A Perspective on Whether Robot Localization Can be Effectively Simulated by Quantum Mechanics

Atiya Masood
Lecturer, Yanbu University College, Royal Commission at Jubail & Yanbu, KSA

***Abstract*– This paper is a theoretical concept that presents a perspective on whether quantum mechanics can be used for robot localization by using concepts of particle filter and quantum computation. A system is made of particles and particle filter is a sequence of algorithms for estimating a state of system. It is a way of taking variable of interest about something you are trying to measure, and whittling those guesses down by using measurements from sensors. Quantum mechanics has the power of atoms. It provides a mean of obtaining information about a system in a micro world associated with various attributes. Quantum states are linear super position of its components states and particles exist in linear combination of superposition. By using quantum power, I propose a new algorithm of particle filter and also its limitation.**

***Keywords*–** Quantum Mechanics, Particle Filter, Superposition, Quantum Computation, De-coherence, Closed System and Complex Number

I. INTRODUCTION

The robot localization is a key problem in mobile robot system [1]. It has also been referred to as "the most fundamental problem of providing a mobile robot with autonomous capabilities" [2]. But the beauty of particle filter is that they provide solution to all mobile robot localization problems that come in different flavors such as "position tracking", "global localization", "kidnap robot problem" and "multi robot localization problem".

A Quantum mechanical system is described by state vectors and this state vector is a function of time $|\varphi(t)>$.Any State space of the system, if the system is closed, we can completely describe by its state vectors, which is a unit vector in a space and the evolution of a closed system is described by a unitary transformed.

$$|\varphi(t1)> = U\,|\varphi(t2)> \qquad (1)$$

The quantum mechanical bit is a quantum bit or qubit. A qubit is a bit of information that can be both zero and one, two level of quantum system with two states |0> and |1>. Physically a qubit may be represented by, e.g., two states in an atom, the polarization of a photon, or the spin of an electron, etc. If a qubit is in one of the two internal states |0> or |1> it may be regarded as a classical bit, but an important difference between bits and qubits is the ability of a qubit to be in a superposition of the two states.

$$\alpha|0> + \beta|1> \qquad (2)$$

Quantum Computer with 500 qubits gives 2^{500} superposition states. Each state would be classically equivalent to a single list of 500 1's and 0's .Such computer could operate on 2^{500} state simultaneously. The potential and power of quantum mechanics can use superposition to make particle guesses of 2^{500} particles.

II. REPRESENTATION

A. Particle Filter

Particle Filter is a sequence of algorithm for localization. It is a hypothesis tracker that approximates the filtered posterior distribution by a set of weighted particles. It weighs particles based on a probability and then propagates these particles according to a motion model. The Particle filter operates in two phases, prediction and update. In prediction stage each particle is modified according to the existing model, including the addition of random noise in order to simulate the effect of noise on the variable of interest. Then each particle weight is reevaluated based on the latest sensory information available, it is called update stage of particle filter [3].

Particle filters estimate the posterior over unobservable state variables from sensor measurements. In the context of the present paper, *state* refers to the position of the robot (location and orientation) relative to its environment, along with the number of obstacles in the robot's proximity. For the sake of the general discussion of particle filters, the total of all those state variables will be denoted by $\boldsymbol{x}$.

In particular, let the state at time $\boldsymbol{t}$ denoted by x_t . Particle filters address situations in which this state is not directly observable. Instead, the robot must rely on sensor measurements and information about the controls it executes to infer the posterior distribution over x .Let z_t denote the sensor measurement acquired at time t and u_t *denote the* control at time t. Thus at time t two types of information relevant to the state x_t is available to the robot.

$$z^t := \{z_1, \ldots\ldots zt\} \qquad (3)$$
$$u^t = \{u_1, \ldots\ldots ut\} \qquad (4)$$

The goal of particle filtering is to estimate the posterior probability over the state variable x at time t

$$p(x_t|z^t,u^t) \quad (5)$$

This posterior is calculated recursively:

$$p(x_t|z^t,u^t)= (z_t|x_t)\ p(x_t |u_t , x_{t-1})\ p(x_{t-1}| z^{t-1},u^{t-1})dx_{t-1} \quad (6)$$

