Time Domain Chatter Stability Comparison of Turning and Milling Processes

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***Abstract*— The delay differential equations describing the turning and milling processes are solved using MATLAB and results compared. The same set of parameter combinations are used for turning tool, one tooth, three tooth and six tooth milling tools in generating graphical trajectories of cutting process. This resulted in the comparison that gave rise to the conclusion that under full-immersion conditions milling stability characteristics get closer to that of turning as the number of teeth of milling tool increases.**

***Keywords*—**Turning Process, Milling Process, Method of Steps and Trajectories

I. INTRODUCTION

The delayed dynamical model for regenerative vibration or chatter of cutting tool is:

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau) \qquad (1)$$

Where the natural frequency and damping ratio of the tool are given in terms of modal parameters k(stiffness) and m(mass) respectively as $\omega_n = \sqrt{\frac{k}{m}}$ and $\xi = \frac{c}{2\sqrt{mk}}$. c is the equivalent viscous damping of the tool. The system is seen to have an infinite dimensional phase space occasioned by the discrete delay $\tau = \frac{60}{n\Omega}$. Ω is the spindle speed and n is the number of cutting edges of the tool. The quantity $h(t)$ is called the specific force variation for the system which is derived in the works by Insperger (2002) and Ozoegwu (2012) for turning to be a constant of form:

$$h = C\gamma(v\tau)^{\gamma-1} \qquad (2)$$

where γ is the power of feed in the non-linear cutting force law, C is the cutting coefficient , v is the prescribed feed speed. $h(t)$ is a time periodic function for milling and has the form:

$$h(t) = \gamma(v\tau)^{\gamma-1}\sum_{j=1}^{n} g_j(t)C sin^{\gamma}\theta_j(t)\left[0.3sin\theta_j(t) + cos\theta jt\right. \qquad (3)$$

in Ozoegwu (2012) where $g_j(t) = \frac{1}{2}\left\{1 + sgn\left[\sin\left(\theta_j(t)\right)\right]\right\}$ the screen function and $\theta_j(t) = \left(\frac{\pi\Omega}{30}\right)t + (j-1)\frac{2\pi}{N}$ is the angular displacement of j.th cutting edge. The implication is that while turning is a time invariant delayed oscillator, milling is a delayed Mathieu system. This is the basis from which the differences of turning and milling processes derive. The time domain analysis of the equation involves determining its solution for known initial conditions and cutting parameters of spindle speed Ω and depth of cut w. It is seen that this type of analysis will result in the determination of stability of a point on a cutting parameter space. This is the limitation of time domain analysis though some useful stability deductions could still be drawn through this method if many of such solutions are obtained at carefully selected points as done in this paper. A much more complete and exhaustive stability analysis is conducted on the cutting parameter space.

The stability analysis of Eq. (1) on the cutting parameter space takes different form for turning and milling processes. The turning operation being a time invariant process, lends itself to the purely analytical method of D-subdivision as seen in Insperger (2002), Stepan (1998), Ozoegwu (2012) and Ozoegwu et al (2012) for its stability analysis that results in stability charts on which the parameter space is demarcated into the stable and the unstable sub domains. On the other hand, the parameter space stability analysis of milling is carried out using methods that are hybrids of analytical and numerical methods. This results in approximately accurate charts for milling as can be seen in Insperger (2002), Ozoegwu (2012), Insperger et al (2003) and Insperger and Stepan (2002).

Fig. 1 is the graph of specific force variation of the systems under consideration having the parameters $= 3.5 \times 10^7 \text{Nm}^{\frac{-7}{4}}$, $\gamma = 0.75$, $v = \frac{150mm}{min} = 0.0025m/s$ and $\Omega = 2000$rpm. The equivalent specific force variation for turning is seen from the relation $h = C\gamma(v\tau)^{\gamma-1}$ to be a constant that equals $2.8207 \times 10^8 Nm^{-2}$. The graphs of specific force variation for milling processes are generated using equation (3). It is clear from the graphs of figure1 that $h(t)$ is $\tau = \frac{60}{N\Omega}$ periodic. It is seen that the one tooth miller spends half of the period of its specific force variation in free flight during which the vibration is damped natural response. The three tooth and six tooth millers have specific force variation that can only have instantaneous zero value thus maintains continuous contact with the workpiece for full immersion operation. This means

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that damped natural vibration is not a component of the tool response of such systems.

