A Theoretical Investigation of SiO_2-Water Nanofluid Heat Transfer Enhancement over an Isothermal Stretching Sheet

Aminreza Noghrehabadi, *Mehdi Ghalambaz and Mohammad Ghalambaz

***Abstract*— The objective of the present study is to analyze the heat transfer enhancement of water in the presence of Sio_2 nanoparticles over an isothermal stretching sheet. To aim this purpose the boundary layer governing differential equations are transformed to a set of ordinary differential equations using similarity transformations, and they are solved numerically. The effect of SiO_2 nanoparticle volume fraction on the heat transfer characteristics is discussed. It is found that the heat transfer rate (reduced Nusselt number) increases with increase of nanoparticle volume fraction.**

***Keywords*—** SiO_2-Water, Nanofluid and Stretching Sheet

I. INTRODUCTION

The flow induced by a moving boundary over a flat plate, known as stretching sheet, is important in the extrusion processes in plastic and metal industries. Sakiadis was the first who carried out the pioneering work in this area [1, 2] and analyzed the boundary layer flow on a continuously stretching sheet with a constant speed. Work of Sakiadis was further verified by Tsou et al. experimentally [3]. Following these works, the various aspects of problem with different boundary conditions and fluids including magnetic flows [4, 5], micropolar fluids [6] and non-Newtonian fluids [7, 8] were generated by other researchers.

Recently, nanofluids are among the most intensively investigated options to enhance heat transfer [9, 10]. A very small amount of nanoparticles, when dispersed uniformly and suspended stably in base fluids, can provide impressive improvements in the thermal properties of base fluids.

Nanofluids are liquids in which the particles in the size of nanometer, ranging from about 10 to 200 nm, are suspended in a base fluid. The general expectation is that the thermal conductivity of such suspensions is significantly enhanced than that of the base fluid. Because of impact of nanoparticles of nanofluid properties, Ho et al. adopted four different models for the effective viscosity and thermal conductivity of nanofluids, demonstrating the importance of dynamic viscosity [11].

Aminreza Noghrehabadi is with Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

*Mehdi Ghalambaz is with the Department of Mechanical Engineering, Dezful Branch, Islamic Azad University, Dezful, Iran (Corresponding author phone: +98-916-341-1491; e-mail: ghalambaz.mehdi@gmail.com)

Mohammad Ghalambaz is with the Department of Mechanical Engineering, Dezful Branch, Islamic Azad University, Dezful, Iran

Very recently, number of researchers analyzed the flow and heat transfer of nanofluids over stretching sheet. Yacob et al. analyzed enhancement of two types of nanofluids, namely, water-Cu and water-Ag nanofluids over an stretching sheet with convective boundary condition [12]. Rana and Bhargava numerically studied the flow and heat transfer of a nanofluid over a nonlinearly stretching sheet [13]. Khan and Pop, examined the boundary-layer flow of a nanofluid past a stretching sheet [14]. Makinde and Aziz analyzed the boundary layer flow of nano fluids over a stretching sheet with convective heat transfer boundary condition [15].

In the present study the effect of volume fraction of SiO_2 on flow and heat transfer characteristics of the boundary layer over a continuous stretching sheet has been investigated numerically.

II. MATHEMATICAL FORMULATION

Consider an incompressible steady two-dimensional boundary layer flow past a stretching sheet in a water-based nanofluid which can contains different volume fraction of SiO_2 nanoparticles. The scheme of the physical model and geometrical coordinates are depicted in Fig. 1.

It is assumed that the induced flow of nanofluid is laminar, and the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the fluid and nanoparticles are given in Table 1 [16]. It is assumed that the sheet surface has constant temperature of T_w, and the temperature of ambient fluid is T_∞. The fluid outside the boundary layer is quiescent, and the stretching sheet velocity is $U(x)=cx$ where c is a constant.

