This cylindrical shell has stainless steel on its outer surface. Fig 1 shows the variations of the volume fractions of Stainless Steel, respectively, in the thickness direction z for a cylindrical shell. In Fig. 1, the volume fraction for decreased from 1 at z=-0.5h to 0 at z=0.5h.

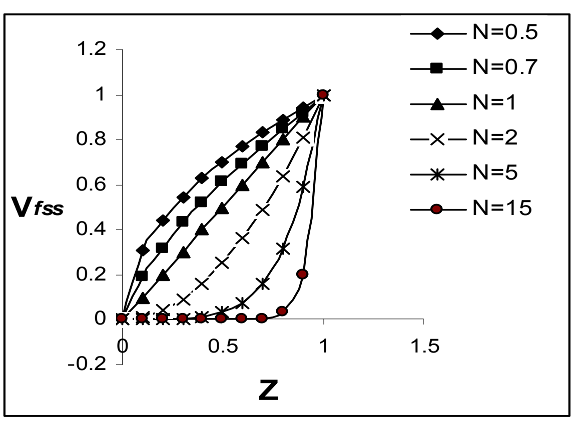

Fig. 1. Variation of volume fraction of stainless steel V_{fss} with radial distance Z in the thickness direction

VI. CONCLUSION

A study on the Cylindrical shell composed of stainless steel has been presented. The results showed that one could easily vary the natural frequency of the cylindrical shell by varying the volume fraction. Cylindrical shell and has properties that vary continuously from stainless steel on its outer surface. The influence of the constituent volume fraction on the frequencies for cylindrical shells has been found to be different. For the cylindrical shells, the natural frequencies decreased when N increased. The present analysis is validated by comparing results with those available in the literature.

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