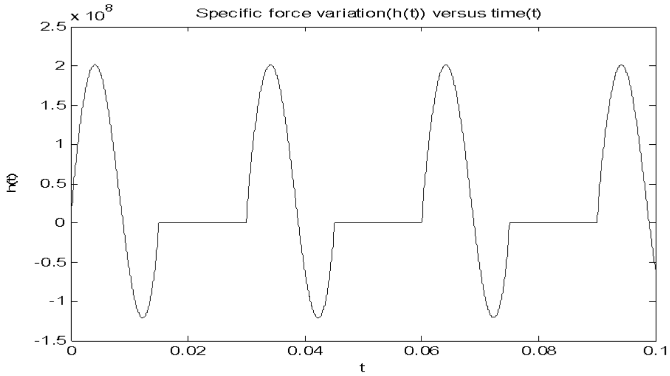

Fig. 1 (a): Specific force variation of fully-immersed one tooth miller

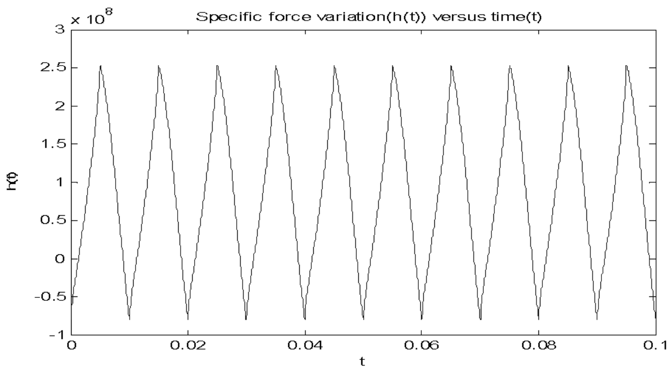

Fig. 1 (b): Specific force variation of fully-immersed three tooth miller

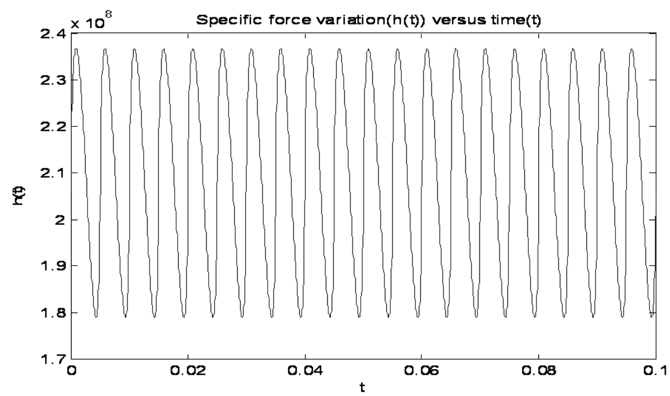

Fig. 1 (c): Specific force variation of fully-immersed six tooth miller

II. TIME DOMAIN CHATTER STABILITY ANALYSIS OF TURNING AND MILLING

The equation governing regenerative vibration of turning or milling tool with single discrete delay can be solve in the time domain if the Myshkis method of steps is evoked such that if the history is known in the time interval $[(n-2)\tau,(n-1)\tau]$ then a unique solution can be found in the interval $[(n-1)\tau, n\tau]$ where $n = 0, 1, 2, 3, \ldots\ldots$. General insight into the Myshkis method of steps can be gained from Myshkis (1989) and Shampine (2000). The governing equation for the nth step solution become:

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau),\ \ t \in [(n-1)\tau, n\tau] \quad (4a)$$

$$z(t) = H(t), \quad t \in [(n-2)\tau,(n-1)\tau] \quad (4b)$$

where $H(t)$ is the known history function in the delayed interval $[(n-2)\tau,(n-1)\tau]$. Making use of the history function, equation (4) is re-written thus:

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}h(t-\tau),\ \ t \in [(n-1)\tau, n\tau] \quad (5)$$

