Numerical Study of Heat Flux and Temperature of Spherical Stagnation Point Using Trajectory Data and Applying Hot Gas Effects

Mahmood Pasandideh fard[1], Hamid Mohammadiun[2], Mohammad Mohammadiun[3] and Omid Mahian[4]
[1]Ferdowsi University of Mashhad, Mashhad, Iran
[2,3]Department of Mechanical Engineering, Shahrood Branch, Islamic Azad University, Shahrood, Iran
[4]Young Researchers Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran
omid.mahian@gmail.com

Abstract— **In this paper, the aero heating of spherical stagnation point is studied. For this purpose, we first calculate the thermodynamic properties behind of the normal shock, for the supersonic flow of the hot gas. After finding these properties, we can calculate the boundary layer properties and then we will be able to determine the heat flux of the spherical stagnation point. This heat flux can be used as a boundary condition in heat transfer code for axial asymmetric case. After computing transient multilayer heat transfer toward the inner side of the shell (which material varies in each layer), we can determine stagnation point temperature and temperature distribution in various layers. To verify the validity of the used numerical procedure in this work, comparisons with the theoretical and experimental results for the X-33 space vehicle have been conducted.**

Keywords— Spherical Stagnation Point, Aero Heating, X-33 Space Vehicle and Hot Gas Effects

I. INTRODUCTION

High temperatures will be generated inside the boundary layer in hypersonic flows, due to high kinetic energy and friction losses. These high temperatures cause vibrating simulation and molecular crack and even ionized air. In this situation, the air can not be assumed as an ideal gas. For example, if we consider a rocket that enters the atmosphere with M=10 and $T_\infty = 283$ K, the hot gas temperature behind shock can be obtained by:

$$T/T_\infty = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_\infty^2 \sin^2\beta \tag{1}$$

where $T = 5502.7$ K , Of course this is not the real temperature because of incorrect assumptions. The errors are due to considering $\gamma = 1.4$. Now, what is the difference between hot gas flow and flow with constant γ? We have answered this question in following expressions:

1- All of the thermodynamic properties are different.
2- Transmission properties are totally different.
3- In hot gas flow condition, γ is variable, thus equations obtained based on a constant γ are not reliable.

According to what we have mentioned above, we should use a convenient process to eliminate errors arise from the assumption of ideal gas for the air.

II. PROBLEM FORMULATION

A. Determination of Hot Gas Flow Condition in the Edge of Stagnation Point Boundary Layer

According to the conditions before the shock, we can use simplified continuity, momentum and energy equations on a streamline:

$$\rho_1 V_1 = \rho_2 V_2 \Rightarrow V_2 = \frac{\rho_1}{\rho_2} V_1 \tag{2}$$

$$P_2 = P_1 + \rho_1 V_1^2 (1 - \frac{\rho_1}{\rho_2}) \tag{3}$$

$$h_2 = h_1 + \frac{V_1^2}{2}\left[1 - (\frac{\rho_1}{\rho_2})^2\right] \tag{4}$$

Because we do not use any simplification due to ideal gas condition and also our assumptions are based on steady state and Newtonian fluid, then the above relations are useful for hot gas flow. As we have said in pervious section, in supersonic flow, the temperature of fluid will increase greatly behind of the shock, so that; O_2 and N_2 molecules can be ionized. In this case, thermodynamic properties of the air vary totally, therefore the γ will be variable. This means that we can not extract γ from differential and integral relations.

Also we can not use thermodynamic relations like sound velocity, temperature, equation of state and relations between enthalpy and temperature in hot gas flow condition. If we consider air as a hot gas, we will be able to use similar relations concluded from fitting curves from statistical thermodynamic data [1]. In new relations, every thermodynamic function is a function of two thermodynamic independent variables:

$$\rho = f_1(P,T) \tag{5}$$

$$h = f_2(P,T) \tag{6}$$

$$a = f_3(P,T) \tag{7}$$

In relation (7), a is the sound velocity. For transforming quantities $\mu, \Pr$, we have following functions:

$$\mu = \mu(P,T) \tag{8}$$

$$\Pr = \Pr(P,T) \tag{9}$$

By choosing $P_1 = P_\infty, T_1 = T_\infty$, and $M_1 = M_\infty$ (flow properties upstream of the shock) we are able to determine ρ_1 and h_1 with using interpolation in relations (5) and (6). We start with an initial guess resulted from ideal gas relation. The values of ρ_1, u_1, P_1 and h_1 are known, so relations (3) and (4) convey P_2 and h_2 as a function of ρ_2 with one unknown component. By using iteration procedure we can determine flow properties after the shock.

