Vibration Two Type Functionally Graded Cylindrical Shell

Mohammad Reza Isvandzibaei [1] and Ali. Moarrefzadeh [2]

***Abstract*— Study on the vibration of cylindrical shell made of a functionally gradient material (FGM) composed of stainless steel and nickel is presented. The effects of the FGM configuration are studied by studying the frequencies of two FGM cylindrical shells. Type I FGM cylindrical shell has Nickel on its inner surface and stainless steel on its outer surface and Type II FGM cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. The study is carried out based on third order shear deformation shell theory. The objective is to study the natural frequencies, the influence of constituent volume fractions and the effects of configurations of the constituent materials on the frequencies. The properties are graded in the thickness direction according to the volume fraction power-law distribution. The governing equations are obtained using energy functional with the Rayleigh-Ritz method. Results are presented on the frequency characteristics, the influence of the constituent various volume fractions on the frequencies for a Type I, II FGM cylindrical shell.**

***Keywords*–** FGM, Vibration and Cylindrical Shell

I. INTRODUCTION

The study of the vibration of cylindrical shells is an important aspect in the successful applications of the cylindrical shells. The study of the free vibrations of cylindrical shells has been carried out extensively. Among those who have studied the vibrations of cylindrical shells include Arnold and Warburton [1], Ludwig and Krieg [2], Chung [3], Soedel [4], Forsberg [5], Bhimaraddi [6], Soldatos and Hajigeoriou [7], Bert and Kumar [8]. The concept of functionally gradient materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan, [9], [10] as a means of preparing thermal barrier materials. Since then FGMs have attracted much interest as heat-Shielding materials. FGMs are made by combining different materials using power metallurgy methods [11].

They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc. and these results in gradients in the mechanical and thermal properties [12], [13]. Studies on FGMs have been extensive but are largely confined to analysis of thermal stress and deformation [14], [15] and [16]. Najafizadeh and Isvandzibaei presented the vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support [17]. The advantage of on FGMs is that desired mechanical properties can be tailored and this holds enormous application potential for FGMs. In this paper a study on the vibration of cylindrical shells composed of functionally gradient material (FGM) is presented. The functionally gradient material considered is composed of stainless steel and nickel where the volume fractions follow a power–law distribution. The objective is to study the natural frequencies, the influence of constituent volume fractions, and the effects of configurations of the constituent materials on the frequencies for two kind of FGM cylindrical shell.

The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. The displacement fields employ consist of some beam eigen-functions of vibrations that guarantee satisfaction of edge boundary conditions.

[1] M.R.Isvandzibaei, Faculty Member of Mechanical Engineering Department, Mahshahr Branch, Islamic Azad University, Mahshahr, Iran.
E-mail: esvandzebaei@yahoo.com
Tel: +989163442982

[2] A.Moarrefzadeh, Faculty Member of Mechanical Engineering Department, Mahshahr Branch, Islamic Azad University , Mahshahr , Iran
E-mail: a_moarrefzadeh@yahoo.com
a.moarefzadeh@mahshahriau.ac.ir
Tel: +989123450936

II. ANALYSIS

Consider a cylindrical shell is shown in Fig.1. R is the radius, L is the length and h is the thickness. The reference surface is chosen to be the middle surface of the cylindrical shell where an orthogonal coordinate system x, θ, z is fixed. The deformations of the shell with reference to this coordinate system are denoted by U_1, U_2 and U_3 in the x, θ and z directions, respectively.

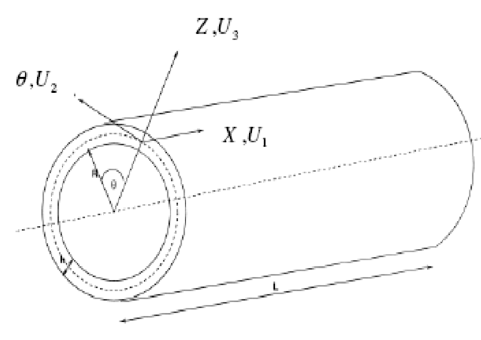

Fig. 1. Geometry of a FGM cylindrical shell

For a thin cylindrical shell, plane stress condition can be assumed. The constitutive relation for a thin cylindrical shell is consequently given by the tow-dimensional Hook's law as

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{1}$$

where, $\{\sigma\}$ is the stress vector, $\{\varepsilon\}$ is the strain vector and

