INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY SCIENCES AND ENGINEERING, VOL. 3, NO. 6, JUNE 2012

Modeling and Optimization of Electricity Distribution Planning System Using Dynamic Programming Techniques: A Case of Power Holding Company of Nigeria (PHCN)

[1]Chinwuko C.E., [2]Chukwuneke J.L., [2]Okolie P.C., and [2]Dara E.J.
[1]Department of Industrial/Production Engineering, Nnamdi Azikiwe University, Awka Anambra, Nigeria
[2]Department of Mechanical Engineering, Nnamdi Azikiwe University, Awka Anambra, Nigeria
(Corresponding Author's email: jl.chukwuneke@unizik.edu.ng)

Abstract– **This paper presents a dynamic approach towards the sustainability of power distribution system using holistic application. The cost of energy losses, substation cost, feeder cost and outage cost was developed using dynamic programming technique and a three stage iterative solution with the aim to optimize the outage costs was presented. The three stages of optimization were carried out to determine, the number of substation sites and exact location, the feeder routes and the load flow in the network, and the outage cost to cover system node reliability evaluation. These three stages are solved sequentially by the optimization toolbox available with MATLAB. The substation and feeder stages were formulated as a quadratic mixed integer programming problem, while the outage stage was established as a non linear mixed integer programming problem. The proposed method has been applied to power distribution system planning for Awka Business Unit (PHCN), Anambra State, Nigeria. The model proposed a reserve substation capacity of approximately 15MVA at Agu-Awka to handle load expansion. Two substations were proposed after optimization, one sub-station at Nibo (existing) 30MVA, Second substation at Agu Awka 120MVA totaling 150MVA.**

Keywords– Power, Optimization, Cost, Programming, Electricity, Substations, Distribution, Generation, Load Demand, Feeder and Outage

I. INTRODUCTION

The Electric Power Sector is one of the most important sectors to national development. The power sector is vital to the developmental reform of any country. For many years in Nigeria, the sector has been plagued by a plethora of problems. These problems included low generation capacity, poor distribution, decaying facilities and many others.

The power sector is unlike most other sectors in any economy. Many countries prior to reform had largely one state owned utility carrying out all the activities in that sector. In Nigeria, the structure was represented through the years by the Electricity Corporation of Nigeria and National Electric Power Authority. These monopolies came with the usual baggage of inefficiency and poor service delivery. To discuss the electric power sector in Nigeria in a rational way, an assessment of its development since independence is necessary in this study.

Electricity supply in Nigeria dates back to 1886 when two small generating sets were installed to serve the then Colony of Lagos. By an Act of Parliament in 1951, the Electricity Corporation of Nigeria (ECN) was established, and in 1962, the Niger Dams Authority (NDA) was also established for the development of Hydro Electric Power. However, a merger of the two was made in 1972 to form the National Electric power Authority (NEPA), which as a result of unbundling and the power reform process, was renamed Power holding Company of Nigeria (PHCN) in 2005 [1], [2].

The Nigerian power sector is controlled by state-owned Power Holding Company of Nigeria (PHCN), formerly known as the National Electric Power Authority (NEPA). In March 2005, President Olusegun Obasanjo signed the Power Sector Reform Bill into law, enabling private companies to participate in electricity generation, transmission, and distribution. The government has separated PHCN into eleven distribution firms, six generating companies, and a transmission company, all of which will be privatized soon [3].

Given global trends in the electricity sector it becomes imperative that in order to bring about significant improvement in the power sector a more holistic approach (dynamic programming techniques) must be adopted towards making changes in the sector. To this end, there is a need for the optimal planning of the distribution system.

The problem of distribution system planning consists of determining the optimum numbers and locations of the distribution substations and the optimum way of connecting the load nodes to these substations through the interconnection of feeders. Solving the exact problem by using classical optimization technique is not possible because of the combinatorial nature of the problem. Dynamic programming method will be applied to optimize the cost function objective of power distribution system planning.

The following steps are involved in dynamic programming technique:

i. The decomposition approach in which a large optimization problem is divided into several smaller sub problems and each one is solved separately.

ii. The alternating policy method which compares a number of alternative policies and selects the best.
iii. The linear programming and mixed integer programming methods where the constraint conditions are linearised.
iv. The dynamics programming method.

In this study, the decomposition approach and Quadratic non linear integer programming method will be employed in solving the power distribution planning problem. The problem will be divided into three stages, namely; substation optimization, feeder optimization and outage cost optimization. These stages are formulated as a quadratic mixed integer programming problem and they are solved sequentially by the optimization toolbox available with MATLAB Subsequently.