The steady two-dimensional boundary layer equations for the nanofluid, in the Cartesian coordinates can be represent as [12],

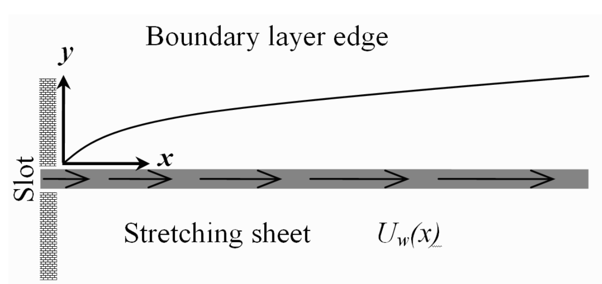

Fig. 1. Scheme of stretching sheet configuration

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Subject to the following boundary conditions,

$$v=0,\qquad u=U_W(x),\quad T=T_W\ ,\ at\ y=0 \tag{4}$$

The boundary conditions at the far field (i.e. $y\rightarrow 0$) are defined as,

$$v=u=0,\quad T=T_\infty\ ,\quad as\ y\rightarrow\infty \tag{5}$$

where the subscript of nf denote the properties of nanofluid. The properties of nanofluids are defined as follows [12],

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_P)_{nf}}, \tag{6}$$

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s\ , \tag{7}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{\frac{5}{2}}}, \tag{8}$$

Therefore,

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_P)_f+\phi(\rho C_p)_s \tag{9}$$

and,

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{10}$$

where subscript of f and s represent the base fluid and nanoparticle suspension, respectively. Here, φ is the nanoparticle volume fraction.

To attain similarity solution of equations (1-3) subject to (4-5), the dimensionless variables can be posited in the following form,

$$\begin{cases}\eta=\left(\dfrac{c}{v_f}\right)^{1/2} y,\ \ \theta(\eta)=\dfrac{T-T_\infty}{T_W-T_\infty},\\ u=cxf'(\eta),\ v=-\sqrt{cv}\,f(\eta),\end{cases} \tag{11}$$

By applying the introduced similarity transforms of (11) on the governing equations of (1-3), the similarity equations are obtained as follows,

$$\frac{1}{(1-\phi)^{\frac{5}{2}}\left(1-\phi+\phi\,\rho_s/\rho_f\right)}f'''+f\,f''-f'^2=0 \tag{12}$$

$$\frac{1}{pr}\frac{\dfrac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}}{\left(1-\phi+\phi(\rho C_p)_s/(\rho C_p)_f\right)}\theta''+f\,\theta'=0, \tag{13}$$

Subject to the following boundary conditions:

$$At\ \eta=0\ :\ f=0,\qquad f'=1,\qquad \theta=1, \tag{14}$$

$$At\ \eta\rightarrow\infty\ :\ f'=0,\qquad \theta=0, \tag{15}$$

For practical purposes, the skin friction coefficient can be introduced as,

$$\begin{gathered}C_f=-\frac{\mu_{nf}}{\rho_f u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}=\frac{1}{(1-\phi)^{\frac{5}{2}}}\sqrt{\mathrm{Re}_x}\,f''(0),\\ \frac{C_f}{\sqrt{\mathrm{Re}_x}}=\frac{f''(0)}{(1-\phi)^{\frac{5}{2}}},\end{gathered} \tag{16}$$

and the reduced Nusselt number as,

$$\begin{gathered}Nu_x=\frac{xk_{nf}}{k_f(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\sqrt{\mathrm{Re}_x}\frac{k_{nf}}{k_f}\theta'(0)\\ Nur=\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-\frac{k_{nf}}{k_f}\theta'(0)\end{gathered} \tag{17}$$

III. RESULTS

The nonlinear ordinary differential equations (12) and (13) subject to the boundary conditions (14) and (15) were solved numerically by the Runge-Kutta-Fehlberg method with shooting technique. The value of η_∞ is taken as 15 to achieve the asymptotically far filed boundary condition. The equations are solved for variation of volume fraction of SiO_2 nanoparticles. The Prandtl number of the base fluid (water) is kept constant at 6.2. It is worth mentioning that when φ = 0, this study reduces to those of a viscous or regular fluid. As the test of the accuracy of numerical solution, the results of present method are compared with results of Hamad [17] and Wang [18] as well as Gorla and Sidawi [19], and they found in good agreement.