After finding ρ_2 from initial guess, P_2 from relation (3) and h_2 from relation (4), by using distance division we calculate P_2, h_2 and T_2:

$$h_2 = h(P_2, T_2) \tag{10}$$

Now, we can calculate new density by using P_2 and T_2 :

$$\rho_{2new} = f_1(P_2, T_2) \tag{11}$$

Now we can determine new values of h_2, P_2, T_2 and repeat steps until the convergence reached. Since we have calculated above properties on the streamline, we can use this method for projectile with lateral booster, only if we determine stagnation point streamline with a secondary program.

B. Solution of flow on Stagnation Point

In this study, Fay and Riddell classic solution have been used because their results are applicable in industry and supersonic projectiles. We have following assumptions:

1) Flow conditions in external edge of boundary layer have local thermodynamic and chemical equilibrium.
2) Inviscid velocity distribution in external edge of boundary layer in stagnation region with incompressible classic results is:

$$u_e = ax \tag{12}$$

$$a = \left(\frac{du_e}{dx}\right)_s \tag{13}$$

In above relation $a = \left(\frac{du_e}{dx}\right)_s$ is the velocity gradient in stagnation point:

$$\left(\frac{du_e}{dx}\right)_s = \frac{1}{R}\sqrt{\frac{2(p_e - p_\infty)}{\rho_e}} \tag{14}$$

Surface heat transfer can be determined with following relation:

$$q_w = \left(k\frac{\partial T}{\partial y}\right)_w + \left(\rho D_{12}\sum_i h_i \frac{\partial C_i}{\partial y}\right)_w \tag{15}$$

In above relation D is the mass fraction and C is the diffusion coefficient. It is notable that surface heat transfer in viscous flow in presence of chemical reaction is not only the result of temperature heat transfer but diffusion is affective too.

Riddell and Fay obtained these results in comparison with gas without reaction:

$$q_w = 0.76\Pr^{-0.6}(\rho_e\mu_e)^{0.4}(\rho_w\mu_w)^{0.1}\sqrt{\left(\frac{du_e}{dx}\right)_S}(h_{Oe}-h_w)\left[1+(Le^{0.52}-1)\left(\frac{h_D}{h_{Oe}}\right)\right] \tag{16}$$

In above equation Le is the levies number and h_o is the stagnation enthalpy and the subscripts e and w introduce boundary layer conditions near the surface condition, respectively. The value of h_D is obtained with following relation:

$$h_D = \sum_i C_{ie}(\Delta h_f)_i^\circ \tag{17}$$

Furthermore, q_w is related to the heat transfer parameter $\left(\frac{Nu}{\sqrt{\mathrm{Re}}}\right)$ as following relation:

$$q_w = \left(Nu\big/\sqrt{\mathrm{Re}}\right)\sqrt{\rho_w\,\mu_w\left(\frac{du_e}{dx}\right)_S}\;\left[(h_S - h_w)/0.76\Pr^{-0.6}\right] \tag{18}$$

C. Temperature Distribution in Transient Condition

The governing equation is multi layer transient heat transfer equation which in cylindrical coordinates with axial asymmetry is given as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\,r\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = \rho C_P\frac{\partial T}{\partial t} \tag{19}$$

After generating a computational flow field, the governing equations are discretized and solved by using Gauss-Sidle algorithm. In each time step, the heat flux obtained from relation (16) is used as boundary condition for above equation. Because of complexity the problem geometry, general coordinate have been used.

For this purpose, the calculations have been done in the rectangular coordinate system $(\xi - \eta)$ initially, and then the results transfer to physical coordinate system $(r - z)$.