In this study quadratic programming will be employed to minimize the installation and operational cost of substations and feeder that will ensure sustained power distribution system in Awka Metropolis and its environs. The formulated model could be applied to other regions of the utility, which has not only affected the stakeholders but has affected the performers of other industrial establishment, thereby creating negative impact and intangible contribution to the national economy. Thus, a close examination of the problems posed by inadequate supply of quality power on the nation's standard of living is significant. The study is therefore very significant for any meaningful attempt at attaining high standard of living and sustainable poverty eradication. Furthermore, the study is significant to the government for policy formulation aimed at resuscitating power distribution.

Finally, the implementation of this study will lead to sustained power distribution which will consequently lead to economic growth and improved socio-economic activities. These will improve the quality of life which is the bedrock of engineering.

A. Distribution System

Distribution network are typically of two types, radial network and interconnected network. A radial network leaves the station and passes through the network area with no normal connection to any other supply. This is a typical of long rural lines with isolated load areas. An interconnected network has multiple connections to other points of supply. These points of connection are normally open but allow various configurations by the operating utility by closing and opening switches. Operation of these switches may be by remote control from a control centre or by a lineman. The benefit of the interconnected model is that in the event of a fault or required maintenance, a small area of network can be isolated and the remainder kept on supply.

Generated power cannot all be utilized at the generating stations and its immediate environ. Therefore, it must be distributed at suitable voltage to points and consumers. Distribution involves primary and secondary transformation of high voltage to the standard medium and low voltage by the appropriate transforming equipment.

Primary Distribution System: These consist of high voltage (11 and 33kV) networks from primary and sub-primary substations. These substations are interconnected with high voltage transmission lines. In most cases, large industries consumers like cement factories, refineries, breweries, flour mills, steel rolling mills and so on take supply at primary distribution system with associated transformers, switchgears and breakers.

Secondary Distribution Systems: These consist of low voltage feeder networks from the secondary transformers that are constructed along main roads and streets. Service connections are made to individual consumers by service cables from these networks feeder lines. The various system of alternating current distribution for domestic consumers include: Single-phase 2-wire system, Single-phase 3-wire system, Three-phase 3-wire system, Three-phase 4-wire system. Of these, the single phase, 2-wires and the three phase 4-wire system are the most widely used in Nigeria.

B. Electric Power Systems

Electric Power Systems are components that transform other types of energy into electrical energy and transmit this energy to a consumer. The production and transmission of electricity is relatively efficient and inexpensive, although unlike other forms of energy, electricity is not easily stored and thus must generally be used as it is being produced.

Components of an Electric Power System: A modern electric power system consists of six main components:

1) The power station,
2) A set of transformers to raise the generated power to the high voltages used on the transmission line,
3) The transmission lines,
4) The substations at which the power is stepped down to the voltage on the distribution lines,
5) The distribution lines, and
6) The transformers that lower the distribution voltage to the level used by the consumer's equipment.

II. PROBLEM FORMULATION

A. Dynamic Programming

In this study, dynamic programming technique will be applied to optimize the objective function of power distribution system planning. In its simplest sense, dynamic programming can be thought of as an attempt to break large, complex problems into a series of smaller problems that are easier to solve separately.

There are number of decision stages at each stage there are several alternate courses of action. The decision generated by stage one, acts as conditions of the problem for stage two and so on. In other words, at each of the several stages there is a choice of decisions and the decisions, initially taken affect the choice of subsequent decisions. The various rules of decision making can be established after considering the effects of each decision (separately) and the optimum policy for further decisions. The basis of dynamic programming is to select the best amongst the final possible alternative decisions. This process is then repeated, ignoring all those alternatives which do not lead to selected best (optimum). The best sequence of decisions can thus be defined, by repeating the above procedure.

B. Quadratic Programming

In linear programming, the values of decision variables are to be determined so as to optimize the value of the linear objective function subject to linear constraints. However, when either objective function or constraints or both are not expressed in terms of linear relationships among decision variables, we take the help of non-linear programming methods to solve such non-linear programming problems.

In this study, the mathematical model developed for optimal power distribution system planning, is a non-linear quadratic mixed integer programming problem. Among several non-linear programming methods available for solving NLP problems, we shall discuss quadratic programming problem. The general structure of quadratic programming problem is as follows:

$$Optimize\ (Max\ or\ Min)Z = \left\{ \sum_{j=i}^{n} c_j\ x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} x_j\ d_{jk}\, x_k \right\} \quad (1)$$

Subject to the constraints

$$\sum_{j=i}^{n} a_{ij}\ x_j \ \le b_i \ and\ x_j \ge 0\ for\ all\ i\ and\ j$$

In matrix notations, the above QP problem is written as;

$$Optimize\ (Max\ or\ Min)Z = \ cx + \frac{1}{2} x^T\ Dx \quad (2)$$

Subject to the constraints; $Ax \le b$ and $x \ge 0$

Where; $x = (x_1, x_2, \ldots, x_n)^T$; $c = (c_1, c_2, \ldots, c_n)$; $b = (b_1, b_2, \ldots, b_m)^T$, $D = [d_{jk}]$ is an n x n symmetric matrix, i.e. $d_{jk} = d_{kj}$; $A = [a_{ij}]$ is an m x n matrix.

