

Application of Laplace Wavelet Kurtosis and Wavelet Statistical Parameters for Gear Fault Diagnosis

Praneeth Chandran¹, M. Loksha², M.C. Majumder³ and Khalid Fathi Abdul Raheem⁴

¹Vellore Institute of Technology, Vellore, Tamil Nadu, India

^{2,4}Department of Mechanical Engineering, Caledonian College of Engineering, Oman

³Department of Mechanical Engineering, NIT, Durgapur, India

¹praneethchandran81@gmail.com, ²loksha@caledonian.edu.om, ³manik_rec@yahoo.com, ⁴khalid@caledonian.edu.om

Abstract– The methodology of vibration analysis for condition monitoring has been evolving at a rapid stage in the recent years. The ability to efficiently detect non-stationary, non-periodic, transient features of the vibration signal makes the wavelet analysis a demanding tool for condition monitoring. In this paper the application of Laplace wavelet kurtosis for processing vibration signal to detect faults in gears is presented. A gear testing apparatus is used for experimental studies to obtain vibration signal from a healthy and faulty gears. An experimental data is processed to compare the fault diagnostic capability of wavelet kurtosis with various wavelet statistical parameters such as Crest Factor, Impulse Factor and Shape Factors as obtained from Laplace wavelet. Further the Laplace wavelet kurtosis method is investigated for various working condition of the gear. Finally application of ANN is used for automated gear fault diagnosis by using the features extracted from wavelet transform.

Keywords– Wavelet, ANN, Laplace Wavelet Kurtosis, Gear, Crest Factor, Shape Factor and Impulse Factor

I. INTRODUCTION

Complex and advanced machines have been largely used for increasing the productivity and profit. The gears are the major component in transmission systems and proper maintenance of gear system is very essential to ensure failure free operation of plant machines. Vibration analysis is one of the major tools used for fault diagnosis of gears. Vibration monitoring works on the principle of healthy gear and faulty gear develop different vibration signals due to presence of fault such as gear tooth crack, gear tooth wear, pitting etc [1]. To analyze vibration signals different techniques such as time domain, frequency domain and time–frequency domain techniques are extensively used [2]. The frequency domain uses Fast Fourier Transform (FFT) of the time domain signal to assess the condition based on the frequency content of the signal. Vibration signals emitting from the gears are considered to be non-stationary and non-periodic signals. In such cases it is difficult to detect the gear fault by conventional FFT analysis [3-6]. Therefore an effective and sophisticated signal processing method like wavelet analysis for feature extraction from noisy gear signal can be used [7].

A number of wavelet functions are being considered and monitored for mechanical fault detection. Morlet and Impulse wavelet are commonly used wavelets for fault diagnosis in bearings. The optimization of these functions is based on the fact that maximum kurtosis increases the quality of fault detection [8]. Laplace wavelet is a complex, single sided damped exponential which finds its application in vibration analysis of an aircraft for aerodynamic and structural testing and to diagnose the wear of the intake valve of an internal combustion engine [9], [10].

Some of the commonly used statistical parameters for vibration signature analysis are Root Mean Square (RMS), Crest Factor, Shape Factor, peak to peak Impulse Factor, kurtosis etc. Kurtosis is said to be a static indicator that finds its application in time history which allows it to define the impulse character of a signal. Kurtosis is defined as the fourth central cumulant divided by the square of the variance of the probability distribution [11]. The representation of the kurtosis of each frequency component of a short time Fourier transform process is known as spectral kurtosis. The Crest Factor (CF) is the ratio of the peak value to the RMS value and hence dimensionless. Crest Factor helps in the study of differentiating a signal produced by a healthy and faulty gear box and also provides a mean to compare these noise measurements against the simulation results measured on the input and output shaft of the model [12], [13]. Shape Factors are a dimensionless quantity which defines the shape of an object or a signal. Shape factors are often normalized and thus its values vary from zero to one. A shape Factor equal to one usually represents an ideal case or maximum symmetry. Shape Factor value should also be adjusted so that the fault generated impulses can be clearly identified from the denoising result [14]. Impulse factor is also used to indicate fault in rotating machinery and it is defined as ratio of peak value to the mean value of the signal [15].