Temperature gradient in physical coordinate system is defined as $\nabla T = T_z\, i + T_r\, j$, in which the components T_r, T_z are:

$$T_z = \frac{1}{J}(r_\eta T_\xi - r_\xi T_\eta) \tag{20}$$

$$T_r = \frac{1}{J}(-z_\eta T_\xi + z_\xi T_\eta) \tag{21}$$

Also, Laplacian T in rectangular coordinate system is given by:

$$\nabla^2 T = \frac{1}{J^2}\left[\alpha T_{\xi\xi} - 2\beta T_{\xi\eta} + \gamma T_{\eta\eta}\right] + \left[(\nabla^2 \xi) T_\xi + (\nabla^2 \eta) T_\eta\right] \tag{22}$$

We can calculate α، β، γ، $\nabla^2 \xi$ and $\nabla^2 \eta$ with following relations:

$$\alpha = z_\eta^2 + r_\eta^2 \tag{23}$$

$$\beta = z_\xi z_\eta + r_\xi r_\eta \tag{24}$$

$$\gamma = z_\xi^2 + r_\xi^2 \tag{25}$$

$$\nabla^2 \xi = \frac{k_1 (r_{\xi\xi} z_\eta - z_{\xi\xi} r_\eta) + k_2 (r_{\xi\eta} z_\eta - z_{\xi\eta} r_\eta)}{J} + \frac{k_3 (r_{\eta\eta} z_\eta - z_{\eta\eta} r_\eta)}{J} \tag{26}$$

$$\nabla^2 \eta = \frac{k_1 (z_{\xi\xi} r_\xi - r_{\xi\xi} z_\xi) + k_2 (z_{\xi\eta} r_\xi - r_{\xi\eta} z_\xi)}{J} + \frac{k_3 (z_{\eta\eta} r_\xi - r_{\eta\eta} z_\xi)}{J} \tag{27}$$

$$k_1 = \frac{1}{J^2}(z_\eta^2 + r_\eta^2) \tag{28}$$

$$k_2 = \frac{-2}{J^2}(z_\xi z_\eta + r_\xi r_\eta) \tag{29}$$

$$k_3 = \frac{1}{J^2}(z_\xi^2 + r_\xi^2) \tag{30}$$

$$\xi_z = \frac{1}{J} r_\eta \tag{31}$$

$$\xi_r = -\frac{1}{J} z_\eta \tag{32}$$

$$\eta_z = -\frac{1}{J} r_\xi \tag{33}$$

$$\eta_r = \frac{1}{J} z_\xi \tag{34}$$

In above relation J is the transformation Jacobin, where:

$$J = z_\xi r_\eta - r_\xi z_\eta \tag{35}$$

Furthermore, according to figure (1), in multi layer problems, we can use following relations in separation place of bodies:

$$J = z_\xi r_\eta - r_\xi z_\eta \tag{36}$$

$$k_A (T_{i,j} - T_{i-1,j}) + \frac{2 k_A k_B}{k_A + k_B}(T_{i,j} + T_{i-1,j}) = k_B (T_{i+1,j} - T_{i,j}) + \frac{2 k_A k_B}{k_A + k_B}(T_{i,j+1} + T_{i,j}) \tag{37}$$

$$k_B (T_{i,j} - T_{i-1,j}) + \frac{2 k_C k_B}{k_C + k_B}(T_{i,j} + T_{i-1,j}) = k_C (T_{i+1,j} - T_{i,j}) + \frac{2 k_C k_B}{k_C + k_B}(T_{i,j+1} + T_{i,j}) \tag{38}$$

III. RESULTS AND DISCUSSION

Because the existing codes have some faults in computing stagnation point heating, the presented code in here along with another codes can be used to compute the stagnation point temperature in general locations and also in the presence of air chemical reactions. Furthermore, since the stagnation point temperature is the maximum value, thus its effect should be concluded in design of thermal shields. Determining heat flux and stagnation point temperature is very important as the first step in the design. Presented code, by using trajectory data, determines the heat flux and stagnation point temperature in a short time and a good accuracy.

To validate the presented code in computing the stagnation point heat flux (q_w), with using relation (18) and according to conditions given in reference [3], the heat transfer parameter $(\frac{Nu}{\sqrt{\mathrm{Re}}})$, have been calculated in velocity range from 1768 m/s up to 6950 m/s.