The matrix D is symmetric and positive-definite (i.e the quadratic term $x^T Dx$ in x is positive for all values of x except at x = 0) in case the problem is of the minimization type, and it is negative definite (i.e. $x^T Dx < 0$ for all values of x except for x = 0) in case the problem is of the maximization type. This also shows that the objective function of the quadratic programming problem is strictly convex in x for minimization and concave in x for maximization. If matrix D is null, then the QP problem reduces to the standard LP problem.

III. MATHEMATICAL FORMULATION

The distribution system planning problem is studied to select the optimum substations, optimum number of feeders and the optimal system node reliability such that the total system cost is minimum, while the voltage and capacity limits are not violated. The objective function may generally be stated as in Eqn.3:

$$\text{Minimize } Z_{Total} = Z_S + Z_F + Z_O \quad (3)$$

Where; Z_{Total} is the total costs to be minimized, Z_s is the cost associated with substations, Z_F is the cost associated with feeders, Z_o is the cost incurred due to outages

A. Stage 1: Substation Cost Formulation (Z_S)

The cost evaluation of Transformers is based on Total Owning Cost (TOC) that considers transformer purchasing price and cost of transformer losses. The data required for each substation are its capacity, location and fixed and variable costs. The fixed cost includes the cost of Transformer (T_x) and other equipment at the substation plus cost of construction. The variable cost includes the cost of power losses in substation T_x, annual operating and maintenance costs. Also to be considered in the formulation of the substation optimization model is the resultant power flow through fth feeder association with the substation, the unit cost of power loss for fth feeder and resistance of the fth feeder.

The total capitalized cost of T_x is expressed as [8]:

$$TTC_{t,r} = PP + A^*P_O + B^*P_K \quad (4)$$

Where; PP = is the purchase price of transformer, A = represents the assigned cost of no-load losses per watt, P_O = is the rated no-load loss, B = is the assigned cost of load losses per watt, P_K = is the rated load loss; P_O and P_K are transformer rated losses. A and B values depend on the expected loading of the transformer and energy prices. Below we propose a relatively simple method for determining the A and B factor for distribution transformers; A and B factors are calculated as follows: (no-load loss capitalization)

$$A = \frac{(1+i)^n - 1}{i(1+i)^n} \times C_{kwh} \times 8760 \quad (5)$$

And (load loss capitalization)

$$B = \frac{(1+i)^n - 1}{i(1+i)^n} \times C_{kwh} \times 8760 \times \left(\frac{I_1}{I_r}\right)^2 \quad (6)$$

Where; I - interest rate [%/year], n - lifetime [years], C_{kWh} - kWH price [₦/kWh], 8760 - number of hours in a year [h/year], I_I - loading current [A], I_r - rated current [A].

Factor A, is directly proportional to electricity price so the A factor can simply be scaled to account for electricity price changes as long as the interest rate and capitalization period remain unchanged. Factor B, as explained previously, is simply the product of factor A and the square of the loading factor. (B = A * (Loading)2). The loading factor used here is the expected average load over the life span of the transformer, possibly taking harmonics into account.

The annual energy losses and cost of these losses [8] can be evaluated as follows:

$$P_{WLoss} = (P_O + P_K L^2)8760 \quad (7)$$

In which; P_{WLoss} - is the annual energy loss in kWh, P_O - is the no-load loss in kW, P_K - is the short-circuit loss (or load loss) in kW, L - is the average per-unit load on the transformer, 8760 -is the number of hours in a year.

To calculate the cost of these losses, they need to be converted to the moment of purchase by assigning capital values, to be able to put them into the same perspective as the purchase price. This is called the Total Capitalized Cost of the losses, TCC_{loss}. This can be calculated using the following formula [8].

$$TCC_{loss} = P_{WLoss}\frac{(1+i)^n-1}{i(1+i)^n} \times C \times 8760 \qquad (8)$$

Where; C - is the estimated average cost per kWh in each year, I - is the estimated interest rate, n - is the expected lifetime of the transformer.

Introducing eqn. (8) into eqn. (4) will result in;

$$TCC_{t,r} = PP + TCC_{Loss}$$

$$TCC_{t,r} = PP + \left[\frac{(1+i)^n-1}{(i(1+i))^n}\right] \times C \times 8760 \qquad (9)$$

Comparing eqns (7) and eqn (9) will result in;

$$TCC_{t,r} = PP + [(P_O + P_K L^2) \times C \times 8760] \qquad (10)$$

In this study of a case of PHCN Awka, 33/11KV, T_x are used in primary/injection substations while 33/0.415KV or 11/0.4151KV transformers with associated feeder pillars are equipment used for distribution substations. Only step down T_{xs} are considered. For selection of appropriate cables, switchboard, feeder pillar and fuse at both sides of upper and down stream of the power supply system, the following design eqn. [5] apply for a 3-phase T_x;

$$Power(P_W) = \sqrt{3}\ IVCos\theta \qquad (11)$$

Where; I – Current, V – Voltage

$$TCC_{TX} = PP + (\sqrt{3}\ IV\ Cos\theta)C_{\in}8760 \qquad (12)$$

Where; PP - Purchase price of T_x, $C_€$ - Estimated energy cost per kwh, Cos θ - Power factor, V – Voltage, I – Current, √3 - For a 3-phase T_X, 8760 - no. of hours per year.

