Nanofluid Effect on the Flow and Heat Transfer Over an Unsteady Stretching Sheet under Consideration of Radiation

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***Abstract*– In this work, the effect of Nanofluid on the heat and fluid flow over an unsteady stretching sheet in presence of thermal radiation is analyzed. The governing nonlinear partial differential equations have been reduced to the coupled nonlinear ordinary differential equations by the similarity transformations. An efficient numerical shooting technique with a fourth-order Runge-Kutta scheme was used to obtain the solution of the boundary value problem. The similarity equations were solved numerically for one typs of nanoparticles, namely copper with water as the base fluid with the Prandtl number Pr = 3.73 to investigate the effect of the solid volume fraction φ and another parameters of the nanofluid. The results of velocity and temperature distributions for different parameters such as the solid volume fraction, the unsteadiness parameter and the radiation parameter were obtained. It is shown that the heat transfer rate is increased with increasing φ. The present results are compared with some reported theoretical results by other investigators and good agreement is found.**

***Keywords*– Nanofluid, Unsteady Stretching Sheet, Heat Transfer and Radiation**

I. INTRODUCTION

The study of flow and heat transfer over a stretching/shrinking sheet is an important problem in many engineering processes with application in industries such as extrusion of plastic sheets, wire drawing, hot rolling and glass fiber production. In particular, in the extrusion of a polymer in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a thin sheet, and then solidified through quenching or gradual cooling by direct contact with water or coolant liquid. The boundary layer flow on a continuously stretching sheet with a constant speed and various aspects of the problem have been investigated (Sakiadis, 1961). The flow of a Newtonian fluid over a linearly stretching surface have been studied (Crane,1970).

The effect of thermal radiation on the flow and heat transfer in a viscous fluid over an unsteady stretching surface was carried out. At this study, three-parameter of problem solved numerically for some representative values of the unsteadiness parameter A, the radiation parameter R and Prandtl number Pr.

It was shown that the heat transfer rate is increased with increasing R, A and Pr. Also the effect of radiation parameter on the heat transfer rate was found to be more noticeable at larger values of A and Pr (El-Aziz, 2009). Hydromagnetic boundary-layer flow over an accelerating permeable surface in the presence of thermal radiation, buoyancy, and heat generation or absorption effects have been investigated (Chamkha, 2000). Thermal radiation and magnetic field of a micropolar fluid past a stretched semi-infinite, vertical and permeable surface in the presence of temperature dependent heat generation or absorption studied. A parametric study illustrating the influence of the various physical parameters on the skin friction coefficient, microrotaion coefficient or wall couple stress as well as the wall heat transfer coefficient or Nusselt number conducted (Khedr, 2009).

The boundary layer flow and heat transfer analysis of electrically conducting viscous fluid over a nonlinearly shrinking sheet have been investigated (Javed, 2011). At this work, the system of equations is solved numerically employing an implicit finite difference scheme known as Keller-box method. The numerical results for the velocity, temperature, wall skin friction coefficient and local rate of heat transfer through the surface for various values of physical parameters both in case of stretching and shrinking sheet were analyzed and discussed for both the solutions. A numerical solution for the magnetohydrodynamic (MHD) non-Newtonian power-law fluid flow over a semi-infinite and non-isothermal stretching sheet with internal heat generation/absorption have been carried out. The governing partial differential equations of momentum and energy were converted into ordinary differential equations by using a classical similarity transformation along with appropriate boundary conditions.

It is important to note that the momentum and thermal boundary layer thickness decrease with increase in the power-law index in presence/absence of variable thermal conductivity (Prasad et.al, 2009). Following these works, the various aspects of problem with different boundary conditions and fluids including micropolar fluids (Ashraf, 2011), moving material with suction or injection (Al-Sanae, 2000), nanofluid past a semi-infinite vertical stretching sheet (Rosmila, 2012), stretching/shrinking sheet in a nanofluid (Norazian, 2012), viscous fluid containing metallic nanoparticles over a nonlinear stretching sheet (Hamad, 2012), suspending metallic nanoparticles in conventional heat transfer fluids (Stephen, 1995), heat transfer of nanofluids (Choi,2011) are considered. Based on the above mentioned investigations and applications, this paper is concerned with a Unsteady, two dimensional Stretching Sheet in the presence of nanofluid and radiation effect.