The delayed system in form of equation (5) has become an inhomogeneous linear ordinary differential equation which could be solved analytically or numerically. Random nature of perturbations will not allow $H(t)$ to be known precisely. Fortunately magnitude and nature of initial or remote perturbation is seen by Ozoegwu (2012) to be unlikely to affect the mathematical stability of an operating point thus constant perturbation history can be assumed for the first step solution and stability preserved. For the first step solution when $n = 0$ equation (4) could choose to have the form

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau),\ \ t \in [0,\tau] \quad (6a)$$

$$z(t) = 0, \dot{z}(t) = \dot{z}(0) \quad t \in [-\tau, 0] \quad (6b)$$

With equation (5) becoming an equation governing transient response in the interval $[0,\tau]$ then

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = 0,\ \ t \in [0,\tau] \quad (7)$$

Eq. (7) for turning is autonomous ordinary differential equation that can be solved analytically while for milling process it is non-autonomous ODE that can only be given either semi-analytical solution using the perturbation method or numerical solution.

The characteristic equation of the tool system for turning is seen from equation (7) to be:

$$\lambda^2 + 2\xi\omega_n\lambda + \left(\omega_n^2 + \frac{hw}{m}\right) = 0$$

with the roots $\lambda_{1,2} = -\xi\omega_n + ir$ where $r = \sqrt{(1-\xi^2)\omega_n^2 + \frac{hw}{m}}$. The first step solution for turning perturbation motion becomes

$$z(t) = A_1 e^{\lambda_1 t} + B_1 e^{\lambda_2 t},\ \ t \in [0,\tau] \quad (8)$$

With the initial conditions prescribed by the history of equation (7), the constants become $A_1 = \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}$ and $B_1 = \frac{-\dot{z}(0)}{\lambda_1 - \lambda_2}$ resulting in the first step solution becoming

$$z(t) = \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right)$$
$$\dot{z}(t) = \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t}\right) \quad (9)$$

Second step solution requires that the equation

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau), \ \ t \in [\tau, 2\tau] \tag{10a}$$

$$z(t) = A_1\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right) \quad t \in [0, \tau] \tag{10b}$$

be solved. Equation (10) is first re-written using equation (5) to become the ODE

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh}{m}\right)z(t) = \frac{wh}{m}A_1\left(e^{\lambda_1(t-\tau)} - e^{\lambda_2(t-\tau)}\right), \ \ t \in [\tau, 2\tau] \tag{11}$$

The complimentary function of $z(t)$ in the interval $[\tau, 2\tau]$ becomes $z_{c2}(t) = A_2e^{\lambda_1 t} + B_2e^{\lambda_2 t}$ while the particular integral reads $z_{p2}(t) = K_{21}e^{\lambda_1 t} + K_{22}e^{\lambda_2 t}$. Putting $z_c(t)$ into equation (11) gives that

$$\begin{aligned} K_{21} &= \frac{whA_1}{mX_1e^{\lambda_1\tau}} \\ K_{22} &= \frac{-whA_1}{mX_2e^{\lambda_2\tau}} \end{aligned} \tag{12}$$

where $X_1 = \left[\lambda_1^2 + 2\xi\omega_n\lambda_1 + \left(\omega_n^2 + \frac{hw}{m}\right)\right]^{-1}$ and $X_2 = \left[\lambda_2^2 + 2\xi\omega_n\lambda_2 + \left(\omega_n^2 + \frac{hw}{m}\right)\right]^{-1}$. The solution $z(t)$ in the interval $[\tau, 2\tau]$ becomes

$$z(t) = A_2'e^{\lambda_1 t} + B_2'e^{\lambda_2 t} \tag{13}$$

where $A_2' = A_2 + \frac{whA_1}{mX_1e^{\lambda_1\tau}}$ and $B_2' = B_2 - \frac{whA_1}{mX_2e^{\lambda_2\tau}}$. Making use of history of equation (10)

$$\begin{aligned} z(\tau) &= A_2'e^{\lambda_1\tau} + B_2'e^{\lambda_2\tau} = 0 \\ \dot{z}(\tau) &= A_2'\lambda_1e^{\lambda_1\tau} + B_2'\lambda_2e^{\lambda_2\tau} = A_1(\lambda_1 - \lambda_2) \end{aligned} \tag{14}$$