Transformer Price: The purchasing price (PPT_X) of a T_X can be estimated from [8]. The price/rating characteristics are roughly described as:

$$C_1 = C_0\left(\frac{S_{tr,1}}{S_{tr,0}}\right)^x \qquad (13)$$

Where; C_1 - is cost of transformer "1", C_O - is cost of transformer "0", $S_{tr.1}$ - is rated power of transformer "1", $S_{tr.0}$ - is rated power of transformer "0", X - exponent (cost factor). The X factor is about 0.4 to 0.5. For more efficient units this factor has a tendency to increase up to 0.6 or even higher. At today's commodity prices, the indicative T_X price for AC class 100KVA typical distribution T_X is around 3000€, 400KVA is around 7000€ and 1000KVA is around 12000€ [8]. The transformer performance equation is:

$$P = \sqrt{(3)}I^2RCos\theta$$

Eqn. 13 can be written as:

$$TCC_{TX} = C_O\left(\frac{S_{tr,1}}{S_{tr,o}}\right)^x + (\sqrt{3}\ IV\ Cos\theta)C_{\in}8760 \qquad (14)$$

$$TCC_{TX} = PP + P_W C_{\in} 8760 \qquad (15)$$

Considering 1, 2, 3, n; number of substations in which Y_i is a candidate substation or selection substation variable, such that the total no. of substations will be $Y_1 + Y_2 + Y_3 + \ldots n = \sum_{i=1}^{n} Y_i$

Therefore, the total cost of substation transformer PPT_X (purchasing price) and number of substations $\sum_{i=1}^{n} Y_i$ over a time planning period T_P, $\sum_{t=0}^{Tp-1}$; becomes

$$\sum_{t=o}^{T_p-1} PP_{TX} \times \sum_{i=1}^{n} Y_i = \sum_{t=o}^{T_p-1}\sum_{i=1}^{n} PP_{TXi}Y_i \qquad (16)$$

Considering the costs of annual energy loss of eqn. 7, over time planning period T_P and substations i = 1, 2, 3 …n, then;

$$\sum_{t=o}^{T_p-1}\sum_{i=1}^{n}\sum_{j=1}^{m} C_{\in}P_{W,ij}8760 \qquad (17)$$

Moreso, formulation of substation optimization model requires the resultant power flow losses through feeder association with T_x having resistance R_{ij} and Length L_{ij} in moving from supply node i to demand node j - expressed mathematically;

$$\sum_{t=o}^{T_p-1}\sum_{i=1}^{n}\sum_{j=1}^{m}(P_{W,ij} - P_{W,ji})C_{\in}P_{ij}L_{ij}8760 \qquad (18)$$

Hence the total costs of substation distribution planning become summation of eqn. 16, 17 and 18 giving substation total cost (Z_S) as:

$$MinZ_s = \sum_{t=0}^{T_p-1}\sum_{i=1}^{n} PP_{TX,i}Y_i + \sum_{t=0}^{T_p-1}\sum_{i=1}^{n}\sum_{j=1}^{m} P_{W,ij}C_{\in}8760 + \sum_{t=0}^{T_p-1}\sum_{i=1}^{n}\sum_{j=1}^{m}(P_{W,ij} - P_{W,ji})^2 C_{\in}R_{ij}L_{ij}8760 \qquad (19)$$

Such that; $C_€$ - Cost coefficient for power loss ₦/kwh, R - Feeder resistance (Ω/km), L - Feeder length (km), P_{Wij} - Power flow from node i to node j in MVA

T_P- $\sum_{t=0}^{Tp-1}$ i.e. time planning period for a dynamic load time variation,

Y_i - Selection substation variable for ith station = 1 if selected otherwise zero

$\sum_{i=1}^{m} Y_i \leq N_{max}$, max. no. of substations is N_{max}, $P_{Wij} \geq 0$, This constraint is specified for mathematical programming and cannot be negative, power cannot flow from demand node to supply node, $P_{Wij} \leq P_{W,\max} Y_i$; direct linked substation maximum capacity,

$\sum_{i=1}^{n} P_{w,ij} \leq Q_{\max} Y_i$; maximum substation capacity

B. Stage 2: Feeder Optimization Cost Formulation (Z_F)

Formulation of feeder costs economic evaluation consists of; cost of loss energy due to power losses in the feeder wire, the cost of loss demand or cost of lost capacity due to power losses and the cost of investment. The investment cost is the largest which includes material and labour costs involving feeders. The distribution system was designed as a mixed integer programming problem with substations as sources and load on feeders as demands.