$[Q]$ is the reduced stiffness matrix. The stress vector for plane stress condition is

$$\{\sigma\}^T = \{\sigma_{11} \quad \sigma_{22} \quad \sigma_{12} \; \sigma_{13} \; \sigma_{23}\} \tag{2}$$

where σ_{11} is the stress in x direction, σ_{22} the stress in the θ direction and σ_{12} is the shear stress on the $x\theta$ plane and σ_{13} is the shear stress on the $x\,z$ plane and σ_{23} is the shear stress on the $\theta\, z$ plane. The strain vector is defined as

$$\{\varepsilon\}^T = \{\varepsilon_{11} \; \varepsilon_{22} \; \varepsilon_{12} \; \varepsilon_{13} \; \varepsilon_{23}\} \tag{3}$$

where ε_{11} is the strain in x direction, ε_{22} the strain in the θ direction and ε_{12} is the shear strain on the $x\theta$ plane and ε_{13} is the shear strain on the $x\,z$ plane and ε_{23} is the shear strain on the θz plane. The reduced stiffness $[Q]$ matrix is given as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \tag{4}$$

For an isotropic cylindrical shell the reduced stiffness Q_{ij} (i, j=1, 2 and 6) are defined as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2} \tag{5}$$

$$Q_{12} = \frac{\nu\, E}{1-\nu^2} \tag{6}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} \tag{7}$$

where E is the Young's modulus and ν is Poisson's ratio. For a thin cylindrical shell the force and moment results are defined as

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} d\alpha_3 \;, \quad \begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{8}$$

$$\begin{Bmatrix} P_{11} \\ P_{22} \\ P_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 \;, \quad \begin{Bmatrix} P_{13} \\ \\ P_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \\ \sigma_{23} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{9}$$

$$\begin{Bmatrix} Q_{13} \\ Q_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} d\alpha_3 \;, \quad \begin{Bmatrix} R_{13} \\ R_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \alpha_3{}^2 d\alpha_3 \tag{10}$$

The constitutive equation is obtained as

$$\{N\} = [S]\{\varepsilon\} \tag{11}$$

where $\{N\}$ and $\{\varepsilon\}$ are, respectively, defined as

$$\{N\}^T = \{N_{11} N_{22} N_{12} M_{11} M_{22} M_{12} \; P_{11} \; P_{22} \; P_{12} \; P_{13} \; P_{23} \; Q_{13} \; Q_{23} \; R_{13} \; R_{23}\} \tag{12}$$

$$\{\varepsilon\}^T = \{\varepsilon_{11}^0 \; \varepsilon_{22}^0 \; \varepsilon_{12}^0 \; k_{11} \; k_{22} \; k_{12} \; k'_{11} \; k'_{22} \; k'_{12} \; \gamma_{23}^0 \; \gamma_{13}^0 \; \gamma_{23}^2 \; \gamma_{13}^2 \; \gamma_{23}^3 \; \gamma_{13}^3\} \tag{13}$$

and $[S]$ is defined as

$$[S] = \begin{pmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \\ [E'] & [G] & [H'] \\ [A'] & [D'] & [E'] \\ [D'] & [F'] & [G] \end{pmatrix} \tag{14}$$

where A, B, E, D, F, H and G are the extensional, coupling and bending stiffness matrices and Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij}, F_{ij}, G_{ij}, H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness. Defining

$$\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\} = \int_{-h/2}^{h/2} Q_{ij} \{1, \alpha_3, \alpha_3^2, \alpha_3^3, \alpha_3^4, \alpha_3^5, \alpha_3^6\}\, d\alpha_3 \tag{15}$$

The strain energy and kinetic energy of a cylindrical shell can be defined as:

$$U = \frac{1}{2} \iiint \{\varepsilon\}^T \{\sigma\} \quad dV \tag{16}$$

$$T = \frac{1}{2} \iiint \rho \left[\left(\frac{\partial u_1}{\partial t}\right)^2 + \left(\frac{\partial u_2}{\partial t}\right)^2 + \left(\frac{\partial u_3}{\partial t}\right)^2 + \left(\frac{\partial \phi_1}{\partial t}\right)^2 + \left(\frac{\partial \phi_2}{\partial t}\right)^2 \right] dV \tag{17}$$

where, ρ is the mass density, $\{\varepsilon\}$ is the strain vector and $\{\sigma\}$ is the stress vector. By substituting from Eq. (1), the strain and kinetic energies can be written as

$$U = \frac{1}{2} \int_0^L \int_0^{2\pi} \{\varepsilon\}^T [S] \{\varepsilon\}\, R d\theta\, dx \tag{18}$$

$$T=\frac{1}{2}\int_0^L\int_0^{2\pi}\rho_T\left[\left(\frac{\partial u_1}{\partial t}\right)^2+\left(\frac{\partial u_2}{\partial t}\right)^2+\left(\frac{\partial u_3}{\partial t}\right)^2+\left(\frac{\partial \phi_1}{\partial t}\right)^2+\left(\frac{\partial \phi_2}{\partial t}\right)^2\right]R\,d\theta\,dx \tag{19}$$

where $\{\varepsilon\}$ is the strain vector defined in Eq. (13) and $[S]$ is the stiffness matrix defined in relation (14). The parameter ρ_T is the density per unit length defined as