from which results $A_2' = \frac{A_1}{e^{\lambda_1\tau}}$ and $B_2' = \frac{-A_1}{e^{\lambda_2\tau}}$. The second step solution becomes

$$\begin{aligned} z(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\frac{e^{\lambda_1 t}}{e^{\lambda_1\tau}} - \frac{e^{\lambda_2 t}}{e^{\lambda_2\tau}}\right) \\ \dot{z}(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\lambda_1\frac{e^{\lambda_1 t}}{e^{\lambda_1\tau}} - \lambda_2\frac{e^{\lambda_2 t}}{e^{\lambda_2\tau}}\right) \end{aligned} \tag{15}$$

For the third step solution the ODE to be solved reads

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh(t)}{m}\right)z(t) = \frac{wh(t)}{m}z(t-\tau), \ \ t \in [2\tau, 3\tau] \tag{16a}$$

$$z(t) = A_1\left(e^{\lambda_1 t} - e^{\lambda_2 t}\right) \quad t \in [\tau, 2\tau] \tag{16b}$$

This becomes

$$\ddot{z}(t) + 2\xi\omega_n\dot{z}(t) + \left(\omega_n^2 + \frac{wh}{m}\right)z(t) = \frac{wh}{m}A_1\left(\frac{e^{\lambda_1(t-\tau)}}{e^{\lambda_1\tau}} - \frac{e^{\lambda_2(t-\tau)}}{e^{\lambda_2\tau}}\right), \ \ t \in [2\tau, 3\tau] \tag{17}$$

The complimentary function of $z(t)$ in the interval $[2\tau, 3\tau]$ becomes $z_c(t) = A_3e^{\lambda_1 t} + B_3e^{\lambda_2 t}$ while the particular integral reads $z_{p3}(t) = K_{31}e^{\lambda_1 t} + K_{32}e^{\lambda_2 t}$. Putting $z_{p3}(t)$ into equation (17) gives that

$$\begin{aligned} K_{31} &= \frac{whA_1}{mX_1e^{2\lambda_1\tau}} \\ K_{32} &= \frac{-whA_1}{mX_2e^{2\lambda_2\tau}} \end{aligned} \tag{18}$$

The solution $z(t)$ in the interval $[\tau, 2\tau]$ becomes

$$z(t) = A_3'e^{\lambda_1 t} + B_3'e^{\lambda_2 t} \tag{19}$$

where $A_3' = A_3 + \frac{whA_1}{mX_1e^{2\lambda_1\tau}}$ and $B_3' = B_3 - \frac{whA_1}{mX_2e^{2\lambda_2\tau}}$. Making use of history of equation (16b)

$$\begin{aligned} z(2\tau) &= A_3'e^{2\lambda_1\tau} + B_3'e^{2\lambda_2\tau} = 0 \\ \dot{z}(2\tau) &= A_3'\lambda_1e^{2\lambda_1\tau} + B_3'\lambda_2e^{2\lambda_2\tau} = A_1(\lambda_1 - \lambda_2) \end{aligned} \tag{20}$$

from which results $A_3' = \frac{A_1}{e^{2\lambda_1\tau}}$ and $B_3' = \frac{-A_1}{e^{2\lambda_2\tau}}$. The third step solution in the interval $[2\tau, 3\tau]$ becomes

$$\begin{aligned} z(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\frac{e^{\lambda_1 t}}{e^{2\lambda_1\tau}} - \frac{e^{\lambda_2 t}}{e^{2\lambda_2\tau}}\right) \\ \dot{z}(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\lambda_1\frac{e^{\lambda_1 t}}{e^{2\lambda_1\tau}} - \lambda_2\frac{e^{\lambda_2 t}}{e^{2\lambda_2\tau}}\right) \end{aligned} \tag{21}$$

