Various Volume Fractions Laws for a Functionally Graded Cylindrical Shell under Hamilton's Principle

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Abstract— **Study of the vibration of thin cylindrical shells made of a functionally gradient material (FGM) composed of stainless steel and nickel is very important. The objective is to study the natural frequencies and the effects boundary conditions on the natural frequencies of the functionally graded cylindrical shell. The study is carried out using third order shear deformation shell theory (T.S.D.T). The analysis is carried out using Hamilton's principle. The governing equations of motion of functionally graded cylindrical shells are derived based on T.S.D.T theory. Results are presented on the frequencies with various power law exponents.**

Keywords– Volume Fraction, Frequencies and Law Exponent

I. INTRODUCTION

Researches on free vibrations of cylindrical shells have been carried out extensively [1-5]. Recently, the present authors presented studies on the influence of boundary conditions on the frequencies of a multi–layered cylindrical shell [6]. In all the above works, different thin shell theories based on Love–hypothesis were used. Vibration of cylindrical shells with ring support is considered by Loy and Lam [7]. The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan [8-9] as a means of preparing thermal barrier materials. Since then, FGMs have attracted much interest as heat-shielding materials. FGMs are made by combining different materials using power metallurgy methods [10]. They possess variations in constituent volume fractions that lead to continuous change in the composition, microstructure, porosity, etc., resulting in gradients in the mechanical and thermal properties [11-12]. Vibration study of FGM shell structures is important. In this paper a study on the vibration of functionally graded cylindrical shells is presented. The FGMs considered are composed of stainless steel and nickel where the volume fractions follow a power-law distribution. The study is carried out based on T.S.D.T theory. The analysis is carried out using Hamilton's principle.

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II. FUNCTIONALLY GRADED MATERIALS

For the cylindrical shell made of FGM the material properties such as the modulus of elasticity E, Poisson ratio ν and the mass density ρ are assumed to be functions of the volume fraction of the constituent materials when the coordinate axis across the shell thickness is denoted by z and measured from the shell's middle plane. The functional relationships between E, ν and ρ with z for a stainless steel and nickel FGM shell are assumed as [13].

$$E=(E_1-E_2)\left(\frac{2Z+h}{2h}\right)^N+E_2 \tag{1}$$

$$\nu=(\nu_1-\nu_2)\left(\frac{2Z+h}{2h}\right)^N+\nu_2 \tag{2}$$

$$\rho=(\rho_1-\rho_2)\left(\frac{2Z+h}{2h}\right)^N+\rho_2 \tag{3}$$

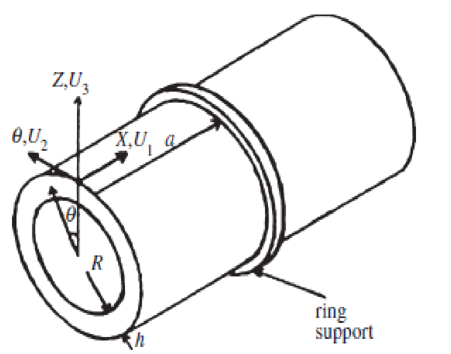

Fig. 1. Geometry FGM Cylindrical Shell

$$A_1=\left|\frac{\partial\bar{r}}{\partial\alpha_1}\right| \quad , \quad A_2=\left|\frac{\partial\bar{r}}{\partial\alpha_2}\right| \tag{4}$$

A_1 and A_2 are the fundamental form parameters or Lame parameters. The third- order theory of Reddy used in the present study is based on the following displacement field:

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) + \alpha_3^2.\psi_1(\alpha_1,\alpha_2) + \alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) + \alpha_3^2.\psi_2(\alpha_1,\alpha_2) + \alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{5}$$

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to $\in_{13} = \in_{23} = 0$ at $Z = \pm\frac{h}{2}$ Thus for third order theory