$$\rho_T=\int_{-h/2}^{h/2}\rho\ dz \tag{20}$$

The displacement fields for a cylindrical shell can be written as:

$$\begin{aligned}
u_1 &= \overline{A}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
u_2 &= \overline{B}\,\phi(x)\sin(n\theta)\cos(\omega t)\\
u_3 &= \overline{C}\phi(x)\cos(n\theta)\cos(\omega t)\\
\phi_1 &= \overline{D}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
\phi_2 &= \overline{E}\phi(x)\sin(n\theta)\cos(\omega t)
\end{aligned} \tag{21}$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions, n denotes the number of circumferential waves in the mode shape and ω is the natural angular frequency of the vibration. The axial function $\phi(x)$ is chosen as the beam function as:

$$\phi(x)=\alpha_1\cosh\left(\frac{\lambda_m x}{L}\right)+\alpha_2\cos\left(\frac{\lambda_m x}{L}\right)-\zeta_m\left(\alpha_3\sinh\left(\frac{\lambda_m x}{L}\right)+\alpha_4\sin\left(\frac{\lambda_m x}{L}\right)\right)$$
(22)

where $\alpha_i\ (i=1,...,4)$ are some constants with value 0 or 1 chosen according to the boundary conditions. λ_m, are the roots of some transcendental equations and ζ_m are some parameters dependent on λ_m. The $\alpha_i\ (i=1,...,4)$, the transcendental equations and the parameters ζ_m considered. To determine the natural frequencies, the Rayleigh-Ritz method is used. The energy functional Π defined by the Lagrangian function as

$$\Pi=T_{\max}-U_{\max} \tag{23}$$

Substituting Eq. (21) into Eqs. (18) and (19) and minimizing the energy functional Π with respect to the unknown coefficients as follows,

$$\frac{\partial\Pi}{\partial\overline{A}}=\frac{\partial\Pi}{\partial\overline{B}}=\frac{\partial\Pi}{\partial\overline{C}}=\frac{\partial\Pi}{\partial\overline{D}}=\frac{\partial\Pi}{\partial\overline{E}}=0 \tag{24}$$

In Eq. (23), T_{max} and U_{max} are the maximum kinetic energy and strain energy, respectively. In Eq. (24), the five governing eigenvalue equations can be obtained. These five governing eigenvalue equation can be expressed in matrix from as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25}\\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55}\end{bmatrix}\begin{Bmatrix}\overline{A}\\ \overline{B}\\ \overline{C}\\ \overline{D}\\ \overline{E}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\end{Bmatrix} \tag{25}$$

The eigenvalue equations are solved by imposing the non-trivial solutions condition and equating the determinant of the characteristic matrix $[C_{ij}]$ to zero. Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ\omega^{10}+\beta_1\omega^8+\beta_2\omega^6+\beta_3\omega^4+\beta_4\omega^2+\beta_5=\circ \tag{26}$$

where $\beta_i\ (i=0,1,2,3,4,5)$ are some constants. Eq. (26) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell in the x, θ and z directions. The smallest of the five roots is the natural angular frequency studied in the present study.

III. RESULTS AND DISCUSSION

In this paper studies are presented on the vibration of simply supported functionally graded (FG) cylindrical shell. The functionally gradient material (FGM) considered is composed of stainless steel and nickel and its properties are graded in the thickness direction according to the volume fraction power-law distribution. The influence of constituent volume fractions is studied by varying the volume fractions of the stainless steel and nickel. This is carried out by varying the value of the power law exponent N. The effects of the FGM configuration are studied by studying the frequencies of two FG cylindrical shells. Type I FG cylindrical shell and Type II FG cylindrical shell. Type I FG cylindrical shell has Nickel on its inner surface and stainless steel on its outer surface and Type II FG cylindrical shell has stainless steel on its inner surface and nickel on its outer surface. The material properties for stainless steel and nickel, calculated at $T=300K$, are presented in table 1

TABLE I
PROPERTIES OF MATERIALS

Coefficients	Stainless Steel			Nickel		
	E	ν	ρ	E	ν	ρ
P_0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

To validate the present analysis, results for cylindrical shells are compared with Chung [20]. The comparisons show that the present results agreed well with those in the literature.