In this code, the wall temperature T_w varies from 300 K to 3000 K. According to figure (2), the results are very similar in references [2] and [3]. The differences between them are because of Prandtl number. In above procedure, Prandtl number must be computed but in the references Prandtl number 0.71 is considered. Also the obtained results have been compared with Hollis and Horvath theoretical and experimental results that have been done on X-33 space vehicle (Figures (3), (4)). In this experimental study, the effect of turbulent and laminar flow has been investigated on an aero heating model for flight trajectory in wind tunnel. Calculations are based on attack angle 30° and heat distribution is based on $\frac{h}{h_{FR}}$ Fraction where h_{FR} is the coefficient of reference heat transfer. In related calculations, h is:

$$h = \dot{q} \big/ (H_{aw} - H_w) \qquad (39)$$

In the above relation, $\dot{q}$ is the heat rate of the wall based on the Fay-Riddle theory for the nose of the model X-33 (a hemisphere with radius 1.6 cm). The Enthalpy at wall is calculated with a wall temperature equal to 300 K. Non dimensional geometric positions are $\frac{Y}{L}, \frac{X}{L}$, in which L is the initial length (25 cm). In this experiment free jet conditions are following:

$$M_\infty = 5.99 \qquad T_\infty = 62.1\ K$$

$$\rho_\infty = 0.0628 kg/m^3 \qquad \mathrm{Re}_{\infty,L} = 3.33\times10^6$$

$$h_{FR} = 0.539 kg/m^2 s \qquad \alpha = 30^0$$

The comparison presented in Fig. 5 belongs to point $\frac{X}{L} = 0.05$. According to space vehicle attack angle 30° the stagnation point occurred in this point. As shown in Fig. 5, the results presented in this paper are very close to the experimental and theoretical results obtained by others. In order to validate the code for multi layer heat transfer, results have been compared with the results obtained using Fluent Software. For this purpose a material composed of three layers is considered: Asbestos $\left(\rho = 1850\frac{kg}{m^3}, C_P = 1200\frac{J}{kg.k}, k = 0.42\frac{w}{m.k}\right)$, Steel $\left(\rho = 7800\frac{kg}{m^3}, C_P = 500\frac{J}{kg.k}, k = 20\frac{w}{m.k}\right)$ and Aluminum $\left(\rho = 2719\frac{kg}{m^3}, C_P = 871\frac{J}{kg.k}, k = 202.4\frac{w}{m.k}\right)$. In this problem inner radius of quadrant is 0.6 cm and outer radius is 1.5 cm. The constant heat flux in top surface is $q_w = 10^5 \frac{w}{m^2 k}$ and other surfaces are insulated, also initial temperature assumed to be 300°K. Contours of temperature distribution after 118^{sec} are compared with Fluent results in Fig, 6 and Fig. 7 which are very close to each other. For this purpose, a quadrant geometry with inner radius $R=2.4^m$ and outer radius $R=2.43^m$ is considered for a ceramic material $\left(\rho = 6000\frac{kg}{m^2}, C_P = 450\frac{J}{kg.k}, k = 1.5\frac{w}{m.k}\right)$.

Fig. 8 and Fig. 9 show height and Mach number variation versus time. Fig. 10 shows comparison of stagnation point temperature distribution in 600^{sec} interval. Differences between the results of reference [5] and presented work are due to approximating the wall heat flux with the heat flux of stagnation point. Since, in this procedure, the heat flux of stagnation point is applied as the boundary conditions for the all grids that are located on outer surface, but in reference [5], the heat flux is calculated for points of outer surface distinctly. Although an approximation is applied in presented problem, but the results are exact.