Putting Z_F = Feeder cost; then we can write;

$$Z_F = Z_S + \left(\begin{matrix}\text{Fixed cost of}\\ \text{Feeder}\end{matrix}\right) + \left(\begin{matrix}\text{Energy Cost of}\\ \text{Feeder}\end{matrix}\right) + \left(\text{Demand Cost Per Unit Length}\right) \qquad (20)$$

Denoting; AIC - Annual Investment Cost per unit length or Fixed cost of feeder, AEC - Annual Energy Cost per unit length, ADC - Annual demand Cost per unit length
Eqn. 20 can be written as;

$$Z_F = Z_S + (AIC + AEC + ADC) \qquad (21)$$

The total annual feeder cost per unit length [15] is expressed as:

$$TAFC = AIC + AEC + ADC \qquad (22)$$

The following constraints are noted both in formulation and subsequent feeder cost optimization:

a. $P_{ij} \geq 0$; this constraint is specified for mathematical programming.
Power cannot flow from demand node to supply node
b. P_{ij} - Power flow from node i to node j in MVA
c. X_i = I or 0; if selected it is I, if zero is rejected
X_i - Is selection variable for feeder ith
d. T_P - Planning period; $\sum_{t=0}^{Tp-1}$
e. $\sum_{i=1}^{n} X_i \geq 1$; i= 1, 2, …N, radiality satisfaction at node i
f. F_X - Fixed cost of feeder
g. V_X - Variable cost of feeder

$$\left.\begin{array}{ll} \text{h.}\quad F_X & = AIC \\ \text{i.}\quad V_X & = AEC + ADC \end{array}\right\} \quad (23)$$

j. $Y_i = 1$; Selected substations
k. $1 \geq X \geq 0$, Upper and lower boundary
l. $P_{ij} = 3I^2R$, power loss in a feeder

The annual investment cost of a feeder includes installation cost capital, taxes, insurance operation maintenance, depreciation and others. It is the installation cost multiplied by fixed cost rate expressed as;

$$AIC = IC_F \times i \qquad (24)$$

The annual energy cost due to power losses in the feeders is calculated using [6]:

$$AEC = 3I^2R_H F_E F_{LL} F_{LS} 8760 \times 10^{-3} \qquad (25)$$

Where; F_E is cost of energy per kwh, F_{LL} is load location factor, F_{LS} is loss factor calculated from the following:

$$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$$

Where; F_{LD} denotes load factor, the annual demand cost maintains an adequate system capacity in order to supply the I^2R_h losses in the feeder conductors expressed as:

$$ADC = 3I^2R\,F_{LL}\,F_R(C_{GIG} + C_{TIT} + C_{SIS}) \times 10^{-3} \qquad (26)$$

Writing the fixed cost (F_X) of feeder as

$$F_x = IC_F \times i \qquad (27)$$

As well as variable costs (V_X) as

$$V_X = AEC + ADC \qquad (28)$$

Since;

$$AEC = 3I^2R_H F_E F_{LL} F_{LS} 8760 \times 10^{-3}$$
$$ADC = 3I^2R\,F_{LL}\,F_R(C_{GIG} + C_{TIT} + C_{SIS}) \times 10^{-3} \qquad (29)$$

$$\therefore V_X = 3I^2R_H F_{LL} \times 10^{-3}[F_R(C_{GIG} + C_{TIT} + C_{SIS}) + (F_E F_{LS} 8760)] \qquad (30)$$

Consideration of feeder ratings to system load projection, relates to power losses in feeder from node i to demand node, j i.e. $P_{ij} = i^2R_h$, The fixed cost (F_X) of feeder investment, over a load time variation planning period and no. of feeders (X_{ij}) in a substation is expressed as;

$$\sum_{t=0}^{T_p-1} \sum_{i=1}^{n} \sum_{j=1}^{m} (F_X X_{ij}) \qquad (31)$$

And the associated variable costs multiplying resultant power flow will give:

$$\sum_{t=0}^{T_p-1} \sum_{i=1}^{n} \sum_{j=1}^{m} V_X(P_{ij} - P_{ji}) \qquad (32)$$

Noting that the load demanded by the customer at each node should be supplied in all conditions, expressed mathematically as constraint

$$\sum_{i=1, i\neq 1}^{n} (P_{ij} - P_{ji}) \leq P_{tj} \quad j = 1, 2, 3, \ldots..N$$