The results of velocity and temperature distributions for different parameters such as the solid volume fraction, the unsteadiness parameter, the radiation parameter were obtained. The obtained results are checked against previously published work for special cases of the problem in order to access the accuracy of the numerical method and found to be in excellent agreement.

II. DESCRIPTION AND FORMULATION

A. *Description of Problem*

Consider the flow of a viscous and incompressible nanofluid on a horizontal sheet ,which issues from a slot at the origin. The nanofluid is considered to be a gray, absorbing–emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equations. The radiative heat flux in the x-direction is negligible in comparison with that in the y-direction. The fluid motion arises due to the stretching of the elastic sheet. As schematic representation of the physical model and coordinates system is depicted in Fig. 1.

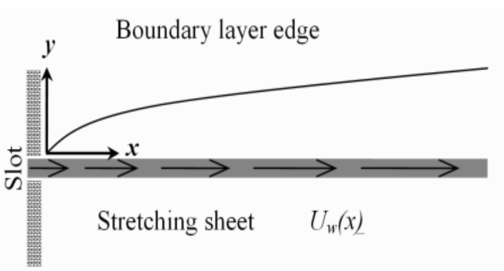


Figure. 1. Scheme of stretching sheet configuration

The continuous sheet aligned with the x-axis at y=0 moves in its own plane with a velocity $U_w(x,t)$ and the temperature distribution $T_w(x,t)$ varies both along the sheet and with time.

B. *Governing Equations*

The velocity and temperature fields in the boundary layer are governed by the two-dimensional boundary layer equations for mass, momentum and thermal energy:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

Equations (1) - (2) and Eq. (3) should be solved by appropriate boundary conditions. In accordance with the problem description, the boundary conditions can be written as follows:

$$u=U_w(x,t), v=0, \mathrm{T}=T_w(x,t) \quad \text{at} \quad y=0 \tag{4}$$

$$u\to 0, T\to T_\infty \quad \text{as} \quad y\to\infty \tag{5}$$

where u and v are the velocity components along the x-and y-axes, the subscript of nf denote the properties of nanofluid. The properties of nanofluids are defined as follows (Yacob, 2011):

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}} \tag{6}$$

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{7}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{5/2}} \tag{8}$$

Therefore,

$$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s \tag{9}$$

and,

$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)} \tag{10}$$

where subscript of f and s represent the base fluid and nanoparticle suspension, respectively. Here, φ is the nanoparticle volume fraction. The thermophysical properties of fluid and nanoparticles are given (Table I) (Abu-Nada & Oztop, 2009).

Table I: Thermophysical properties of fluid and nanoparticles

Physical properties	Fluid phase (water)	Cu
$C_p\left(J/Kg^\circ K\right)$	4179	385
$\rho\left(kg/m^3\right)$	997.1	8933
$k\left(W/m^\circ K\right)$	0.613	400

The radiative heat flux q_r under Rosseland approximation (Brewster, 1992) has the form:

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T^4}{\partial y} \quad (11)$$

where σ is the Stefan–Boltzmann constant and k_1 is the mean absorption coefficient. Assuming that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in Taylor series about the free stream temperature T_∞ to yield

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (12)$$

Where the higher-order terms of the expansion are neglected. In view of Eqs. (12) and (11), Eq. (3) reduces to:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k_1(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} \quad (13)$$

The stretching velocity $U_w(x,t)$ is assumed to be of the form (Andersson, 2000):

$$U_w = bx/(1-\alpha t) \quad (14)$$

where b and α are constants (with $b \geq 0$ and $\alpha \geq 0$ where $\alpha t < 1$), and both have dimension t^{-1} , we have b as the initial stretching rate $b/(1-\alpha t)$ and it is increasing with time. In the context of polymer extrusion, the material properties, in particular the elasticity of the extruded sheet may vary with time even though the sheet is being stretched by a constant force. With unsteady stretching, however, α^{-1} becomes the representative time scale of the resulting unsteady boundary layer problem. The adopted formulation of the sheet velocity $U_w(x,t)$ in Eq. (14) is valid only for times $t < \alpha^{-1}$ unless $\alpha = 0$. We assume the surface temperature $T_w(x,t)$ of the stretching sheet to vary with the distance x and time t in the form:

$$T_w(x,t) = T_\infty + T_0[bx^2/2\nu](1-\alpha t)^{-3/2} \quad (15)$$

Where T_0 is a (positive or negative; heating or cooling) reference temperature. Introducing the similarity variable η and the dimensionless variables f and θ as follows:

$$\eta = \left(b/\nu_f\right)^{1/2}(1-\alpha t)^{-1/2} y \quad (16)$$

$$\psi = (\nu_f b)^{1/2}(1-\alpha t)^{-1/2} . x . f(\eta) \quad (17)$$

$$T = T_\infty + T_0\left[bx^2/2\nu_f\right](1-\alpha t)^{-3/2}\theta(\eta) \quad (18)$$

where, $\psi(x,y,t)$ is a stream function which automatically assures mass conservation. The velocity components are readily obtained as:

$$u = \partial\psi/\partial y = U_w f'(\eta) ,$$

$$v = -\partial\psi/\partial x = -(\nu_f b)^{1/2}(1-\alpha t)^{-1/2} f(\eta) \quad (19)$$

The mathematical problem defined in Eqs. (1), (2) and (13) are then transformed into a set of ordinary differential equations and their associated boundary conditions:

$$\beta f''' + ff'' - f'^2 - A\left[f' + \eta/2 f''\right] = 0 \quad (20)$$

$$[3R+4]\theta'' + 3R.\Pr.\lambda\left[f\theta' - 2f'\theta - A/2(3\theta + \eta\theta')\right] = 0 \quad (21)$$

Where :

$$\beta = \frac{1}{(1-\varphi)^{5/2}\left[(1-\varphi) + \varphi\rho_s/\rho_f\right]} \quad (22)$$

$$\lambda = \frac{\left[k_s + 2k_f + \varphi(k_f - k_s)\right]}{\left[k_s + 2k_f - 2\varphi(k_f - k_s)\right]}\left[(1-\varphi) + \varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right] \quad (23)$$

the unsteadiness parameter $A = \alpha/b$, non-dimensional temperature $\theta = (T - T_\infty)/(T_w - T_\infty)$, the radiation parameter $R = k_{nf} k_1/4\sigma T_\infty^3$. Their associated boundary conditions:

$$f(0) = 0 \;,\; f'(0) = 1 \;,\; \theta(0) = 1 \quad (24)$$

$$f'(\infty) = 0 \;,\; \theta(\infty) = 0 \quad (25)$$

From the engineering point of view, the important characteristics of the flow are the skin-friction coefficient and the Nusselt number, respectively defined as :

$$C_{fx} = \frac{2\mu(\partial u/\partial y)_{y=0}}{\rho_f U_w^2} = 2\,\mathrm{Re}_x^{-1/2} f''(0) \quad (26)$$

$$Nu_x = \frac{x}{T_0}(\partial T/\partial y)_{y=0} = -\frac{1}{2}(1-\alpha t)^{-1/2}\,\mathrm{Re}_x^{3/2}\,\theta'(0) \quad (27)$$

where $\mathrm{Re}_x = U_w x / \nu_f$ is the local Reynolds number based on the sheet velocity U_w .

III. RESULTS AND DISCUSSIONS

In order to get the physical insight into the flow problem, comprehensive numerical computations are conducted for various values of the parameters that describe the flow characteristics, and the results are illustrated graphically. Figures 2(a) and 2(b) illustrate the effect of nanoparticle volume fraction φ on the nanofluid velocity profile for steady (A=0) and unsteady (A=0.2) test case, respectively. In this case the Cu nanoparticles and water base fluid (Pr = 3.73) when $\varphi = 0, 0.05, 0.1, 0.5$ and R = 5 is considered. It is clear that, as the nanoparticles volume fraction increases, the nanofluid velocity decreases. In addition, the results show that the velocity decreases with the distance from the stretching sheet for all φ .