IV. FIGURES

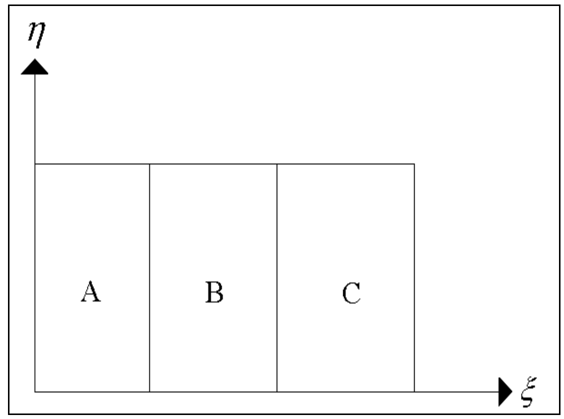

Fig. 1. Geometry of multi layer in rectangular coordinate system

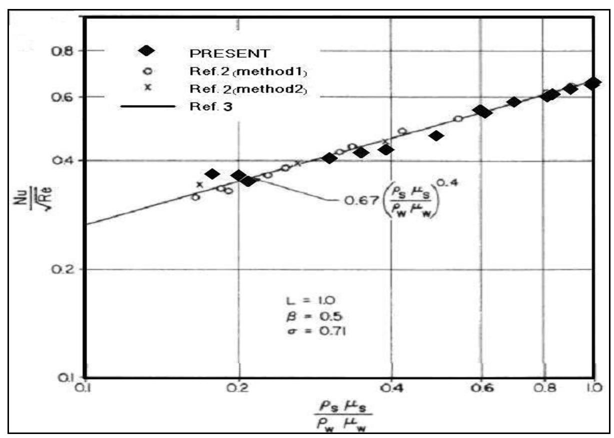

Fig. 2. Heating of stagnation point

Fig. 3. Overall view of X-33

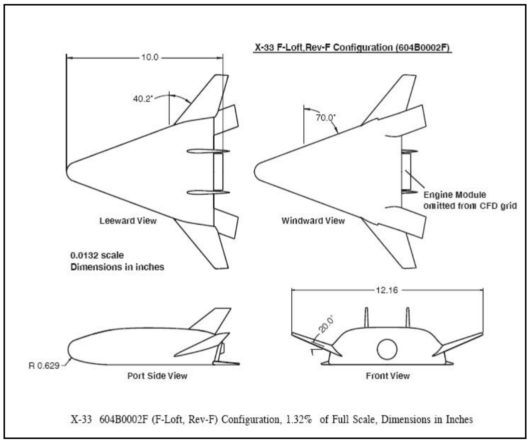

Fig. 4. Different views of X-33 model

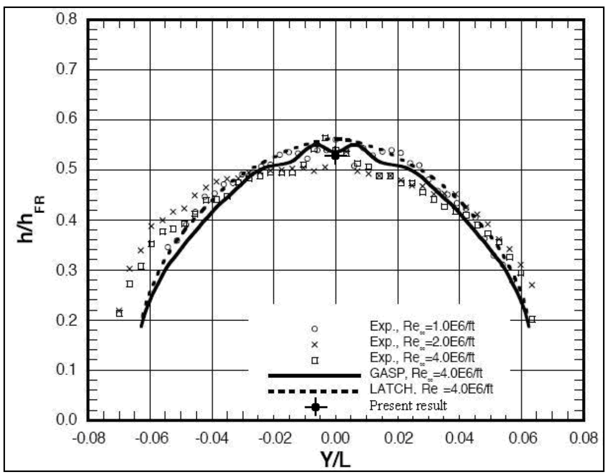

Fig. 5. Heat flux of stagnation point and Comparison with Ref [4]

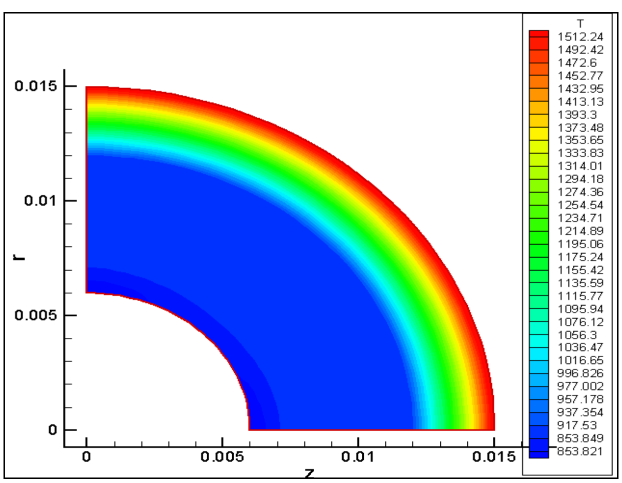

Fig. 6. Contours of temperature for multi layer condition (presented code)