It is seen from equations (9), 15 and (21) that generally the nth step solution becomes

$$\begin{aligned} z(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\frac{e^{\lambda_1 t}}{e^{(n-1)\lambda_1\tau}} - \frac{e^{\lambda_2 t}}{e^{(n-1)\lambda_2\tau}}\right) \\ \dot{z}(t) &= \frac{\dot{z}(0)}{\lambda_1 - \lambda_2}\left(\lambda_1\frac{e^{\lambda_1 t}}{e^{(n-1)\lambda_1\tau}} - \lambda_2\frac{e^{\lambda_2 t}}{e^{(n-1)\lambda_2\tau}}\right) \end{aligned} \tag{22}$$

The Myshkis method of steps as applied to the turning process so far is seen to be cumbersome and will get rather much more cumbersome when applied to the milling process. In order to get speedy time domain solutions equation (1) is solved for both turning and milling processes using the numerical integration code of MATLAB called MATLAB dde23.

Initial perturbation velocity and zero initial perturbation displacement are assumed. Graphical solution portrays asymptotic stability when chatter is seen to decay with time at an operating point while chatter instability is implied when vibration is seen to rise with time. Based on this time domain analysis, magnitude of assumed initial perturbation velocity does not have any effect on judgment of a machining stability in large Ozoegwu (2012). Time histories for the studied systems with parameters; $m = 0.0431kg, \omega_n = 5700rad/sec \ and \ \xi = 0.02 \ and \ C = 3.5 \times 10^7 \text{Nm}^{\frac{-7}{4}}$ are shown in figures 2, 3, 4 and 5. Each time history is titled with the determining cutting parameter combination of spindle speed Ω and depth of cut w. Initial perturbation velocity assumed for generation of these time histories is arbitrarily assumed to be one tenth of the tool feed speed v. The spindle speeds selected for investigation are made to fall within the low speed region so that full immersion conditions persist.

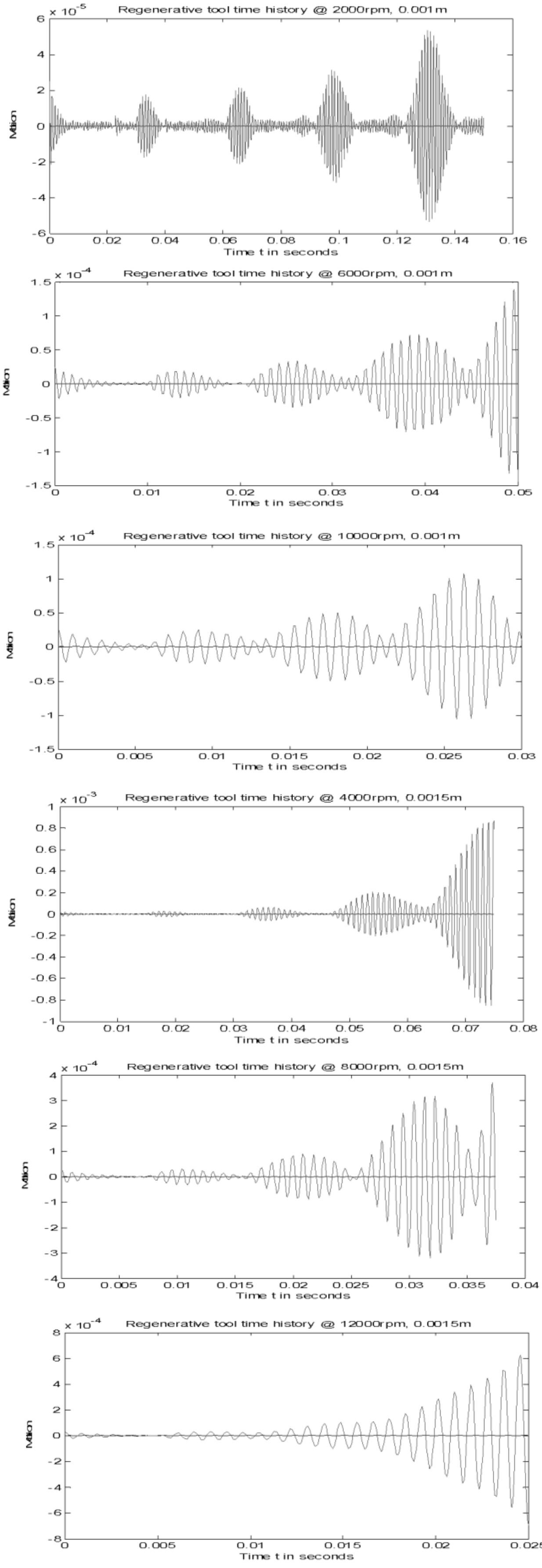

Fig. 2: Time histories of turning tool with initial history $z(t) = 0$ and $\dot{z}(t) = 0.00025m/s$, $t \in [0, -\tau]$