Hence combination of eqns. 31 plus 32 gives total cost of feeder distribution network in addition;

$$Z_F = 28 + 29 \qquad (33)$$

To substation cost (Z_S) of eqn. 19, because the distribution system is modeled as a mixed integer quadratic problem with the substation transformer as sources and the loads on the feeders as demands, mathematically presented as:

$$Z_T = \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} PP_{Tx,i} Y_i + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} P_{W,ij} C_{\in} 8760 + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} (P_{W,ij} - P_{W,ji})^2 C_{\in} R_{ij} L_{ij} 8760 + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} (F_X X)_{ij} + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} \left[V_X (P_{ij} - P_{ji})^2 \right] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (30)$$

C. Stage 3: Formulation of Power Outage (Z_O)

The outage cost is a responsible indicator for the power supply availability in a distribution system node. If the reliability is improved, the value of outage cost falls, and if it deteriorates, the value of outage cost rises. The outage cost varies depending on the duration of the outage. Constraints in stage 1 and stage 2 apply in this stage 3.

The term reliability is closely associated with outages, interruptions, failures and availability. It is associated with switch gear, protection and control equipment, absolute 100% reliability and availability of generation systems, transmission and distribution systems cannot be guaranteed. High reliability is possible with the following (more than 99.8%):

- Availability of generation, transmission and distribution system
- Reserve capacity (margin) between installed capacities and expected max. load
- Design and quality aspects
- Operation and maintenance aspects

Reliability describes the ability of continuous service without outages/failure/ interruptions. It is expressed as [7], [8]:

Reliability index (β_r) =

$$\frac{\text{Total service hours - Interruption hours/Yr}}{\text{Total service hours}} \qquad (34)$$

Outage - is state of a component when it is not available to perform its intended function, Interruption – loss of electric power supply to one or more loads, Failure rate ($n_{i,w}$) – The mean number of failures per unit of exposure time for a component. Usually exposure time is expressed in years and failure rate is given in terms of failure per year. Repair time: The clock time from the time of component failure to the time when the component is restored to service either by repair of the failed components or substitution of a spare component. It includes time for diagnosing the trouble locating the failed component, waiting for parts, repairing or preplanning, testing and restoring the component to service. The terms repair time and forced outages can be used synonymously.

Quantitative evaluation calls for precise definitions of terms. For quantitative evaluation, the reliability performance of constituent components of the system should be known [7], [8]. Component – a piece of the equipment a line or circuit or a section of a line or circuit or a group of items which is viewed as an entity for purposes of reliability evaluation. $t_{i,w}$ - outage duration r repair time per outage at ith load node and wth component in hour, C_w - outage cost co-efficient at ith load node in ₦/Kwh, PT_i - Load demand at ith load node in MVA, Repair time restriction $n_{i,w}\ t_{i,w} \geq 0$

For outage duration ($t_{i,w}$) and failure rate ($n_{i,w}$) for ith load node; for wth component. Maximum repair time ($T_{i,w}$)is calculated as;

$$T_{i,w} = n_{i,w}\ t_{i,w} \qquad (35)$$

The total outage duration time $T_{i,w}$ over a series if wth component failure from w = 1, 2, 3, …n becomes;

$$T_{i,w} = \sum_{w=1}^{n} n_{i,w}\ t_i \qquad (35a)$$

The outage cost $C_{i,w}$, over a load $P_{T,i}$ at ith load node i =

$$\sum_{i=1}^{n} P_{T,i}\ C_{i,w} \qquad (36)$$

To obtain the outage cost, multiply eqn. 35a and eqn. 36 to bring into the correct cost unit =

$$\sum_{w=1}^{n} \sum_{l=1}^{n} P_{T,i}\ C_{i,w}\ T_{i,w} \qquad (37)$$

Introducing the reliability index factor for the system to be continuous in service;

$$Z_o = \sum_{t=0}^{Tp-1} \sum_{i=1}^{n} \sum_{w=1}^{m} P_{T,i}\ C_{i,w} T_{i,w} \beta_r \qquad (38)$$

The total cost of power distribution system planning becomes;

$$Z_T = Z_s + Z_f + Z_o \qquad (39)$$

$$Z_T = \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} PP_{Tx,i} Y_i + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} P_{W,ij} C_{\in} 8760 + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} (P_{W,ij} - P_{W,ji})^2 C_{\in} R_{ij} L_{ij} 8760 + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} (F_X X)_{ij} + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{j=1}^{m} \left[V_X (P_{ij} - P_{ji})^2 \right]. + \sum_{t=0}^{T_{p-1}} \sum_{i=1}^{n} \sum_{w=1}^{m} P_{T,i}\ C_{i,w} T_{i,w} \beta_r \qquad (40)$$

IV. SOLUTION OF OPTIMIZATION

The final solution indicates that an efficient and cost effective power supply can be achieved in the case studied by establishing a transmission substation of 120MVA capacity in Agu-Awka. The sub-station would be fed through the turning-in-and-out of the existing Onitsha-Oji River 132KV single circuit line.