Here η is a constant normalizer. The conditional probability distribution $p(z_t | x_t)$ is a *measurement model* Similarly, $p(x_t |u_t , x_{t-1})$ is a *motion model.* The recursive update "equation (6)" is equivalent to the well-known Bayes-filters.The key idea of the particle filter is to approximate this posterior by a set of hypothesized states called *particles*, which are distributed according to $p(x_t|z^t,u^t)$. Put mathematically $p(x_t|z^t,u^t)$ is represented by a set of particles.

$$Xt:=\{x_t^{[i]}\}\ i=1,\ldots.N \quad (7)$$

Where N is a size of particles. It is well-known that such a set of particles x_t can be obtained via the following sampling procedure, which is directly derived from the recursive update "equation (5)".

$x_t=\emptyset$
for *i*=1 to N do
take $x_{t-1}^{[i]}$ *from xt*$_{-1}$
draw$x_t^{[i]} \sim p(x_t |u_t , x_{t-1}^{[i]})$
calculate (non –normalized) weight
$w_t^{[i]}=p(z_t|x_t^{[i]})$
endfor
for *i*=1 to N do
draw k with probability $w_t^{[k]}/\sum_t^N wt[i]$
add $x_t^{[k]}$ *to* x
endfor

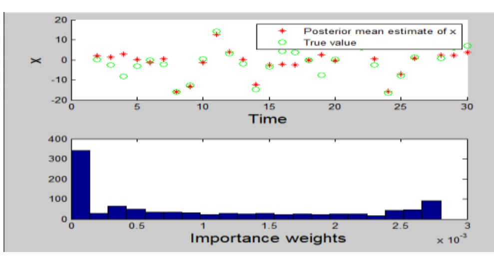

Fig. 1. Imporatnt weights of particles and posterior mean estimate of x

B. Prediction

In the environment there are particles, each particle is structured as an x coordinate, a y coordinate and a heading direction .These three values comprise of a single guess. In order to predict the probability distribution of a pose of the robot after a motion needs to have a model of the effect of noise on the resulting pose. A single point will carry three values (X,Y,Θ) for X position, Y position, and Θ heading .Robot can drive forward along the heading, or rotate its heading. Each particle has three dimension vectors.

$$X' = X + V\Delta t \cos\theta \quad (8)$$

$$Y' = Y + V\Delta t \sin\theta \quad (9)$$

$$\theta' = \theta + \omega t \quad (10)$$

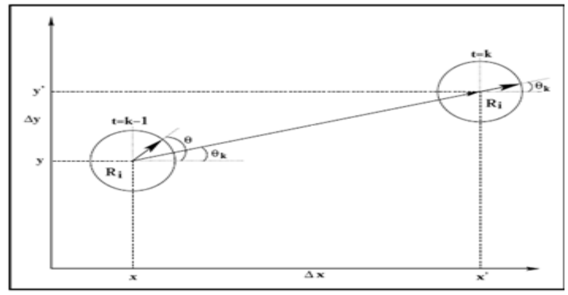

Fig. 2. Arbitrary motion of robot Ri. At time t [x,y,θ], after the motion the pose is [x',y' θ']

C. Quantum Mechanics

Quantum physics is branch of science that deal with discrete, invisible unit of energy called quanta as described by quantum theory. Quantum theory attempts to describe the behavior of very small objects, generally speaking the size of atom or smaller in much the same way as relativity describes the law of larger everyday objects. Hence the assumption that quantum mechanics is applicable only in microscopic domain it is only partially true. It applies to everything, such as galaxies, universe presumably, photon and electrons.

Superposition is a principle of Quantum theory that describes the nature and behavior of matter and forces at the sub atom level .Superposition claims that we do not know the state of any object, it is actually in all possible state simultaneously.