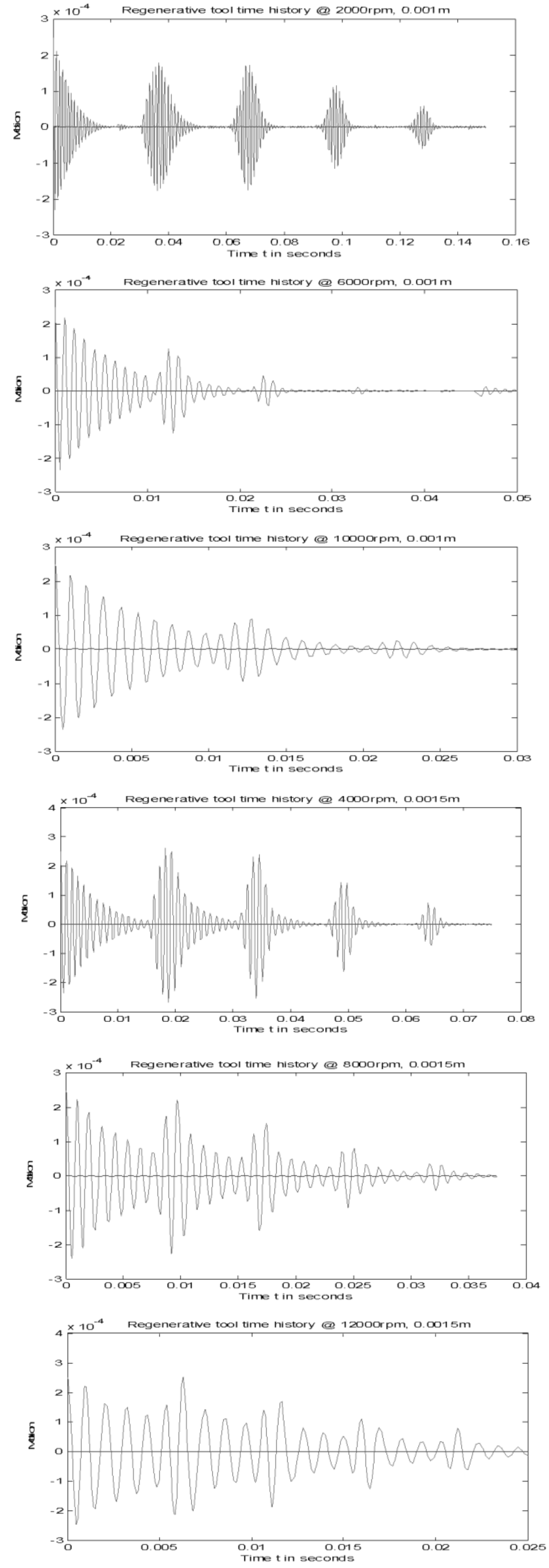

Fig. 3: Time histories of one tooth end-miller with initial history $z(t) = 0$ and $\dot{z}(t) = 0.00025m/s$, $t \in [0, -\tau]$

