An Insight into the Birth-Death Process with Reference to the Yeast Population

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***Abstract*– A biological or physical process faithfully described by one form of linear homogenous or non homogenous differential equation or relevant analytic model can be explicitly analyzed. The birth-death process with delineation of appropriate models is discussed. Essential reference is made to the logistic curve illustrating the yeast population.**

***Keywords*– Analytic Solution, Birth Process, Birth-Death Process and Logistic Curve**

I. INTRODUCTION

Here is the discussion of birth-death process. The pure death process follows some exponential law or mathematical expression which can be formulated as a pure deterministic model or otherwise. The pure birth process is extended to the logistic curve of the yeast population and quite well explained. The birth process, logistic curve and the birth-death process constitute the discussion of this investigation.

II. DISCUSSION

A. The Pure Birth Process

Suppose that a population of organisms develops over a short period of time (relative to their life span) in crowd-free conditions and with unlimited food resource. Making the assumptions that (i) Organisms do not die (ii) they develop without interacting with each other (iii) the birth rate(λ) is the same for all organisms, regardless of their age, and does not change with time, the last assumption is particularly appropriate to single –celled organisms that reproduce by dividing.

B. A Deterministic Model

N(t) denotes the population size at time t, then in the subsequent small time interval of length h the increase in size due to a single organism is $\lambda \times t$,(i.e rate × time), so the increase in size due to all N(t) organisms is $\lambda \times h \times N(t)$.

Thus, $N(t+h)=N(t)+\lambda hN(t)$, …………………………….(i)

$[N(t+h)-N(t)]/h=\lambda N(t)$; dividing by t. …………………..(ii)

As h approaches zero, we have the differential equation;

$\frac{dN}{dt} = \lambda N(t)$, on integrating gives $N(t)=N(0)\exp(\lambda t)$, where N(0) denotes the initial population size at time t=0. This form of N(t) is known as the Malthusian expression for population development.

A plot of $\log_e[N(t)]$ against t should be approximately linear viz;

$\log_e[N(t)]=\log_e[N(t)]+ \lambda t$. ……………...…………….….(iii)

This relation gives a useful way of seeing whether a data exhibits exponential growth and $\log_e[N(t)]$ against t is linear.

C. Growth of a Yeast Population

The amount of yeast N(t) can be plotted against t from the logistic curve N(t) given as:

$$N(t) = {}^{665}/[1 + \exp(4.16 - 0.531t)]. \quad \text{……………..(iv)}$$

Another expression for the logistic curve is also:

$Y(t)=\log_e[(k-N)/N]=\log_e[(k-n_0)/n_0]-rt$ …….……………..(v)

K is the carrying capacity and can be chosen as k=665 (Carlson data). Y(t) against t plotted gives a straight line of negative slope (-r) and an intercept at t=0 of $Y(0)=\log_e[(k-n_0)/n_0]$.

D. A birth death process

$$\frac{dN(t)}{dN(t)} = B[N(t)] - D[N(t)] \quad \text{……………………..(vi)}$$

$$\frac{dN(t)}{dt} = B[N(t - t_G) - DN(t)] \quad \text{…………………….(vii)}$$

$\frac{dN(t)}{dt} = rN(t - t_G)[1 - \frac{N(t-t_D)}{k}]$, also allows for a reproductive time-lag(t_G) which may be measured by the gestation time or its equivalent; in early stages of population growth this reproductive time-lag may be important in slowing down the rate of population increase.

In many situations the birth rate may depend not just on one particular time t_D, but on a weighted average of previous times s, whence (vii) becomes:

$\frac{dN(t)}{dt} = r\int_0^\infty z(s)N(t-s)ds - D[N(t)]$ (Nisbet and Gurney, 1982)………………………………………….(viii)

The weighting function z(s) is normalized to ensure that;

$\int_0^\infty z(u)du = 1\,.$ …………………………………..(ix)

z(u) is called a 'lag-window' and gives rise to the subject of 'window-carpentry' (see, for example, Chatfield, 1980).

Consider z(u) :Mean value of t_D=1.

(a) Z(u) is a spike at $u=t_D$ (b) $z(u)=(1/t_D)\exp(-u/t_D)$
(c) $Z(u)=(4u/t_D^2)\exp(-2u/t_D)$ (d) $z(u)=(\pi/4t_D)\sin(\pi u/2t_D)$

III. FURTHER DISCUSSION

The birth-death process has been discussed citing appropriate models. Based on the logistic curve given by equation (vi) above, the yeast population over time is plotted as shown in Fig. 1.

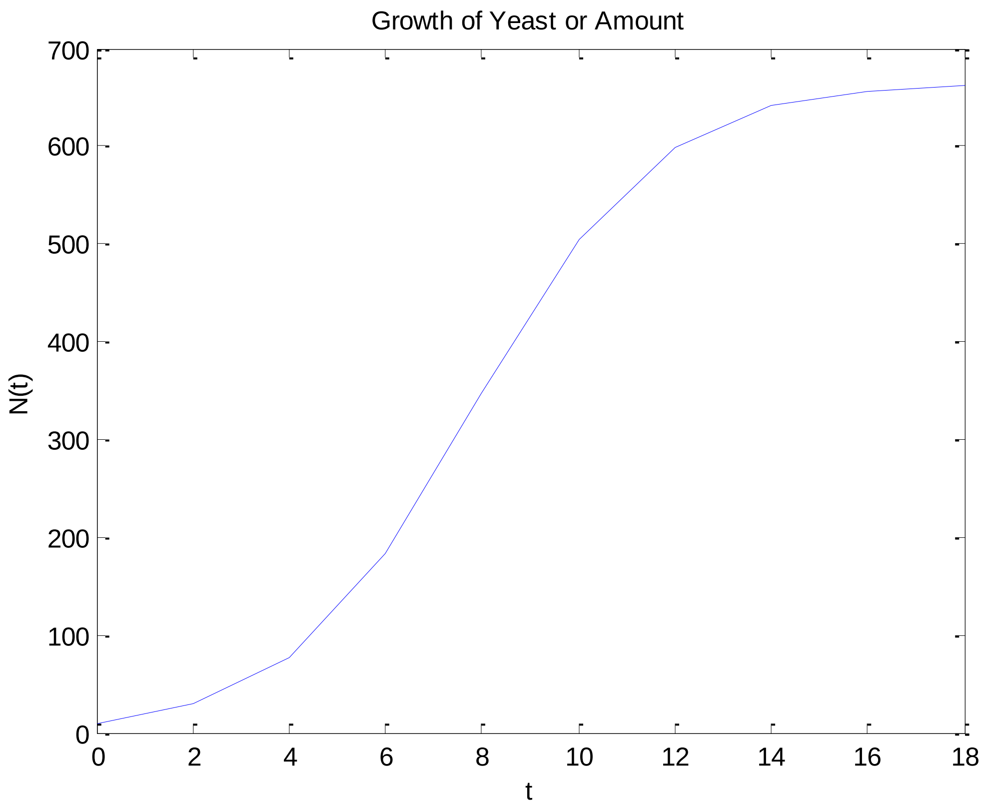

Fig. 1: Growth of Yeast or Amount

IV. CONCLUSION

The extension of this discussion to the birth-death process has been extensively enumerated. Available analytic expressions describing pure birth process, birth-death process and the logistic curve based on the birth process for the yeast population are explained.

The growth of yeast and the amount or population size over time has been plotted based on expression of the logistic curve.

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