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{6}$$

where $C_1 = \frac{4}{3h^2}$. Obtained for the third-order theory of Reddy is

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{7}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} \tag{8}$$

where

$$\begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial u_1}{\partial\alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{u_3}{R_1}) \\ (\frac{1}{A_2}\frac{\partial u_2}{\partial\alpha_2} + \frac{u_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1} + \frac{u_3}{R_2}) \\ \frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{u_1}{A_1}) \end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial\phi_1}{\partial\alpha_1} + \frac{\phi_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}) \\ (\frac{1}{A_2}\frac{\partial\phi_2}{\partial\alpha_2} + \frac{\phi_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}) \\ (\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1})) \end{Bmatrix} \tag{10}$$

$$\begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} = -C_1 \begin{Bmatrix} (\frac{1}{A_1}(-\frac{\partial u_1}{R_1\partial\alpha_1} + \frac{\partial\phi_1}{\partial\alpha_1} + \frac{\partial^2 u_3}{A_1\partial\alpha_1^2} - \frac{\partial A_1}{A_1^2\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_1}) + \frac{\partial A_1}{\partial\alpha_2}\frac{1}{A_1A_2}(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})) \\ (\frac{1}{A_2}(-\frac{\partial u_2}{R_2\partial\alpha_2} + \frac{\partial\phi_2}{\partial\alpha_2} + \frac{\partial^2 u_3}{A_2\partial\alpha_2^2} - \frac{\partial A_2}{A_2^2\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_2}) + \frac{\partial A_2}{\partial\alpha_1}\frac{1}{A_1A_2}(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})) \\ (\frac{A_2}{A_1}(-\frac{\partial}{R_2\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{1}{A_2^2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_2^4}\frac{\partial A_2^2}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_2}) + \\ +\frac{A_1}{A_2}(-\frac{\partial}{R_1\partial\alpha_2}(\frac{u_1}{A_1}) + \frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1}) + \frac{1}{A_1^2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_1^4}\frac{\partial A_1^2}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_1})) \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} = \begin{Bmatrix} (\phi_1 - \frac{u_1}{R_1} + \frac{1}{A_1}\frac{\partial u_3}{\partial\alpha_1}) \\ (\phi_2 - \frac{u_2}{R_2} + \frac{1}{A_2}\frac{\partial u_3}{\partial\alpha_2}) \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} = 3C_1 \begin{Bmatrix} (-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ (-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} = C_1 \begin{Bmatrix} \dfrac{(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})}{R_1} \\ \dfrac{(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})}{R_2} \end{Bmatrix} \tag{14}$$

III. FORMULATION

The equations of motion for a generic shell can be derived by using Hamilton's principle which is described by:

$$\delta\int_{t_1}^{t_2}(\Pi - K)dt = 0 \ , \ \Pi = U - V \tag{15}$$

Where K, Π, U and V are the total kinetic, potential, strain and loading energies, t_1 and t_2 are arbitrary time. The kinetic, strain and loading energies of a cylindrical shell can be written as:

$$K = \frac{1}{2}\int_{\alpha_1}\int_{\alpha_2}\int_{\alpha_3}\rho(\dot{U}_1^2 + \dot{U}_2^2 + \dot{U}_3^2)dV \tag{16}$$

$$U = \iiint_{\alpha_1\alpha_2\alpha_3}(\sigma_{11}\in_{11} + \sigma_{22}\in_{22} + \sigma_{12}\in_{12} + \sigma_{13}\in_{13} + \sigma_{23}\in_{23})dV \tag{17}$$

$$V = \int_{\alpha_1}\int_{\alpha_2}(q_1\delta U_1 + q_2\delta U_2 + q_3\delta U_3)A_1A_2 d\alpha_1 d\alpha_2 \tag{18}$$

The infinitesimal volume is given by

$$dV = A_1A_2 d\alpha_1 d\alpha_2 d\alpha_3 \tag{19}$$

with the use of Eqs. (16)-(18) and substituting into Eq. (15), we get the equations of motions a generic shell.

$$\begin{aligned} &\frac{\partial(N_{11}A_2)}{\partial\alpha_1} + N_{22}\frac{\partial A_2}{\partial\alpha_1} - \frac{\partial(N_{12}A_1^2)}{A_1\partial\alpha_2} - \frac{Q_{13}}{R_1}A_1A_2 - \frac{\partial}{\partial\alpha_1}(\frac{P_{11}C_1A_2}{R_1}) + \frac{P_{22}C_1}{R_1}\frac{\partial A_2}{\partial\alpha_1} - \\ &\frac{\partial}{\partial\alpha_2}(\frac{P_{12}C_1A_1^2}{R_1})\frac{1}{A_1} + \frac{3C_1R_{13}}{R_1}A_1A_2 - \frac{C_1P_{13}A_1A_2}{R_1^2} = -(\ddot{u}_1I_\circ + \ddot{\phi}_1I_1 + [-C_1(-\frac{\ddot{u}_1}{R_1} + \\ &\ddot{\phi} + \frac{\partial\ddot{u}_3}{A_1\partial\alpha_1}) + \frac{C_1\ddot{u}_1}{R_1}\ ]I_3 + \frac{C_1\ddot{\phi}_1}{R_1}I_4 - \frac{C_1^2}{R_1}(-\frac{\ddot{u}_1}{R_1} + \ddot{\phi}_1 + \frac{\partial\ddot{u}_3}{A_1\partial\alpha_1})I_6) \end{aligned} \tag{20}$$