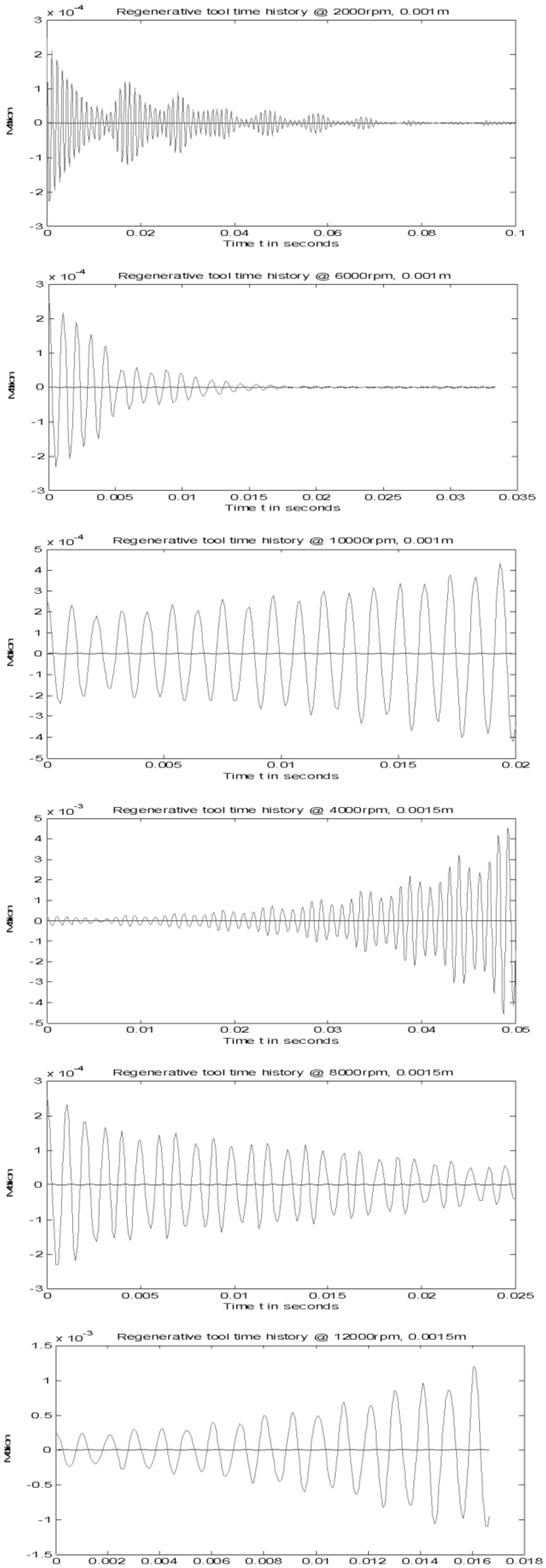

Fig. 4: Time histories of three tooth end-miller with initial history $z(t) = 0$ and $\dot{z}(t) = 0.00025m/s$, $t \in [0, -\tau]$