Artificial Neural Network is a computational or a mathematical model which closely resembles to biological neural network. These are non-linear statistical data modeling tools that are adaptive in nature. They tend to change their structures based on the external and internal information given to it during training process. It consists of an interconnected group of artificial neurons, and it processes

information using a connectionist approach to computation. ANN has been widely used in automated health detection and diagnosis of machine condition using features extracted from vibration signals.

This paper presents the application and use of Laplace wavelet kurtosis (LWK) for gear fault diagnosis. Further, it is also investigated how the wavelet parameters can be optimized so as to maximize the kurtosis of the wavelet coefficients in order to render the wavelet coefficients sensitive to the generated fault signals. The statistical parameters from Laplace wavelet transform namely the Crest Factor, Impulse Factor and Shape Factor are obtained and compared under various fault condition in order to understand their potential of being used for gear fault diagnosis. The study is also extended to on the behavior of Laplace wavelet kurtosis. Further it discusses the application of ANN for an effective classification of vibration data to analyze gear faults.

II. WAVELET KURTOSIS

A continuous wavelet is given by the following equation:

$$W(a, b) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|a|}} \psi \left(\frac{t-b}{a} \right) dt \tag{1}$$

Where ‘b’ acts to translate the function across x(t) and the variable ‘a’ acts to vary the time scale of the probing function ψ. If ‘a’ is greater than 1, the wavelet function ψ is stretched along the time axis and if ‘a’ is less than 1 then it contracts.

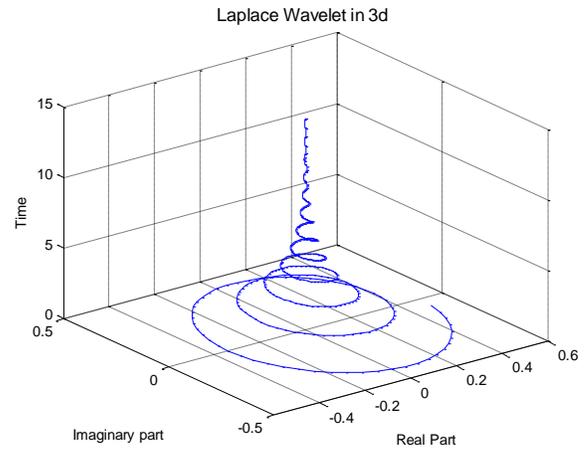
The Laplace wavelet is a complex, analytical and single-sided damped exponential, and it is given by,

$$\psi(t) = A e^{-\left(\frac{\beta}{\sqrt{1-\beta^2}} + j \right) \omega_c t} \tag{2}$$

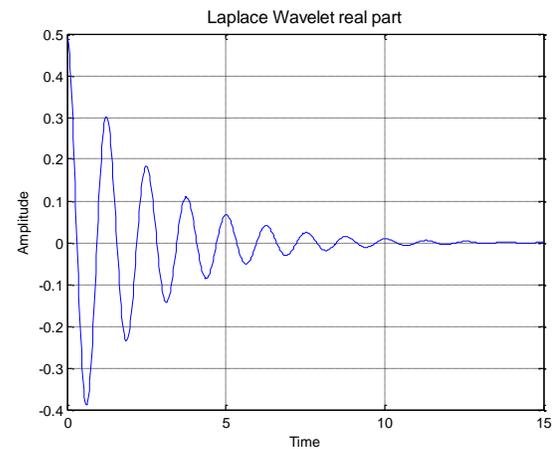
Where, β is a factor that controls the decay rate of the exponential envelope in the time, known as damping factor and regulates the resolution of the wavelet. It simultaneously corresponds to the frequency band width of the wavelet in the frequency domain. Frequency ω_c determines the number of significant oscillations of the wavelet in the time domain and corresponds to the wavelet centre frequency in frequency domain. A is an arbitrary scaling factor. Fig. 1 depicts the 3-D view, the real and imaginary part of a Laplace wavelet.

The wavelet transform (WT) of the signal x(t) with the mother wavelet ψ(t) is the inner product of x(t) with a scaled and conjugate wavelet ψ*_{a,b}. Since the wavelet used is analytical and as we employed complex wavelet to calculate the wavelet transform, the result of the wavelet transform obtained will also be analytical signal as shown in equation (3) and (4).