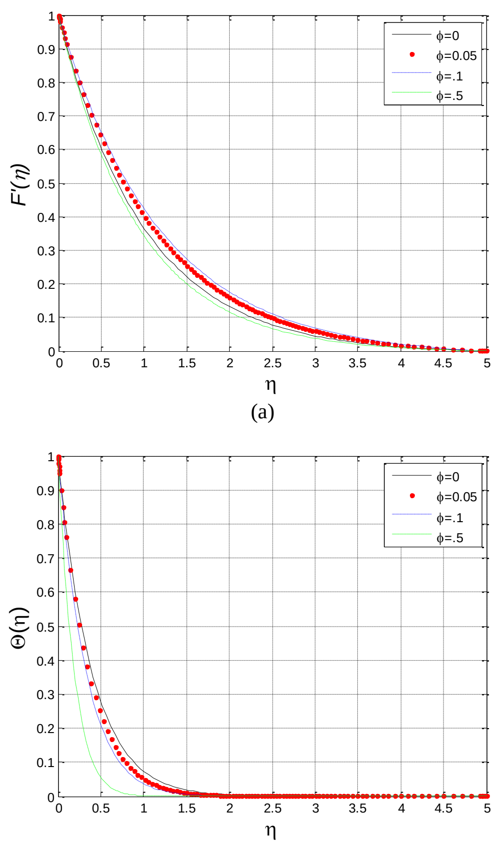


Figure 2: Velocity profiles for various values of φ with R=5 and Pr =3.73 for Cu-water working fluid (a) steady state condition (A=0), (b) unsteady state condition (A=0.2)

The values for Velocity profiles and Temperature profiles are compared with the available results in the previous literature (El-Aziz, 2009) for the steady case (A = 0) and $(\varphi = 0)$ presented. The results are found to be in good agreement. Figures 4 and 5 depict the effect of the volume fraction φ on the nanofluid temperature profile $\theta(\eta)$ at steady and unsteady conditions, respectively. Figure 3(a) illustrates that increases of volume fraction tends to decrease the nanofluid temperature in the case of Cu-water when A=0, and R = 5. Furthermore, Figure 3(b) shows that increasing the volume fraction φ tends to decrease the temperature distribution the same values, thus leading to higher heat transfer rate between the nanofluid and the surface. These figures show the good agreement with the physical behavior of nanofluids. Because when the volume of nanoparticles increases, the thermal conductivity increases and then the thermal boundary layer thickness increases.

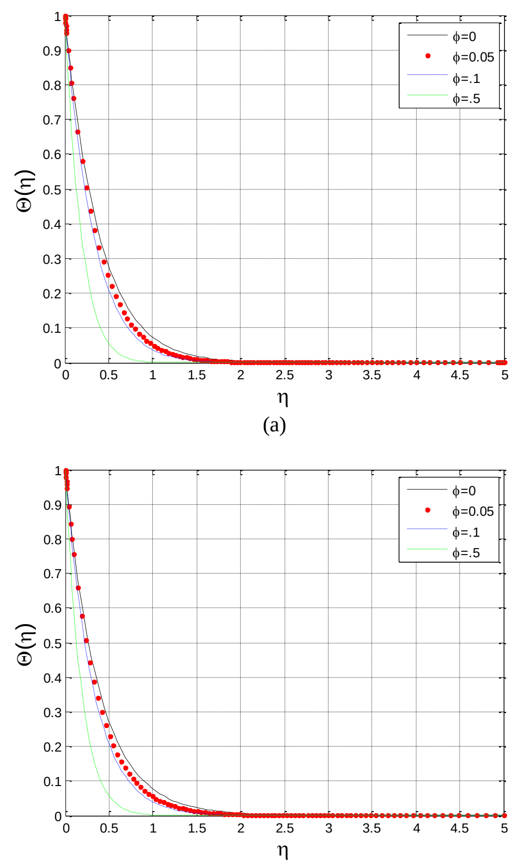
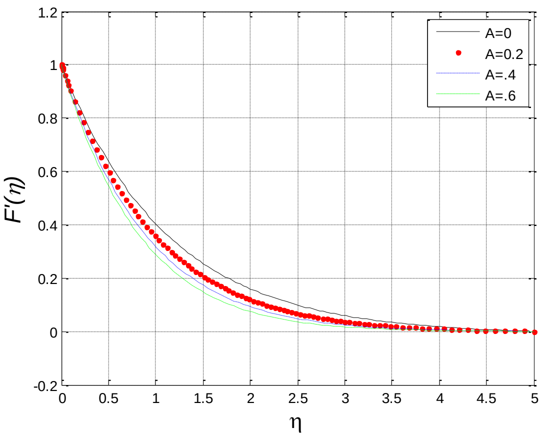
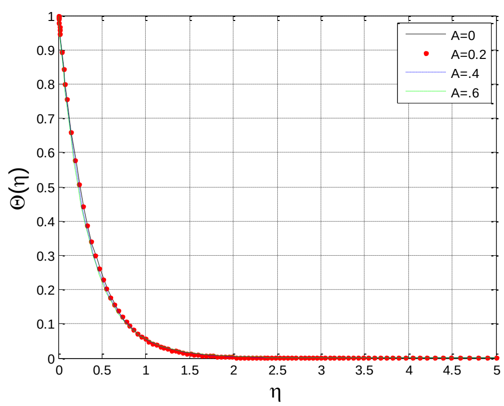