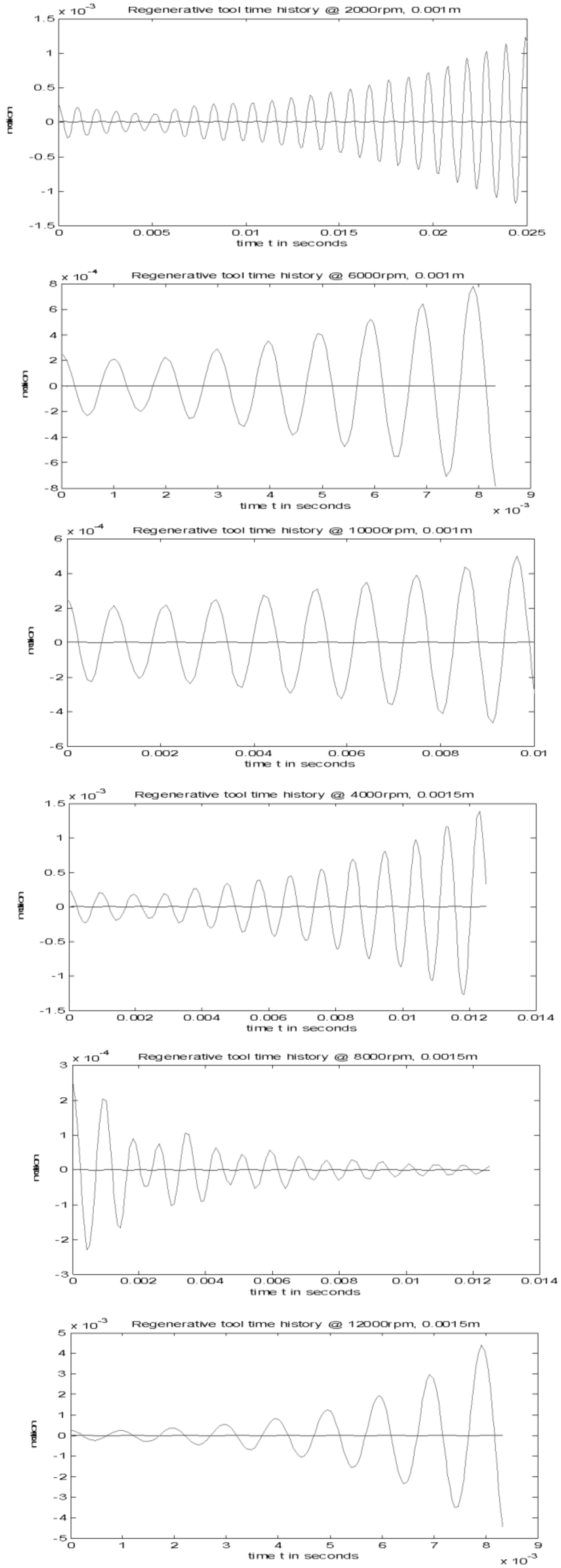

Fig. 5: Time histories of six tooth end-miller with initial history $z(t) = 0$ and $\dot{z}(t) = 0.00025m/s$, $t \in [0, -\tau]$

III. COMPARISON OF MILLING AND TURNING

Differences between turning and milling processes derive from low-speed full-immersion turning process being a continuous process with constant specific force variation $h(= C\gamma(v\tau)^{\gamma-1})$ while low-speed full-immersion milling is a disturbed process with time-dependent specific force variation $h(t)(= \gamma(v\tau)^{\gamma-1}q(t))$ capturing the effect of chip thickness variation during teeth pass. Six graphical solutions is presented for each tool using the same set of parameter combinations. For the turning tool, all operations considered are mathematically unstable as shown in Fig. 2.

The turning operations are judged unstable since regenerative vibration rises with time. It is seen in figure3 that the operation of a one tooth end-milling tool is entirely stable for the same set of parameter combinations. Three operations which are at a parameter combinations of spindle speeds $2000,\ 4000\ and\ 8000rpm$ and depths of cut $1, 1\ and\ 1.5mm$ are seen to be stable while the rest are unstable for the three tooth end-miller. Only one milling operation is seen to be stable for the six tooth end miller thus all things being equal reducing the number of teeth seems to stabilize the milling process. It can be seen that unstable turning condition could become a stable milling condition. This observation is acceptable when it is assumed that parametric excitation that exists in milling suppressed the chatter in the turning equivalent.

This conforms to the theoretical result of stabilizing an inverted pendulum by parametric excitation of its base. Further conclusion could be drawn that an unstable turning condition more likely becomes a stable milling condition as the number of milling tooth reduces. If this result is used with the observation that one tooth milling is more stable than three tooth milling while the latter is more stable than the six tooth milling it could be extrapolated that relatively low frequency parametric excitation would be more effective in suppressing turning chatter. The conclusion is that under full-immersion conditions milling stability characteristics get closer to that of turning as the number of teeth of milling tool is increased. This conclusion is also expected to mean that on the parameter plane of spindle speed and depth of cut, stability chart of milling approaches that of turning in form as the number of end milling teeth increases.

IV. CONCLUSION

It is found in this work that relative to common cutting parameters of spindle speed Ω and axial depth of cut w that the milling process has better stability characteristics than turning with the gulf between their stability characteristics widening as the no of teeth of the milling tool decreases. Point is made that this could be partly due to periodic disturbance existing in the milling process. Based on the observation that one tooth milling is more stable than three tooth milling while the latter is more stable than the six tooth milling, it is suggested that in relative terms low frequency periodic excitation would be more effective in suppressing turning chatter. The ultimate conclusion drawn is that under full-immersion conditions milling stability characteristics get closer to that of turning as the number of teeth of milling tool is increased. It is thus expected that the stability chart of milling should approach that of turning in form as the number of end milling teeth increase.

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