$$\begin{aligned} &\frac{\partial(N_{22}A_1)}{\partial\alpha_2} - N_{11}\frac{\partial A_1}{\partial\alpha_2} + \frac{\partial(N_{12}A_2^2)}{A_2\partial\alpha_1} + \frac{Q_{23}}{R_2}A_1A_2 + \frac{\partial}{\partial\alpha_2}(\frac{P_{22}C_1A_1}{R_2}) - \frac{P_{11}C_1}{R_2}\frac{\partial A_1}{\partial\alpha_2} + \\ &\frac{\partial}{\partial\alpha_1}(\frac{P_{12}C_1A_2^2}{R_2})\frac{1}{A_2} - \frac{3C_1R_{23}}{R_2}A_1A_2 + \frac{C_1P_{23}A_1A_2}{R_2^2} = (\ddot{u}_2I_\circ + \ddot{\phi}_2I_1 + \frac{C_1\ddot{\phi}_2}{R_2}I_{42} + \\ &\left[-c_1(-\frac{\ddot{u}_2}{R_2} + \ddot{\phi}_2 + \frac{\partial\ddot{u}_3}{A_2\partial\alpha_2}) + \frac{C_1\ddot{u}_2}{R_2}\right]I_3 - \frac{C_1^2}{R_2}(-\frac{\ddot{u}_2}{R_2} + \ddot{\phi}_2 + \frac{\partial u_3}{A_2\partial\alpha_2})I_6) \end{aligned} \tag{21}$$

$$(-\frac{\partial^2(P_{11}C_1A_2/A_1)}{\partial\alpha_1^2} + N_{11}\frac{A_1A_2}{R_1} + \frac{\partial}{\partial\alpha_2}(\frac{C_1P_{11}}{A_2}\frac{\partial A_1}{\partial\alpha_2}) + N_{22}\frac{A_1A_2}{R_2} - \frac{\partial^2(P_{22}A_1C_1/A_2)}{\partial\alpha_2^2} +$$