The proposed transmission substation would be three 33KV feeder lines and three 11KV feeder lines. The three 11KV feeder lines would serve Awka Township and environs. Two of the 33KV feeder lines would feed 33/11KV injection substations at Abagana and Enugu-Agidi respectively. These two proposed injection substation would serve Abagana, Enugu-Ukwu, Nawfia, Abba, Ukwuru, Nneogu, Awkuzu, Enugu- Agidi and their environs. The remaining 33KV feeder line would feed Nibo transmission substation in event of emergency, the establishing of the new substation will eliminate load shading problems and improve power supply.

In general, distribution utility companies should organize a planning process for the distribution system that identifies the locations on the local grid that could benefit most from targeted addition of demand resources. In this work, Agu Awka was proposed for citing of a new substation because it minimizes both power losses and costs without compromising future growth.

Distribution companies should also demand side resource that would improve the reliability, operation and economics of the local distribution system. They should focus on the distribution system that considers all available resources to meet distribution needs. Investment at distribution level should be guided by the principles of efficient reliability least cost and resource parity.

A. Solution Substation Optimization

The substation optimization problem is started with four feasible substation locations as possible candidates for optimum locations. The quadratic programming problem is formulated and after each solution one substation is eliminated from the corresponding Y_i to zero value. A new problem is formulated with the remaining candidate substation and the quadratic programming problem is solved. This process is repeated until the required number of substation is eliminated which is checked by verifying the condition:

$$\sum (\text{Capacity of The Selected Substation} = \text{Total Load Demand} + \text{Capacity Margin}) \qquad (41)$$

The capacity margin is for future provision for load growth of the system. The solution of the problem eqn. 19, gives value of Y_i, the substation selection variable, may have a value 0 or 1 in reality. The solution of the substation optimization is shown in Table 1.

B. Solution of Feeder Optimization

The feeder optimization problem determines the optimum radial configuration of the distribution network. The optimization process started by formulating a quadratic programming problem with the optimum substation locations and the associated feeders obtained by the substation optimization problem. This process is adopted because the radical configuration of the feeder through each selected substation can be obtained easily during alteration process in Matlab optimization tool box.

After each quadratic programming solution of the feeder optimization problem one or more feeder are eliminated by forcing the value of $X_k^{'s}$ to zero value. This process is repeated until the optimum radical network is obtained. The feeder optimization is obtained. The feeder optimization problem therefore can be relaxed to assume any value from '0' to '1' which however actually will be '0' or '1'. A '0' or '1' value to X_k is forced after the solution of the quadratic programming problem is obtained. The solution of the feeder optimization is shown in Table 2.