TABLE II

COMPARISON OF FREQUENCY $\left(\frac{rad}{s}\right)$ PARAMETER $\Omega = \omega R\sqrt{\frac{((1-\nu^2)\rho)}{E}}$ FOR A ISOTROPIC CYLINDRICAL SHELL

Case	$\omega\left(\frac{rad}{s}\right)$	Ω Chung [18]	Ω Present
$(L/R)=10$, $(R/h)=500$, $n=4$	327.5406	0.01508	0.0154656
$(L/R)=10$, $(R/h)=20$, $n=2$	1254.2173	0.05787	0.0592211
$(L/R)=2$, $(R/h)=20$, $n=3$	1380.3668	0.3117	0.235887

Tables 3 and 4 show the variations of the volume fractions V_f of Nickel and Stainless Steel, respectively, in the thickness direction z for a Type I FG cylindrical shell. the volume fraction for Nickel V_{fN} decreased from 1 at $z=-0.5h$ to 0 at $z=0.5h$ and the volume fraction of Stainless Steel V_{fss} increased from 0 at $z=-0.5h$ to 1 at $z=0.5h$.

TABLE III

VARIATION OF THE VOLUME FRACTION V_{fss} IN THE THICKNESS DIRECTION z FOR A TYPE I FG CYLINDRICAL SHELL

z	V_{fss} N=0.5	N=0.7	N=1	N=2	N=5	N=15
-0.5h	0	0	0	0	0	0
-0.4h	0.3162	0.1995	0.1	0.01	0.00001	$10^{-15}\times 1$
-0.3h	0.4472	0.3241	0.2	0.04	0.00032	$10^{-11}\times 3.27$
-0.2h	0.5477	0.4305	0.3	0.09	0.00243	$10^{-8}\times 1.43$
-0.1h	0.6324	0.5265	0.4	0.16	0.01024	0.00000107
0	0.707	0.6155	0.5	0.25	0.03125	0.00003051
0.1h	0.7745	0.6993	0.6	0.36	0.07776	0.0004701
0.2h	0.8366	0.7790	0.7	0.49	0.1680	0.004747
0.3h	0.8944	0.8553	0.8	0.64	0.3276	0.03518
0.4h	0.9486	0.9289	0.9	0.81	0.5904	0.20589
0.5h	1	1	1	1	1	1

TABLE IV

VARIATION OF THE VOLUM FRACTION V_{fN} IN THE THICKNESS DIRECTION z FOR A TYPE I FG CYLINDRICAL SHELL

z	V_{fN} N=0.5	N=0.7	N=1	N=2	N=5	N=15
-0.5h	1	1	1	1	1	1
-0.4h	0.6837	0.8004	0.9	0.99	0.9999	1
-0.3h	0.5527	0.6758	0.8	0.96	0.9996	1
-0.2h	0.4522	0.5694	0.7	0.91	0.9975	0.9999
-0.1h	0.3675	0.4734	0.6	0.84	0.9897	0.9999
0	0.2928	0.3844	0.5	0.75	0.9687	0.9999
0.1h	0.2254	0.3006	0.4	0.64	0.9222	0.9995
0.2h	0.1633	0.2209	0.3	0.51	0.8319	0.9952
0.3h	0.1055	0.1449	0.2	0.36	0.6723	0.9648
0.4h	0.0513	0.0710	0.1	0.19	0.4095	0.7941
0.5h	0	0	0	0	0	0

IV. CONCLUSIONS

A study on the vibration of functionally graded (FG) Cylindrical shell composed of stainless steel and nickel has been presented. The study was carried out for two types of functionally graded cylindrical shells where the configurations of the constituent materials in the functionally graded cylindrical shells are different. One is termed as a Type I FG cylindrical shell and has properties that vary continuously from nickel on its inner surface to stainless steel on its outer surface. The other is termed as a Type II FG cylindrical shell and has properties that vary continuously from stainless on its inner surface to nickel on its outer surface. The analysis of the functionally graded cylindrical shell is carried out using third order shear deformation shell theory and solved using Rayleigh-Ritz method with energy functional, obtained using an energy approach. Studied were made on study the natural frequencies, the influence of constituent volume fractions, the effects of configurations of the constituent materials on the frequencies for two kind of FG cylindrical shell and the influence of boundary conditions simply support on the frequencies. The study showed that the constituent volume fractions and the configurations of the constituent materials affect the natural frequencies. However, because of the functionally graded cylindrical shells exhibit interesting frequency characteristics when the constituent volume fractions are varied. This is done by varying the power law exponent N. The influence of the constituent volume fraction on the frequencies for Type I and II FG cylindrical shells has been found to be different. For the Type I FG cylindrical shells, the natural frequencies decreased when N increased, and for the Type II FG cylindrical shells, the natural frequencies increased when N decreased.

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Biography

M.R.Isvandzibaei, Department of Mechanical Engineering, Mahshahr Branch, Islamic Azad University, Mahshahr, Iran.
E-mail: esvandzebaei@yahoo.com
Tel: +989163442982

Ali Moarrefzadeh, Faculty member of Department of Mechanical Engineering, Mahshahr Branch, Islamic Azad University , Mahshahr , Iran
E-mail: a_moarrefzadeh@yahoo.com
a.moarefzadeh@mahshahriau.ac.ir
Tel: +989123450936