TABLE 1. THERMOPHYSICAL PROPERTIES OF WATER AND SiO_2

Physical Properties	Fluid Phase (water) [12]	SiO2[16]
Cp (J/kg.K)	4179	765
ρ (kg/m3)	997.1	3970
k (W/m.K)	0.613	36
α ×107 (m2/s)	1.47	118.536

TABLE 2. COMPARISON OF RESULTS FOR $-\theta'(0)$ WITH PREVIOUS WORKS

Pr	Hamad [17]	Gorla and Sidawi [19]	Wang [18]	Present Results φ=0
0.7	0.45391	0.4539	0.4539	0.4539
2	0.91136	0.9114	0.9114	0.9114
7	1.89540	1.8905	1.8954	1.8954
20	3.35390	3.3539	3.3539	3.3539
70	6.46220	6.4622	6.4622	6.4623

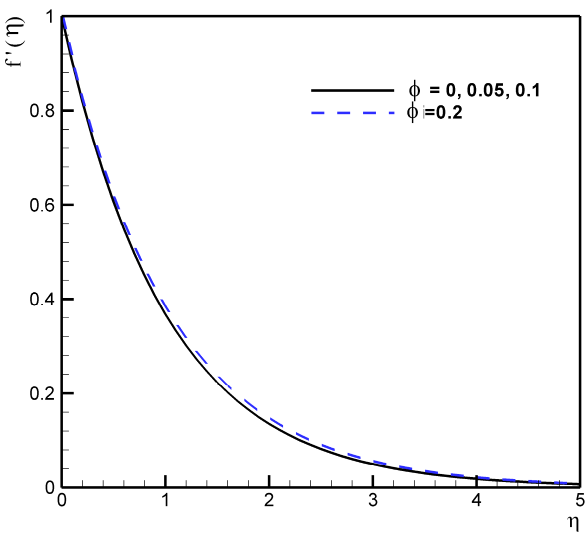

Fig. 2. Profiles of non-dimension velocity for selected values of volume fraction of φ

Fig. 2 shows the profiles of dimensionless velocity for selected values of SiO_2 nanoparticle volume fraction. This figure reveals that increase of nanoparticle volume fraction increases the magnitude of velocity for comparatively high values of nanoparticles. The low volumes fractions of nanoparticles have not significant effect on the velocity profiles. Fig. 3 shows profiles of $f''(\eta)$ over the stretching sheet. This figure depicts that increase of nanoparticle volume fraction in low concentrations has not any significant effect on the shear stresses in the fluid, but comparatively large values of concentration results in increase of $f''(\eta)$ near the wall.

Fig. 4 depicts the variation of non-dimensional wall shear stress with variation of nanoparticle volume fraction. As seen, increase of φ increases the wall shear stress.

The profiles of dimensionless temperature are plotted in Fig. 5 for different values of nanoparticle volume fractions. The profiles are begun from $\theta(\eta)=1$ for the $\eta=0$ and asymptotically tend to zero as η tends to its asymptotical value.

This figure (Fig. 5) reveals that increase of nanoparticle volume fraction in any concentration has significant effect on the temperature profile. As seen, the magnitude of temperature increases with increase of concentration. Furthermore, it is clear that increase of concentration increases the thickness of the thermal boundary layer, but it has not significant effect on the thickness of hydrodynamic boundary layer which is depicted in Fig 2.

Fig. 6 shows the profiles of $\theta'(\eta)$ for selected values of nanoparticle concentrations. This figure depicts that increase of nanoparticle volume fraction first decreases the values of $\theta'(\eta)$ near the wall and then increases them.

Fig. 7 shows the relation of nanoparticle volume fraction of SiO_2 with variation of reduced Nusselt number. This figure reveals that increase of nanoparticle volume fraction increases the non-dimensional heat flux, reduced Nusselt number, approximately linearly.