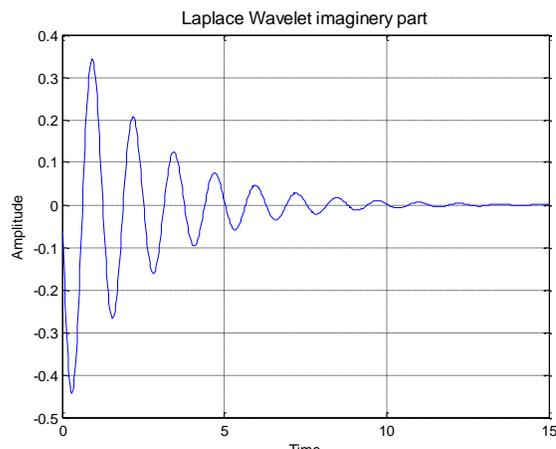
$$WT\{x(t), a, b\} = \langle x(t), \psi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi_{a,b}^*(t) dt \tag{3}$$



a)



b)



c)

Fig. 1: a) 3 D view b) real part c) Imaginary part

$$= \text{Re} [WT(a, b)] + j \text{Im} [WT(a, b)] \tag{4}$$

Where ψ*_{a, b} is a family of wavelet with a as scale parameter and b as translation parameter.

The Laplace wavelet kurtosis are calculated by the following steps:

- a. Time domain values are collected using an accelerometer and data collector from the experimental setup
- b. Time domain values are transformed into wavelet transform using Laplace wavelet function.
- c. Laplace wavelet kurtosis is calculated from wavelet transform

The schematic representation of above process for a vibration signal is depicted in Fig. 2.

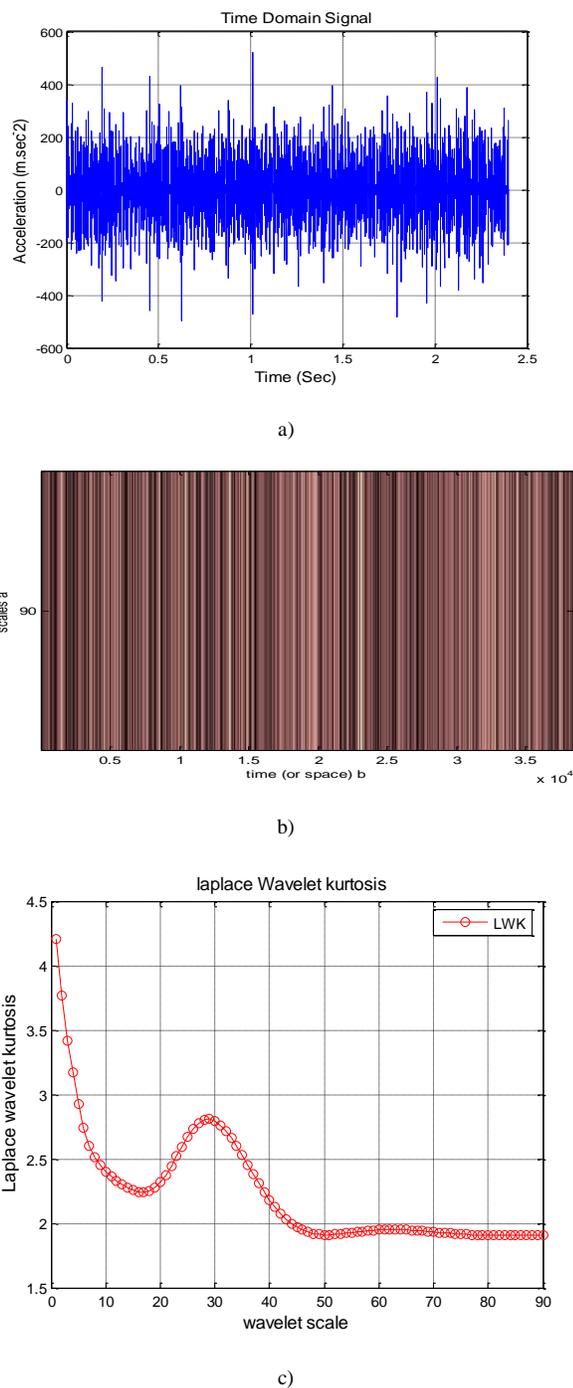


Fig. 2: The Methodology for calculation of wavelet kurtosis (a) Vibration signal collected from experimental setup, (b) The Wavelet Transform (c) Plot of Wavelet Kurtosis vs. wavelet scale.