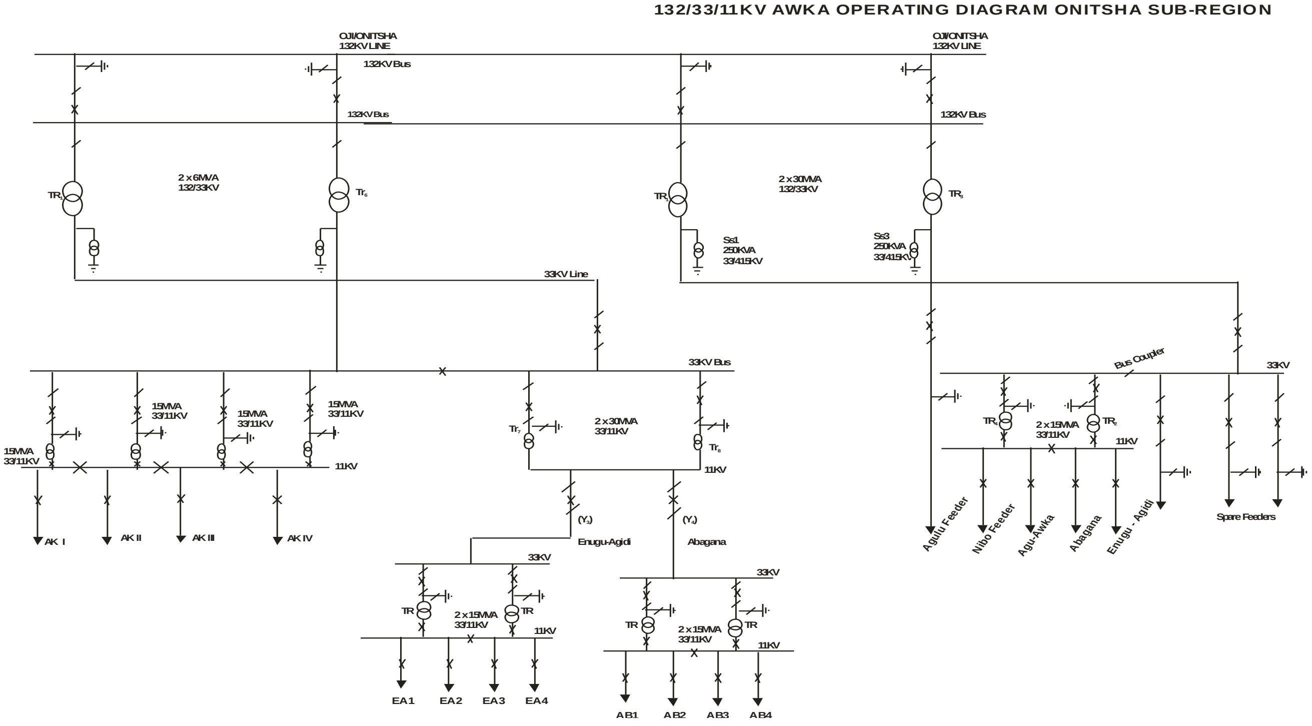

Fig. 1: Proposed Awka/Nibo transmission substation PHCN

C. Solution of the Outage Cost Optimization

The system node outage indicates component when it is not available to perform its intended function adequate due to some event directly associated with the component. The outage cost optimization needs single iteration.

Table 1: Results obtained from substation optimization

Iteration	Selection variable	Substation fixed selection variable value after solution	Available system capacity (MVA)
1	*$Y_1 = 1$ $Y_2 = 0.4703$ $Y_3 = 0.3528$ $Y_4 = 0.4383$	$Y_1 = 1$ $Y_2 = 1$ $Y_3 = 0$ $Y_4 = 1$	210
2	$Y_1 = 1$ $Y_2 = 0.5710$ $Y_4 = 0.4980$	$Y_1 = 1$ $Y_2 = 1$ $Y_4 = 0$	150

*Nibo substation exists already

Table 2: Result obtained from feeder optimization

Iteration	Minimum selection index	Feeder eliminated
First	0.3	X_2, X_3, X_7
Second	1.0	X_6,

The minimum selection index of the feeders calculated during different iterations' and the feeders eliminated at different stages of the solution are shown in Table 2.

V. CONCLUSION

The model developed has been applied to a practical case. The obtained solutions indicate that the energy losses mainly determine the optimal feeder size, and the investment cost always dictates the optimal routing of the power distribution. In this work, a three stage solution algorithm has been proposed to solve the problem of power distribution network. The problem was solved iteratively with the help of the optimization toolbox of Matlab. Heuristic rules are applied to force the integer variables to 0 or 1. The proposed technique can easily solve up to 30 substation nodes provided the computer have a sufficient memory size. The optimization program minimized the total cost of the distribution system as the objective function by determining the optima of the number locations and power of the substations, the routes of the feeders and the power losses within the network subject to a set of constraints.

Equally, the model recommended that the energy losses cost be included together with the investment cost in the planning stage. The results indicate that the more complex alternatives can lead to less expensive distribution expansion in the optimal solution.

The conclusions deduced from this research are:

- The optimization program minimized the total cost of the distribution system as the objective function by determining the optima of the number locations and power of the substations, the routes of the feeders and the power losses within the network subject to a set of constraints.
- The Power distribution system of Awka business unit have a capacity of 150MVA; this means that the distribution system should be able to handle the present load demand of Awka Business Unit of 135.12MVA with a margin capacity of 4.88MVA for future expansion and growth as well as unauthorized connection.
- The final solution indicates that an efficient and cost effective power supply can be achieved in the case studied by establishing a transmission substation of 120MVA capacity in Agu-Awka. The sub-station would be feed through the turning-in-and-out of the existing Onitsha-Oji River 132KV single circuit line.

REFERENCES

[1] Ibitoye F.I and Adenikinju A. (2006) Future demand for electricity in Nigeria, Elsevier Ltd.

[2] Sandeep Chakravorty & Smarajit Ghosh (2009) Power Distribution Planning using Multi-Criteria Decision Making Method. International Journal of Computer and Electrical Engineering, Vol. 1 No. 5.

[3] Vaziri, M., Tomsovic, K., Bose A., and Gonen T.; Distribution Expansion Problem: Formulation and Practicality for a Multistage Globally Optimal Solution, School of Electrical Engineering and Computer Science, Washington State University, Pullman U.S.A

[4] SEEDT Project No: EIE/05/056/S12.419632 June 2008, National Technical University of Anthens NTUA, SEEDT Coordinator Greece.

[5] Deshpande M.V. (1993) Elements of Electrical Power Station Design, Wheeler Publishing, Allahad India.

[6] Tùrkay B. (1998) Distribution System Planning using mixed Integer Programming, Elektrik, Vol6, No 1.

[7] IEEE Committee Report (1974), Report on Reliability Survey of Industrial Plants, Part 1; Reliability of Electrical Equipment, IEEE Transactions on Industry Applications, pp. 213-235, March/April.

[8] IEEE Committee Report (1975), Reliability of Electric Utility Supplies to Industrial Plants, Conference Record & CPS Technical Committee, pp. 131-133.