D. Quantum Computation

Building block of a classical computer is bit, either 0 or 1, whereas the basic building block of quantum mechanical is qubits, a qubit may be represented by two states in an atom, the polarization of a photon, or the spin of an electron, etc.. If a qubit is in one of the two internal states|0> and |1>, it may be regarded as a classical bit, but an important difference arises between bits and qubits is due to the ability of e qubit to be in two states simultaneously.

$$|\varphi >= |0 > + |1 > \quad (11)$$

Where α and β are complex numbers fulfilling

$|\propto|^2 + |\beta|^2=1$.It is important to realize that because the state in "equation (11)",may exhibit interferences between the states |0> and |1> there is a major difference between the coherent superposition in "equation (11)",and statistical mixture of |0> and |1>.The classical bit obeys the rules of classical logic and is either 0 or 1, whereas the qubit act according to quantum logic and may be in both |0> and |1> at the same time. The difference between classical and quantum logic is at the heart of many of the 'paradoxes' in quantum mechanics, eg.Schrö dinger cat.

In quantum mechanics, the evolution of a closed system is described by unitary operators, and the action of the quantum computer is to implement a given unitary operation U on the initial state $|\varphi t1>$, so that it produces the final state $|\varphi t2>$ which may be measured by the experimentalist

$$\varphi t1>=U|\varphi t2> \qquad (12)$$

The construction of a general purpose quantum computer thus requires the ability to implement any unitary operator on the mutual state of n qubits in the computer. Any unitary operator on n qbits may be implemented by applying rotations on single qubits and two-qubit C-Not gates.

$$\alpha|00|>+\beta|01>+\Upsilon|11>+\delta|10> \qquad (13)$$

The control-not gate acts on two qubits and flips the state of second qubit if the first qubit is in |1>, i.e., it implements the unitary transformation.

|00>→|00> |01>→|01>
|10>→|11> |11>→|01>

For matrix representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Quantum state of any system is not directly observable but process of measurement in computational basis can extract information from quantum computers. This process gives classical bit and its probability of occurrence.

$$\alpha|0|>+\beta|1> \quad 0 \text{ with probability of } |\propto|^2 \qquad (14)$$

1 with probability of $|\beta|^2$

Where α and β are complex numbers fulfilling $|\propto|^2 + |\beta|^2 = 1$ this is normalization condition of quantum state.

II. QUANTUM MECHANICS AND PARTICLES FILTERS

System is made-up of particles and quantum mechanics also deals with microscopic elements such as atom, particles. A Quantum mechanical system is described by state vectors and its evolution in a closed system .According to James B. Hartle, in quantum mechanics, generally the universe as a whole can be taken as a closed system [4]. All quantum systems show wave-like and particle-like properties. Particle filter also used particles for localization of robot by assigning important weight or probability to particles and re-sampling. Following is mathematical transition of particle filter algorithms in quantum mechanics.

According to Hamiltonian formulism in classical mechanics:

$$\dot{q} = \frac{dH}{dp} \quad \text{And } \dot{p} = -\frac{dH}{dq} \qquad (15)$$

There is a set of differential equations known as the *Hamilton equations* which give the time evolution of the system

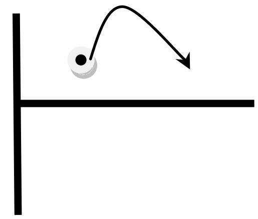

Fig. 2. Phase space specifies particle and its future value

In phase space "equation 15" specifies particles and its future values.

In Quantum mechanics no more phase space it is replaced by Hilbert space of states .Hence particles in a system are represented by a vector state and time evolution of these particles is determined as **$|\varphi t>$.**After each measurement **of p**osition of these particles has Eigen values a_1 ,a_2,a_3 ... Eigen vectors |∅1>, |∅2>, |∅3> forms basis set of particles in Hilbert Space of the system (assumption).