Let $x(n)$ be a real discrete time random process, and WT_a its N point Laplace wavelet transform at scale a . The Laplace wavelet kurtosis (LWK) for $x(n)$ is defined as the kurtosis of the magnitude of WT_a at each wavelet scale a as given by [8]:

$$LWK_{(a)} = \frac{\sum_{n=1}^N abs(WT_a^4(x(n))/\beta, \omega_c)}{[\sum_{n=1}^N abs(WT_a^2(x(n))/\beta, \omega_c)]^2} \quad (5)$$

III. EXPERIMENTAL SETUP

The setup used for experimentation is shown in Fig. 3. It consists of a motor, simple (one stage reduction) gear box and loading system. The input side of gearbox was connected to 0.5 HP, 2900 RPM electric motor through coupling and the output side of the gearbox was connected to a loading system.

All drive shafts are supported at its ends with antifricition bearings. The vibration data is collected from the drive end bearing of gear box using the accelerometer (model 621B40, IMI sensors, sensitivity is 1.02 mV/m/s² and frequency range up to 18 kHz) with a NI Data Acquisition Device. The healthy gears are depicted in Fig. 4. The vibration data collected are processed in MATLAB for signal processing.

The vibration signals from a healthy gear were collected at a shaft speed of 2850 RPM. Faults were induced in four different stages as shown in Table 1 and the corresponding vibration readings were taken. The various fault stages are shown in Fig. 5.

TABLE 1: STAGES OF INDUCED FAULT

Stage of fault	Condition of the gear	Fault description
Stage 0	Healthy gear	Without any induced fault
Stage 1	Faulty gear	A crack of 3mm is induced at the root of the tooth
Stage 2	Faulty gear	Tooth was partially broken
Stage 3	Faulty gear	Fault was further increased.
Stage 4	Faulty gear	Tooth was completely removed

IV. IMPLEMENTATION OF LWK

This section provides implementation of the proposed approach of gear fault diagnosis. It is normal that the increase in the magnitude of the wavelet kurtosis value indicates the presence of fault. As the fault size progresses the corresponding wavelet kurtosis value is also expected to increase in magnitude. Fig. 6 shows a typical time domain signal obtained from the experimental setup.

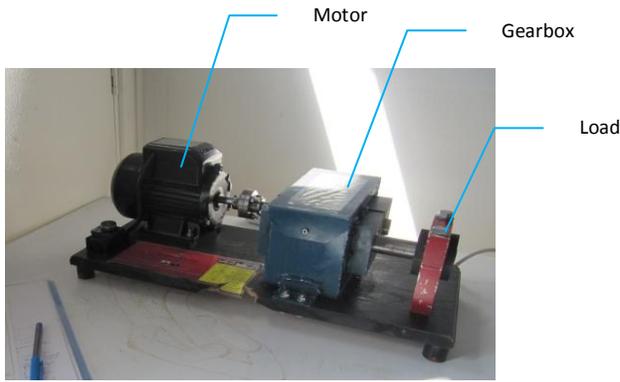


Fig. 3. Fault Simulator set up



Fig. 4: View of healthy gears



a)



b)



c)



d)

Fig. 5: Stages of induced crack (a) Stage 1, (b) Stage 2, (c) Stage 3, and (d) Stage 4

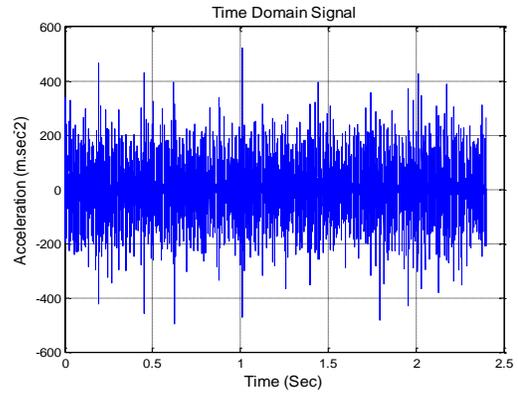


Fig. 6: Time domain data

This is further processed using various signal processing techniques based on spectral kurtosis (SK) principle and wavelet kurtosis based on Laplace wavelet function.