Figure 3: Temperature profiles for various values of φ with R=5 and Pr =3.73 for Cu-water working fluid (a) steady state condition (A=0), (b) unsteady state condition (A=0.2)

In Figures 4(a) and 4(b), the typical velocity and temperature distributions are plotted respectively for R=5, Pr =3.73, φ =0.05 and for different values of the unsteadiness parameter A. This parameter considered here is A=0, 0.2, 0.4 and 0.6. It is observed that when the value of A increases, then the velocity profiles decrease, while the temperature profile will be constant. The results show that the velocity and temperature decrease with the distance from the stretching sheet for all of A. In addition, increasing the value of A tends to decrease the velocity in the boundary layer without any flow reversal.



(a)



(b)

Figure 4: (a) Velocity and (b) temperature profiles for various values of A with $\varphi = 0.05$, R=5 and Pr =3.73 for Cu-water working fluid

The effect of the radiation parameter R on the velocity and temperature profiles in the case of Cu-water when the radiation parameter R = 0.05, 0.5, 1 and 5 with A = 0.2, Pr = 3.73 and φ =0.05 are shown in Figures 5(a) and 5(b), respectively. It is clear that the velocity will be constant by increasing the value of R while, temperature distribution decreases with an increase in the radiation parameter R. The results show that the temperature decreases with the distance from the stretching sheet for of the all R.

IV. CONCLUSION

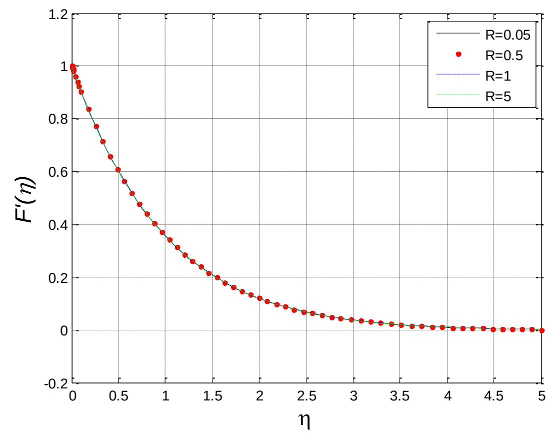
An unsteady forced convection boundary-layer flow of a nanofluid due to a stretching sheet is studied with the influence of thermal radiation. The similarity technique has been employed as a solution technique to complete the formulation of the unsteady model. For both the steady and unsteady case, the behavior of nanofluid is analyzed. The results are presented for the effect of various parameters. The velocity and temperature effects on the sheet are studied and shown graphically. Some of the interesting conclusions are as follows:

(i) It is observed that an increase the solid volume fraction φ is to decrease the thermal boundary layers.

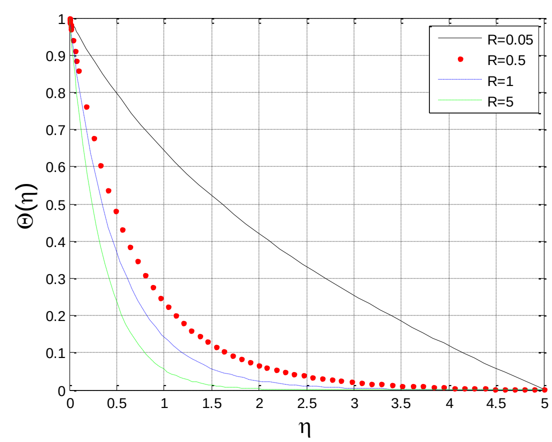
(ii) the velocity boundary layers depends on the material and solid volume fraction.

(iii) the values of the unsteadiness parameter A has no influence on Temperature profiles.

(iv) the values of the radiation parameter R has no influence on velocity profiles.



(a)



(b)

Figure 5: (a) Velocity and (b) temperature profiles for various values of R with A=0.2, φ =0.05 and Pr =3.73 for Cu-water working fluid

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