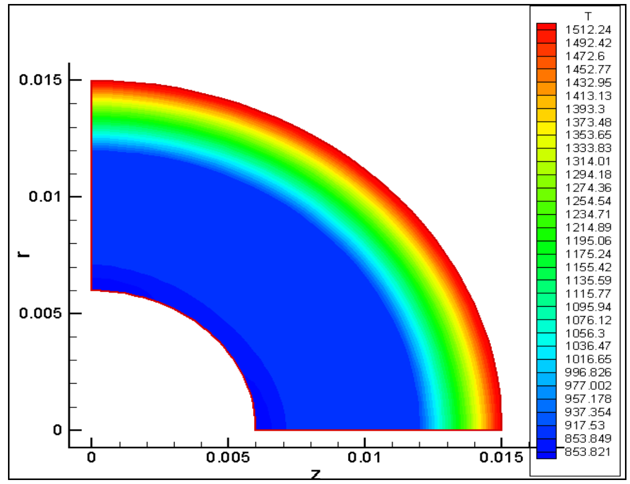

Fig. 7. Contours of temperature for multi layer condition(Fluent)

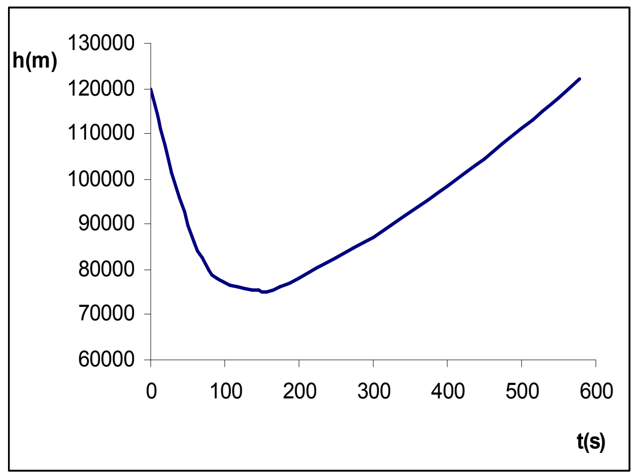

Fig. 8. Height variation vs. time

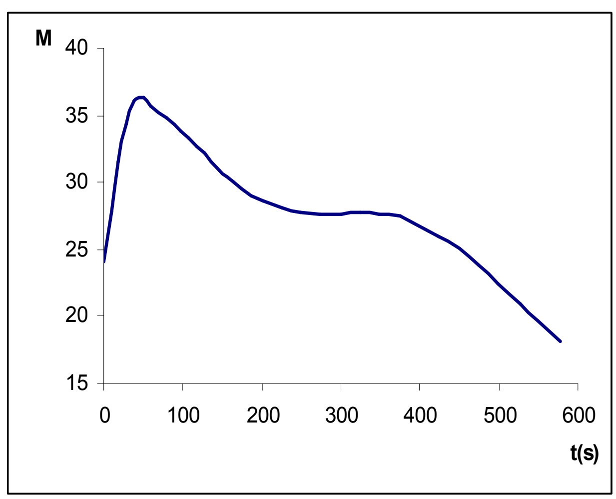

Fig. 9. Mach variation vs. time

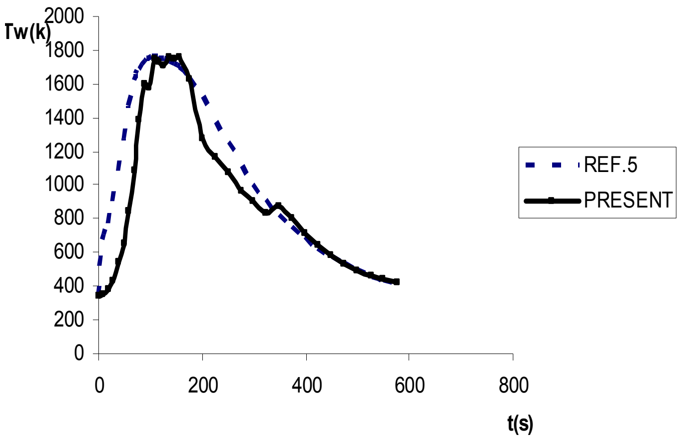

Fig. 10. curves of stagnation point temperature

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