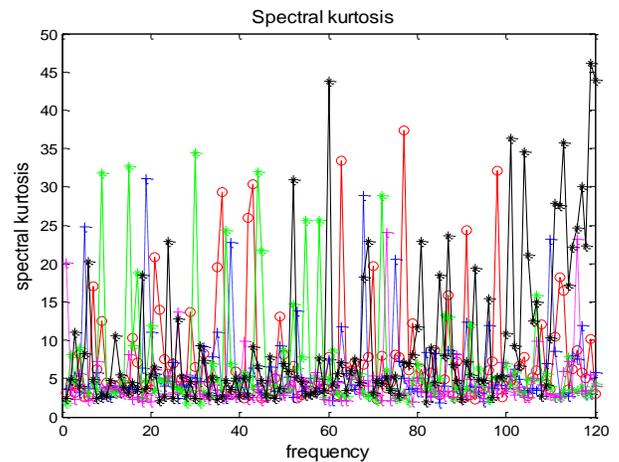


Fig. 7: SK for different stages of fault

The Fig. 7 depicts the spectral kurtosis to determine the gear faults. Spectral kurtosis with various fault condition provides prominence of peak with stage 4 fault at higher frequency. However, it is difficult to analyze and isolate peaks corresponding to the healthy and faulty signals due to complex inter mixing of signals at constant window size. The implementation of equation.5 results in Laplace wavelet

kurtosis with wavelet scale of 90 for the gear with healthy conditions and at 4 different stages of crack as shown in Fig. 8. The Laplace wavelet shape parameters $\beta=0.3$ and $\omega_c=8.1$ are selected based on maximum kurtosis.

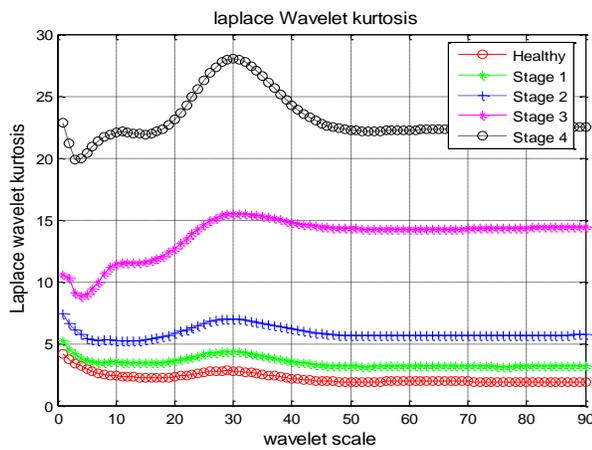


Fig. 8: Laplace wavelet kurtosis

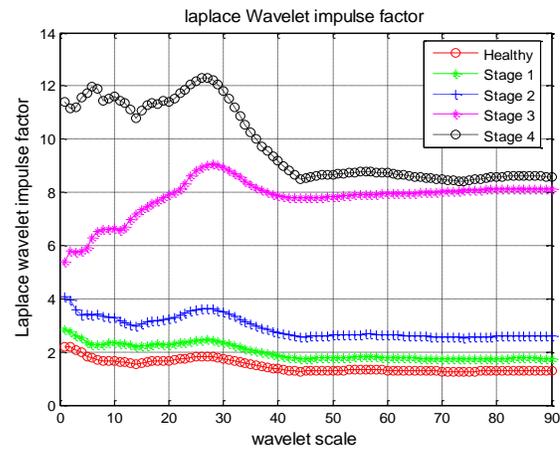
The healthy and faulty conditions of the gear at different stages are shown with increasing magnitude of LWK value with distinct correlation between them. As the fault progresses in size, corresponding LWK values also increase in magnitude. The wavelet scale number and frequency relationship is given as:

$$F_a = \frac{F_0}{a * \Delta} \tag{6}$$

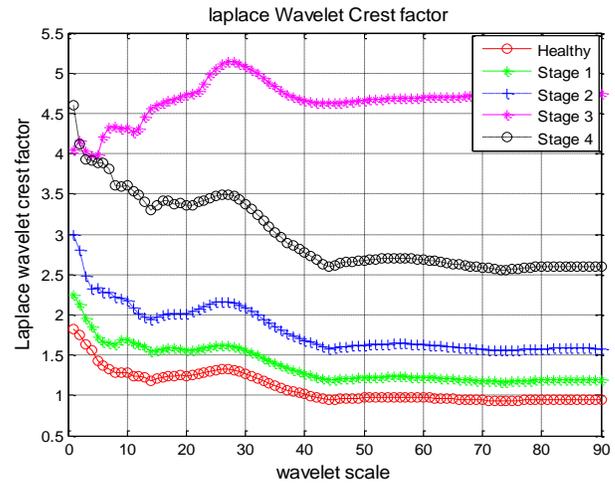
Where F_a is frequency, F_0 = wavelet central frequency, a = wavelet scale, Δ = sampling frequency. The wavelet scale number of 30 is corresponding to the frequency of 24Hz, which is equal to gear rotation frequency. It is evident, LWK results in a prominent magnitude at gear rotation frequency with all fault conditions and increase in magnitude is significant for stage 3 and 4 fault condition at all wavelet scale.

Fig. 9 shows the comparison of Laplace wavelet kurtosis with other Laplace wavelet transforms statistical parameters like Crest Factor, Impulse Factor, and Shape Factor. As seen from the Fig. 9, the Laplace wavelet Crest Factor depicts changes in the magnitude, but fails to show the distinction of fault at different stages. However, the Crest Factor at wavelet scale 30 is dominant with stage 3 and 4 faults. This means that the scale factor is sensitive to the fault at the last stage of degradation. We observe that Laplace wavelet Shape Factor provides useful insight into the indication fault particularly at the later stage of fault development, but often fails to give proportional changes with degree of fault. Impulse Factor provides considerable changes in relation to magnitude of fault, but not essentially showing evidence of consistency, might be attributed to the error in data. Fig. 9(c) shows the significant changes in shape factor, particularly with later stage of fault (stage 4). Again there is a moderate changes in Shape Factors at the initial stage of fault. More or less, the

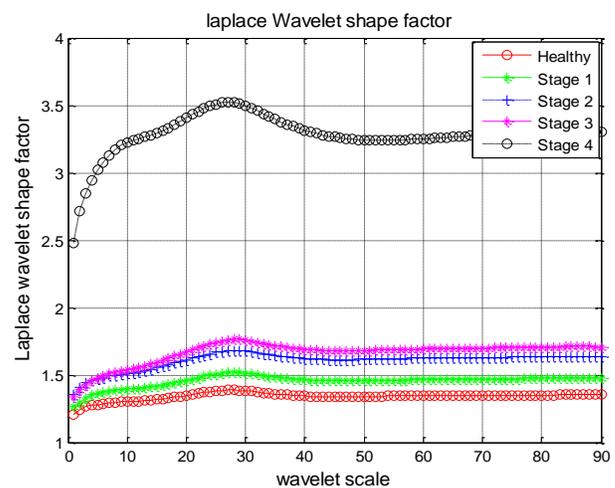
Impulse Factor and Shape Factor and Crest Factor parameters are more prominent when the defect in the gears reaches the final stage of degradation even though, these parameters provide an indication of fault at gear mesh frequency with all fault conditions.



a)



b)



c)

Fig. 9: (a) Laplace wavelet Crest Factor (b) Laplace wavelet Impulse Factor (c) Laplace wavelet shape factor

The proposed Laplace wavelet kurtosis method is also implemented for the different working condition of gears (change in load, speed and lubrication). Fig. 10 shows the Laplace wavelet kurtosis for varying speed, load and lubrication conditions.