$$
\begin{aligned}
&+\frac{\partial}{\partial\alpha_1}\left(\frac{P_{22}C_1}{A_1}\frac{\partial A_2}{\partial\alpha_1}\right)-\frac{\partial^2(P_{12}C_1)}{\partial\alpha_1\partial\alpha_2}-\frac{\partial}{\partial\alpha_2}\left(\frac{P_{12}C_1}{A_2^2}\frac{\partial A_2^2}{\partial\alpha_1}\right)-\frac{\partial^2(P_{12}C_1)}{\partial\alpha_1\partial\alpha_2}-\frac{\partial}{\partial\alpha_1}\left(\frac{P_{12}C_1}{A_1^2}\frac{\partial A_1^2}{\partial\alpha_2}\right)\\
&-\frac{\partial(Q_{13}A_2)}{\partial\alpha_1}+\frac{\partial(3C_1R_{13}A_2)}{\partial\alpha_1}-\frac{\partial}{\partial\alpha_1}\left(\frac{P_{13}C_1A_2}{R_1}\right)-\frac{\partial(Q_{23}A_1)}{\partial\alpha_2}+\frac{\partial(3C_1R_{23}A_1)}{\partial\alpha_2}-\\
&-\frac{\partial}{\partial\alpha_2}\left(\frac{C_1P_{23}A_1}{R_2}\right)-\frac{\partial}{\partial\alpha_1}\left(\frac{P_{11}C_1A_2}{A_1^2}\frac{\partial A_1}{\partial\alpha_1}\right)-\frac{\partial}{\partial\alpha_2}\left(P_{22}C_1\frac{A_1}{A_2^2}\frac{\partial A_2}{\partial\alpha_2}\right)\Big)=-\Big\{\ddot{u}_3I_\circ+\\
&+C_1\left[\frac{\partial}{\partial\alpha_1}\left(\frac{u_1}{A_1}\right)+\frac{\partial}{\partial\alpha_2}\left(\frac{u_2}{A_2}\right)\right]I_3+C_1\left[\frac{\partial}{\partial\alpha_1}\left(\frac{\ddot{\phi}_1}{A_1}\right)+\frac{\partial}{\partial\alpha_2}\left(\frac{\ddot{\phi}_2}{A_2}\right)\right]I_4-C_1^2I_6\Big(\Big(-\frac{\partial}{R_2\partial\alpha_2}\left(\frac{\ddot{u}_2}{A_2}\right)\\
&+\frac{\partial}{\partial\alpha_2}\left(\frac{\ddot{\phi}_2}{A_2}\right)+\frac{1}{A_2}\frac{\partial^2\ddot{u}_3}{\partial\alpha_2^2}-\frac{\partial A_2}{A_1^2\partial\alpha_2}\frac{\partial\ddot{u}_3}{\partial\alpha_2}\Big)+\Big(-\frac{\partial}{R_1\partial\alpha_1}\left(\frac{\ddot{u}_1}{A_1}\right)+\frac{\partial}{\partial\alpha_1}\left(\frac{\ddot{\phi}_1}{A_1}\right)+\\
&+\frac{1}{A_1}\frac{\partial^2\ddot{u}_3}{\partial\alpha_1^2}-\frac{\partial A_1}{A_1^2\partial\alpha_1}\frac{\partial\ddot{u}_3}{\partial\alpha_1}\Big)\Big)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&-\frac{\partial(M_{11}A_2)}{\partial\alpha_1}+\frac{\partial(C_1P_{11}A_2)}{\partial\alpha_1}+M_{22}\frac{\partial A_2}{\partial\alpha_1}-C_1P_{22}\frac{\partial A_2}{\partial\alpha_1}-\frac{\partial(M_{12}A_1^2)}{A_1\partial\alpha_2}+\frac{\partial(P_{12}C_1A_1^2)}{A_1\partial\alpha_2}-\\
&-3C_1R_{13}A_1A_2+A_1A_2Q_{13}+\frac{C_1P_{13}}{R_1}A_1A_2=-\Big[\ddot{u}_1I_1+\ddot{\phi}_1I_2-C_1\ddot{u}_1I_3+\Big(-2C_1\ddot{\phi}_1+C_1\frac{\ddot{u}_1}{R_1}-\\
&-\frac{C_1}{A_1}\frac{\partial\ddot{u}_3}{\partial\alpha_1}\Big)I_4+C_1^2\Big(-\frac{\ddot{u}_1}{R_1}+\ddot{\phi}_1+\frac{\partial\ddot{u}_3}{A_1\partial\alpha_1}\Big)I_6\Big]
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&-\frac{\partial(M_{22}A_1)}{\partial\alpha_2}+\frac{\partial(C_1A_1P_{22})}{\partial\alpha_2}+M_{11}\frac{\partial A_1}{\partial\alpha_2}-C_1P_{11}\frac{\partial A_1}{\partial\alpha_2}-\frac{\partial(M_{12}A_2^2)}{A_2\partial\alpha_1}+\frac{\partial(P_{12}C_1A_2^2)}{A_2\partial\alpha_1}-\\
&-3C_1R_{23}A_1A_2+A_1A_2Q_{23}+\frac{C_1P_{23}}{R_2}A_1A_2=-\Big[\ddot{u}_2I_1+\ddot{\phi}_2I_2-C_1\ddot{u}_2I_3+\Big(-2C_1\ddot{\phi}_2+\\
&+C_1\frac{\ddot{u}_2}{R_2}-\frac{C_1}{A_2}\frac{\partial\ddot{u}_3}{\partial\alpha_2}\Big)I_4+C_1^2\Big(\frac{\ddot{u}_2}{R_2}+\ddot{\phi}_2+\frac{\partial\ddot{u}_3}{A_2\partial\alpha_2}\Big)I_6\Big].
\end{aligned}
\tag{24}
$$

For Eqs. (20) – (24) are defining as

$$I_i=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\alpha_3^i\,d\alpha_3 \tag{25}$$

The displacement fields for a FG cylindrical shell and the displacement fields which satisfy these boundary conditions can be written as

$$
\begin{aligned}
u_1&=\overline{A}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
u_2&=\overline{B}\,\phi(x)\sin(n\theta)\cos(\omega t)\\
u_3&=\overline{C}\phi(x)\prod_{i=1}^{N}(x-a_i)^{\xi i}\cos(n\theta)\cos(\omega t)\\
\phi_1&=\overline{D}\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\cos(\omega t)\\
\phi_2&=\overline{E}\phi(x)\sin(n\theta)\cos(\omega t)
\end{aligned}
\tag{26}
$$

where, $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ and $\overline{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, ϕ_1 and ϕ_2 are the displacement fields for higher order deformation theories for a cylindrical shell, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions. The axial function $\phi(x)$ is chosen as the beam function as

$$\phi(x)=\gamma_1\cosh\left(\frac{\lambda_m x}{L}\right)+\gamma_2\cos\left(\frac{\lambda_m x}{L}\right)-\zeta_m\left(\gamma_3\sinh\left(\frac{\lambda_m x}{L}\right)+\gamma_4\sin\left(\frac{\lambda_m x}{L}\right)\right) \tag{27}$$