At any instance of time

$$|\varphi t> = \sum_n c_{n(t)} |\phi_n> \qquad (16)$$

Position of a particle x is not explicitly time dependent; it may be dependent after the motion of particles.

$$<\phi_n|\varphi t> = C_{n(t} \qquad (17)$$

$$<\varphi t|\emptyset n> = {}^*C_{n(t)} \qquad (18)$$

Probability or weight of each particles is $|C_{n(t)}|^2$.This is the probability of the particles at time t After motion re-sampling the particle at any instance t .The probability or weight of the particle can be extracted by using same process as quantum computation**.**

$$\frac{\sum_n a_n |C_{n(t)}|^2}{|C_{n(t)}|^2} \qquad (19)$$

$$\frac{\sum_n a_n <\phi_n|\varphi t><\varphi t|\emptyset n>}{\sum_n <\phi_n|\varphi t><\varphi t|\emptyset n>} \qquad (20)$$

$$\frac{|\varphi t>\sum_n a_n|\phi_n><\phi_n|\varphi t>}{|\varphi t>\sum_n |\phi_n><\phi_n|\varphi t>} \qquad (21)$$

$$\frac{|\varphi t>\sum_n a_n|\phi_n><\phi_n|\varphi t>}{<\varphi t|\varphi t>} \qquad (22)$$

So, "equations (16) to (22)" represent particle filter algorithms in quantum mechanics. Particle filter is one of the simplest algorithms but if quantum implementation is used then it also affects the efficiency of the algorithm because of the superposition principal.

III. MAJOR PROBLEMS FOR CREATING NEW ALGORITHIM

Robot localization cannot be effectively simulated by Quantum particles filters because of many factors, de-coherence, closed system, and probability amplitude.

Measurement of superposition quantum state will collapse it into classical state. This is called de-coherence. [5].Any kind of measurement of quantum state parameter considers interaction process with environment, which causes a change of some parameter of this quantum state. For instance the original state of qubits gets disturbed during the measurement process.

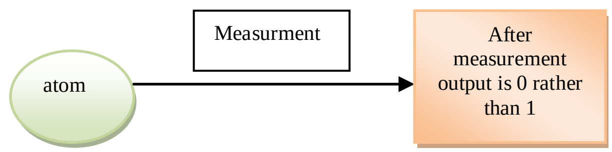

Fig. 3. De-coherence of atom state

Every real-life quantum system is coupled to an environment ("bath") [6].

$$H = H_s + H_B + H_{int} \quad (23)$$

$$H_{int} = \sum_\alpha S\alpha \otimes B\alpha \quad (24)$$

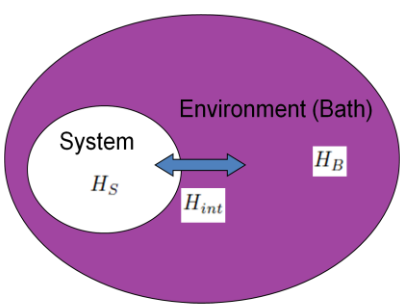

Fig. 4. Quantum system is coupled to an environment

De-coherence is one of the major obstacles in a process of particles interacting with other particles. If de-coherence problem cannot be solved, algorithms of robot localization cannot be implemented.

α and β are complex numbers fulfilling $|\alpha|^2 + |\beta|^2 = 1$ by referring "equation (2)" Amplitudes are complex numbers. Physical process cannot run if amplitude is negative or complex. Hence complex number implies that localization cannot be effectively simulated by Quantum particles filters.

A Quantum mechanical system is described by state vectors and its evolution in a closed system. A closed system is a physical system which doesn't exchange any matter with its surroundings and isn't subject to any force whose source is external to the system [7]. Open quantum systems are *not* described by the Schrodinger equation but in quantum mechanics violation of the Schrodinger equation is not acceptable.

IV. CONCLUSION

This paper presented the new algorithms of quantum particle filter by using quantum mechanics for robot localization. Quantum mechanics deals with microscopic domain, such as particles. Hence algorithms of particle filters can be described by quantum mechanics by using the power of superposition, the robot can guess all the particles simultaneously. De-coherence is one of the major problems in creating new algorithms of quantum particle filters. In order to use quantum mechanics for localization, many operations must be performed before quantum coherence is lost.

ACKNOWLEDGMENT

I gratefully acknowledge the Yanbu University and College, Royal Commission for Yanbu, Kingdom of Saudi Arabia for every support of this research.

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