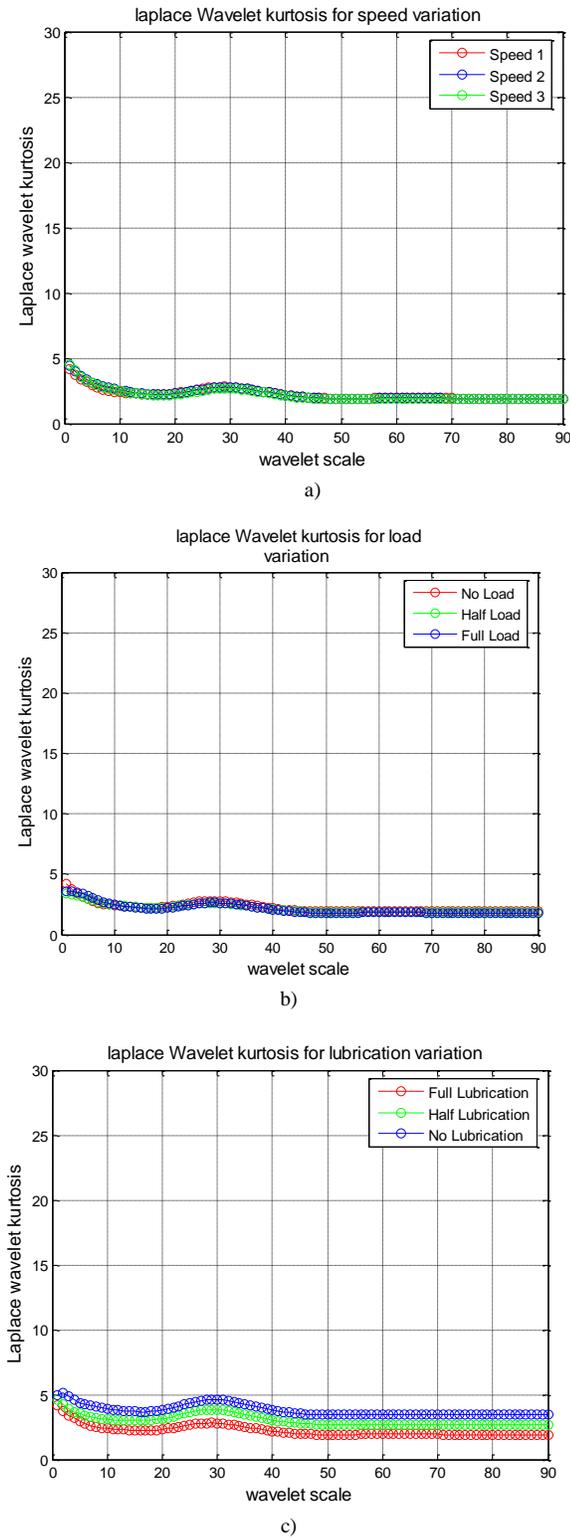


Fig. 10: Laplace wavelet kurtosis for different working conditions (a) varying speed condition (b) varying load conditions (c) varying lubrication condition

The result obtained shows that the Laplace wavelet kurtosis does not show significant changes with respect to varying work conditions.

V. ARTIFICIAL NEURAL NETWORK (ANN)

A feed forward multilayer perceptron (MLP) neural network has been developed with 3 layers. ANN consists of one input layer with 4 source nodes and a hidden layer of 5 computation nodes. The output layer with 2 nodes, which classifies the working condition of gear as healthy (0 1) and faulty (1 0) for the gear signals has been developed. Fig. 11 shows the architecture of ANN implemented for the application of fault diagnosis in gears.

ANN training and testing was created using MATLAB Neural Network toolbox with maximum iterations (epochs) of 1000, MSE of 10E-10, minimum gradient of 10E-10 were used. The training process would stop, if any of these restrictions are met. The initial weights and biases of the network are generated by the program.

Training of an MLP network is achieved by modifying the connection weights and biases iteratively to optimize the performance criterion. Statistical features like Standard deviation and Kurtosis and in frequency domain such as peak frequency (f_{max}) to the shaft rotational frequency (f_{rpm}) ratio (f_{max}/f_{rpm}), and the maximum amplitude (A_{max}) to the overall amplitude (Sum (A_i)) ratio ($A_{max}/\text{sum}(A_i)$) of vibration signals obtained from wavelet transform with Laplace wavelet as base function are used as an input to ANN. The training process is shown in Fig. 12. The ANN needs only 18 epochs to reach the 5×10^{-3} MSE. The MSE for the testing process is shown in Fig. 13.