Substituting Eq. (26) into Eqs. (20) - (24) for third order theory we can be expressed

$$\det\left(C_{ij}-M_{ij}\,\omega^2\right)=0 \tag{28}$$

Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ\omega^{10}+\beta_1\omega^8+\beta_2\omega^6+\beta_3\omega^4+\beta_4\omega^2+\beta_5=\circ \tag{29}$$

where $\beta_i\,(i=0,1,2,3,4,5)$ are some constants. Eq. (29) is solved five positive and five negative roots are obtained. The five positive roots obtained are the natural angular frequencies of the cylindrical shell based third-order theory. The smallest of the five roots is the natural angular frequency studied in the present study. The FGM cylindrical shell is composed of Nickel at its inner surface and Stainless steel at its outer surface. The material properties for stainless steel and nickel, calculated at $T=300K$, are presented in table 1.

TABLE I
PROPERTIES OF MATERIALS

Coefficients	Stainless Steel			Nickel		
	E	ν	ρ	E	ν	ρ
P_0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

Where $P_\circ, P_{-1}, P_1, P_2$ and P_3 are the coefficients of temperature $T(K)$ expressed in Kelvin and are unique to the constituent materials. The material properties P of FGMs are a function of the material properties and volume fractions of the constituent material.

IV. RESULTS AND DISCUSSION

In this paper, studies are presented for a FGM cylindrical shell is considered. Table II, shows the variation of the natural frequency with the circumferential wave number n for a functional graded cylindrical shell with two rings support. The frequencies increased with the circumferential wave number.

TABLE II
THE NATURAL FREQUENCIES FOR A FGM CYLINDRICAL SHELL
(m = 1, h / R=0.01, L / R=20, a1/L=0.3, a2/L=0.6).

m	n	ω (HZ)
1	1	0.376687
	2	0.472224
	3	0.496101
	4	0.506079
	5	0.513007
	6	0.520445
	7	0.530317
	8	0.544065
	9	0.562919
	10	0.587923

For simplicity, we actually vary the value of power law exponent whenever we need to change the volume fractions. Varying the value of power law exponent N of the FGM cylindrical shell with two rings support, natural frequencies are computed for this conditions. Results are also computed for pure stainless steel and pure nickel shells. All these results are plotted in Fig. 2.

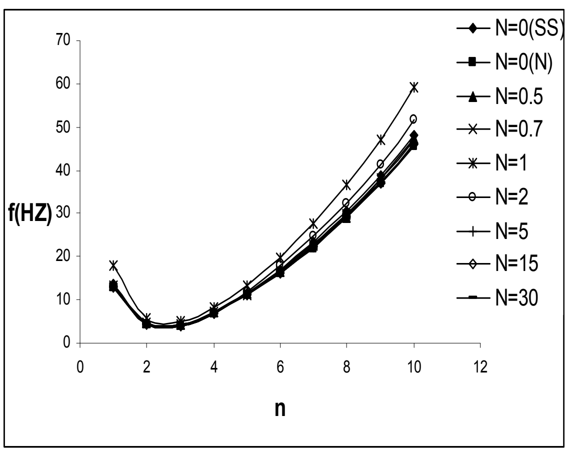

Fig. 2. Natural frequencies FGM cylindrical shell associated with various volume fractions laws for 2 rings support m=1, h/R=0.01, L/R=20, a1/L=0.3, a2/L=0.6

V. CONCLUSION

A study on the vibration of functionally graded (FG) Cylindrical shell composed of stainless steel and nickel has been presented. The study showed that in this conditions the frequencies first decreases and then increases as the circumferential wave number n increases. The minimum frequency occurs in between n equals 2 and 3 for these boundary conditions. The results showed that one could easily vary the natural frequency of the FGM cylindrical shell with two rings support by varying the volume fractions laws.

ACKNOWLEDGMENT

The work described in this paper was fully supported by research grants Andimeshk Branch, Islamic Azad University, Andimeshk, Iran.

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