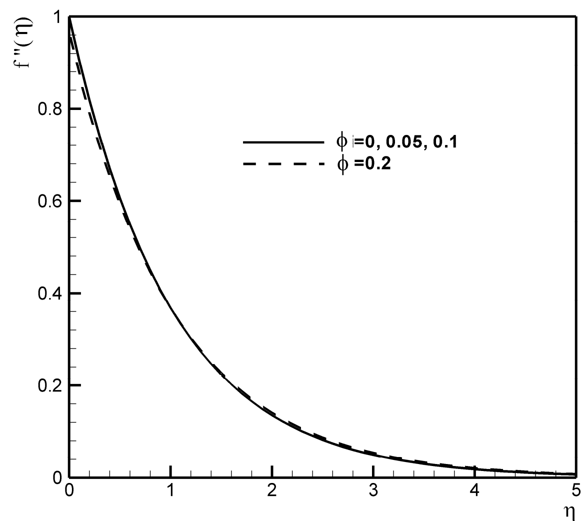

Fig. 3. Profiles of $f''(\eta)$ for selected values of volume fraction of φ

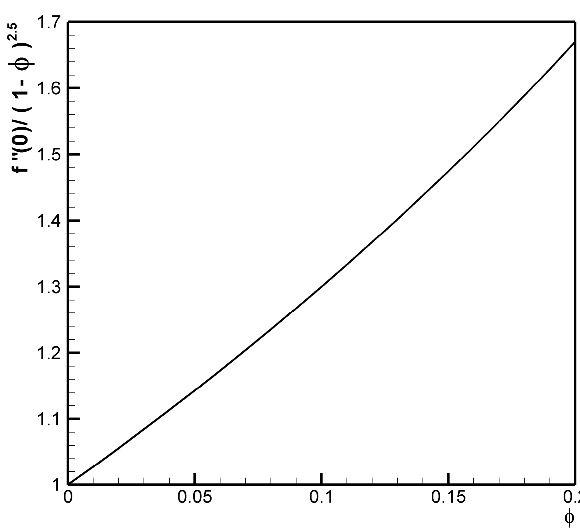

Fig. 4. Profiles of $f''(0)/(1-\varphi)^{2.5}$ for selected values of volume fraction of φ

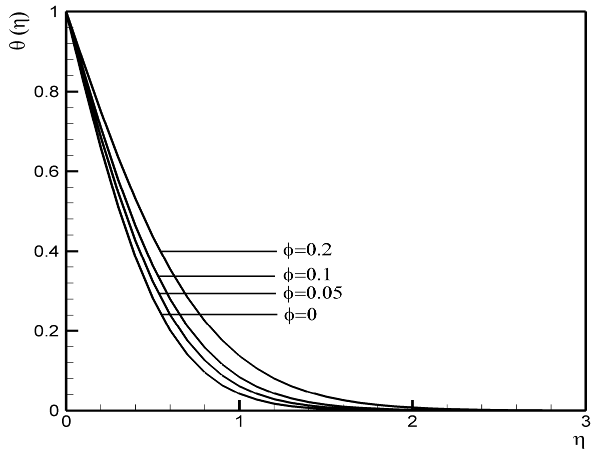

Fig. 5. Profiles of θ (η) for selected values of volume fraction of φ

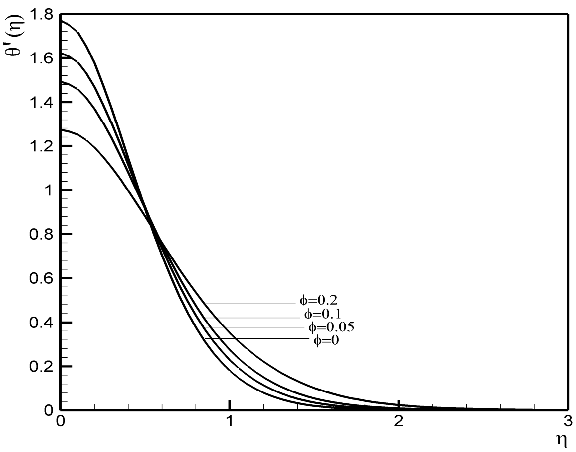

Fig. 6. Profiles of θ' (η) for selected values of volume fraction of φ

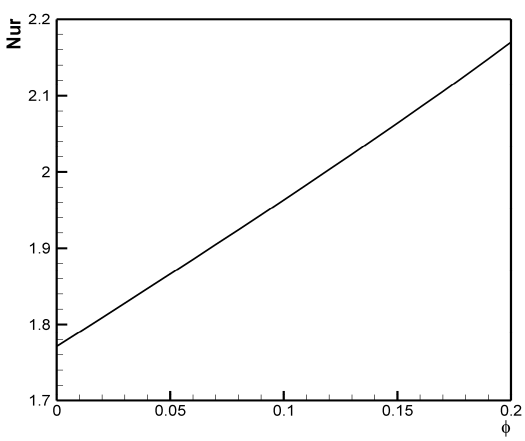

Fig. 7. Variation of reduced Nusselt number with variation of nanoparticle volume fraction of φ

VII. CONCLUSIONS

In the present work, the effect of variation of SiO_2 nanoparticles volume-fractions on the boundary layer flow and heat transfer over a stretching sheet has been analyzed. The governing differential equations are transformed to ordinary differential equations and solved numerically. The results show that, increase of nanoparticle volume fraction results in increase of wall shear stress as well as reduced Nusselt number. Furthermore, any increase of nanoparticle volume fraction increased the magnitude of dimensionless temperature profiles, but in low volume fractions of nanoparticles the increase of dimensionless velocity profiles in not very significant.

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