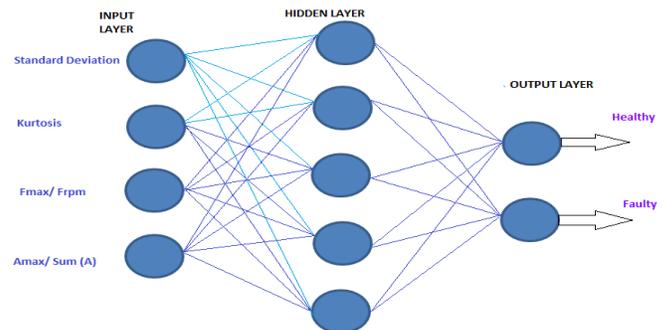


Fig. 11: Architecture of ANN

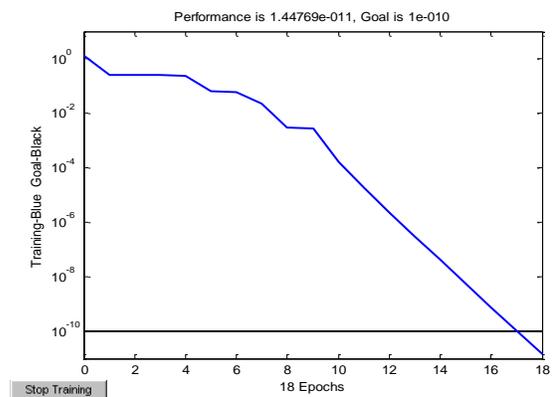


Fig. 12: Training process

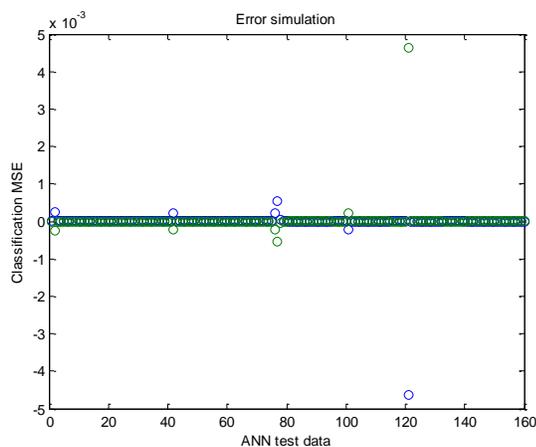


Fig. 13: Testing process

VI. CONCLUSION

Various signal processing methods adopted using wavelet transform for the gear fault diagnosis is presented in this paper. Gear fault diagnosis using Laplace wavelet kurtosis is implemented and the results are studied for the various stages of induced fault conditions in the experimental setup.

Further, statistical parameters like Laplace wavelet crest factor, Impulse Factor and shape factors are compared with kurtosis parameter. The proposed Laplace wavelet kurtosis method depicts increasing magnitude of LWK value along with increase in the size of fault and hence shows prominence as a useful tool to show the correlation between healthy and faulty gears. It was also observed that Laplace wavelet kurtosis proved to be a better tool for vibration analysis than other Laplace statistical parameters. The study also shows that Laplace wavelet kurtosis has less influence for varying work condition. All these factors enhance the use of this proposed method for gear fault diagnosis. Further, wavelet coefficients provide provision for statistical features of signal which can be used as inputs to ANN. The result of the learning process of the proposed ANN shows that the training with 18 iterations met the MSE stopping criteria (MSE less than 5×10^{-3}). The test process for unseen vibration data of the trained ANN combined with the ideal output target values indicates the high success rate for automated gear fault detection.

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Mr. Praneethchandran is pursuing his Bachelor degree in Mechanical engineering at Vellore Institute of Technology University, Vellore, Tamil Nadu, and India. His research interest is on machine fault diagnosis using vibration analysis and signal processing.



Mr. M. Lokesha received his B.E. degree in Mechanical Engineering from Mysore University, Mysore, India in 1989 and M.TECH. Degree in Maintenance engineering from Mysore University, Mysore, India in 1994. Presently, he is working as a Senior Lecturer in the Department of Mechanical and Industrial Engineering at Caledonian College of Engineering, Muscat. He has been working for 20 years in engineering institutions His research focuses on vibration instrumentation & measurement, condition monitoring of rotating machinery. He is currently PhD research student at National Institute of Technology, Durgapur, India.



Professor Manik Chandra Majumdar is a Professor in the Department of Mechanical Engineering and Member, Board of Governors of NIT Durgapur India. He has a PhD from the Indian Institute of Technology, Kharagpur, India. He has guided many Ph.D. scholars. His prime area of research is Tribology /Design & Production



Professor Khalid Fatihi Abdul-Raheem, B.E., M.Tech from University of Technology, Iraq and PhD from Glasgow Caledonian University, UK. Presently he is working as a Associate professor in the Department of Mechanical and Industrial Engineering at Caledonian College of Engineering, Muscat. He has been working for more than 22 years in engineering institutions and many industries. He has many publications to his credit. He has research interest in condition monitoring and signal analysis, artificial intelligence, automatic control system, mechanical vibration analysis and control.